Complexité avancée - Homework 3

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Due at 8.30 a.m., October 14, 2020

PSPACE and games   The Geography game is played as follows:

- The game starts with a given name of a city, for instance *Cachan*;

- the first player gives the name of a city whose first letter coincides with the last letter of the previous city, for instance *Nice*;

- the second player gives then another city name, also starting with the last letter of the previous city, for instance *Evry*;

- the first player plays again, and so on – with the restriction that no player is allowed to give the name of a city already used in the game;

- the loser is the first player who does not find a new city name to continue.

This game can be described using a directed graph whose vertices represent cities and where an edge (X, Y) means that the last letter of the city X is the same as the first letter of the city Y. This graph has also a vertex marked as the initial vertex of the game (the initial city). Each player choses a vertex of the graph, the first player choses first, and the two players alternate their moves. At each move, the sequence of vertices chosen by the two players must form a simple path in the graph, starting from the distinguished initial vertex.

Player 1 wins the game if, after some number of moves, Player 2 has no valid move (that is no move that forms a simple path with the sequence of previous moves).

Generalized Geography (GG for short) is the following problem:

- INPUT: a directed graph G and an initial vertex s.

- QUESTION: does Player 1 have a winning strategy for a GG game played on G from s?

1. Show that GG is in PSPACE

2. Exhibit a logarithmic space reduction f from QBF to GG. Carefully prove that the reduction is logspace and the equivalence $w \in \mathsf{QBF} \Leftrightarrow f(w) \in \mathsf{GG}$.

3. What can you deduce about GG?