Complexité avancée - Homework 1

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Exercise 1, follow up: a new representation. With the same alphabet as in Exercise 1, we give a third representation of the graph $G = (V, E)$:

- By its lists of edges $e_G \in \Sigma^*$:

$$e_G \overset{\text{def}}{=} n \bullet v_{k_1}/v_{k_2} \bullet \cdots \bullet v_{k_{j-1}}/v_{k_j}\#$$

  where n is a unary word giving the number of vertices $|V|$, where for all $i = 1, \ldots, j$, v_{k_i} is a binary word denoting a vertex of V so that $E = \{(v_{k_1}, v_{k_2}), (v_{k_3}, v_{k_4}), \ldots, (v_{k_{j-1}}, v_{k_j})\}$.

1. Explain why n is part of e_G.
2. Show that it is possible to check in logarithmic space that a word $w \in \Sigma^*$ is a well-formed description of some G in list-of-edges representation.
3. - Describe a logarithmic space bounded deterministic Turing machine taking as input a graph G, represented by its adjacency list, and computing the list of edges representation of G.

   OR

   - Describe a logarithmic space bounded deterministic Turing machine taking as input a graph G, represented by its list of edges, and computing the adjacency matrix representation of G.
4. Explain why, with both of these constructions, we can conclude that any pair of the three constructions that was presented are logspace reducible.