Rare Event Handling
in Statistical Model Checking

Benoît Barbot, Serge Haddad and Claudine Picaronny

LSV, ENS Cachan, CNRS & INRIA

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Plan

1. Introduction
2. Theoretical framework
3. Experimentation
4. Conclusion and Perspectives
Rare Event

Critical systems

- Plane, rocket (failure of the fuel control system)
- Nuclear power plant (failure of all the redundant security systems)
- Security device like an airbag (delayed deployment)
- Telecommunication (overflow)
- Banking system (ruin of an insurance)
- Biology
- etc.

In common

- Consequences of failure are dramatic.
- The probability of failure is very small.
Rare Event

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In common

- Consequences of failure are dramatic.
- The probability of failure is very small.

Estimation of this probability is critical.
Model checking

\[ M \otimes A \]

Process algebra
Petri net
...

LTL
CTL
...

Transition system

\[ M \models \varphi \quad M \not\models \varphi \]

Buchi automaton
Alternating automaton
Model checking for stochastic system

Stochastic Process algebra
Stochastic Petri net

\[ \mathcal{M} \]

\[ C \otimes A \]

\[ p = \Pr(\mathcal{M} \models \varphi) \]

PCTL
HASL

Markov chain

Hybrid automaton
Numerical and Statistical Approaches

- Numerical Approach
  - Branching logic (based on CTL)
  - Exact value (but subject to numerical error)
  - Efficiently implemented in many tools
    (PRISM, MRMC, GreatSPN)
  - Strong probabilistic hypotheses
  - Memory space
    proportional to the size of the stochastic process

- Statistical Approach
  - Linear Logic (based on LTL)
  - Confidence interval: probabilistic framing
  - Very small memory space
  - Easy to parallelize
  - Weak probabilistic hypothesis (only an operational semantic)
  - Unsuitable for rare events' probability

Objective: Develop a dedicated method for rare events.
Numerical and Statistical Approaches

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Rare Event Problem

Illustration

- **Objective:** Estimation of the probability $p$ of an event $e$ with a confidence level of 0.99

- **Hypotheses:**
  1. Computation of $10^9$ trajectories
  2. $p \leq 10^{-15}$
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Possible outcomes

- With probability $\approx 1 - 10^{-6}$, $e$ does not occur in any trajectory
  
  We obtain as confidence interval: $[0, 7 \, 10^{-9}]$

  $\Rightarrow$ Confidence interval too large

- With probability smaller than $10^{-6}$, $e$ occurs in one trajectory
  
  We obtain as confidence interval: $[7 \, 10^{-10}, 2 \, 10^{-9}]$

  $\Rightarrow$ Value outside the confidence interval

- With a tiny probability, $e$ occurs in more than one trajectory
  
  $\Rightarrow$ Value outside the confidence interval
Rare Event as a Reachability Problem

A Discrete Time Markov chain \( C \)
Two absorbing states \( s_-, s_+ \)
reached with probability 1

Let \( \sigma = s \to s_1 \to s_2 \to \cdots \to s_\pm \)
be a random trajectory in \( C \)

\[
V_s = \begin{cases} 
1 & \text{if } \sigma \text{ ends in state } s_+ \\
0 & \text{if } \sigma \text{ ends in state } s_- 
\end{cases}
\]

Objective:
Estimate \( \Pr(\sigma \text{ ends in state } s_+) = E(V_{s_0}) \)
when \( E(V_{s_0}) \ll 1 \)

Difficulty:
\( V(V_{s_0}) \) too big to have an accurate estimation
Importance Sampling

Principle: Substitute $W_s$ to $V_s$ with same expectation but reduced variance.

1. Substitute $P'$ to $P$ such that $P(s, s') > 0 \Rightarrow P'(s, s') > 0 \forall s = s_-$
2. For each trajectory $\sigma = s \rightarrow s_1 \rightarrow s_2 \cdots s_k \rightarrow s_\pm$
   We define
   
   $$W_s = \begin{cases} 
   \frac{P(s, s_1)}{P'(s, s_1)} \cdot \frac{P(s_1, s_2)}{P'(s_1, s_2)} \cdots \frac{P(s_k, s_\pm)}{P'(s_k, s_\pm)} & \text{if } \sigma \text{ ends in state } s_+ \\
   0 & \text{if } \sigma \text{ ends in state } s_- 
   \end{cases}$$

3. Statistically estimate $E(W_{s_0})$
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   0 & \text{if } \sigma \text{ ends in state } s_-
   \end{cases}
   \]

3. Statistically estimate $E(W_{s_0})$

Expectation is unchanged

\[
\forall s \in S, \ E(W_s) = E(V_s)
\]

Objective: reduction of the variance

\[
V(W_{s_0}) \ll V(V_{s_0})
\]
Optimal Importance Sampling

A non effective result

There exists an importance sampling with variance equal to zero.

Let $\mu(s) = E(V_s)$
Let $P'(s, t) = \frac{\mu(t)}{\mu(s)} \cdot P(s, t)$

\[
W_s = \frac{P(s, s_1)}{P'(s, s_1)} \cdot \frac{P(s_1, s_2)}{P'(s_1, s_2)} \cdots \frac{P(s_k, s_+)}{P'(s_k, s_+)} = \frac{\mu(s)}{\mu(s_1)} \cdot \frac{\mu(s_1)}{\mu(s_2)} \cdots \frac{\mu(s_k)}{1} = \mu(s)
\]

Problem: Need to know $\mu$ which is what one wants to compute.

An help to design good importance sampling.
State of the art

Asymptotically optimal importance sampling
*(P. Dupuis, A.D. Sezer, H. Wang 2007)*

Reduced to an optimization problem (Cross Entropy Method)
*(E. Clarke, P. Zuliani 2011)*
*(C. Jegourel, A. Legay, S. Sedwards 2012)*

Use of heuristic
*(P.E Heegaard, W. Sandmann 2007)*

Case by case analysis
*(Rubino, Tuffin 2009)*
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Problems

- None of these methods is fully automatic.
- None of these methods produces a true confidence interval.
1 Introduction

2 Theoretical framework
   - General Method
   - Guaranteed variance reduction
   - Method for Guaranteed Variance Reduction
   - Bounded Reacheability Discrete Case
   - Bounded Reacheability Continuous Case

3 Experimentation

4 Conclusion and Perspectives
Principle of efficient importance sampling

Design a reduced model $\mathcal{M}^\bullet$ of $\mathcal{M}$ and an abstraction function $f : S \rightarrow S^\bullet$.

Numerically compute $\mu^\bullet$.

Substitute $\mu^\bullet$ to $\mu$ in the optimal importance sampling.
Rare event: The are at least $N$ clients between two idle periods.

From a tandem queues to a bounded capacity tandem queues ($R \ll N$).

The clients in excess are moved back to the first queue.

$$f(n_1, n_2) = \begin{cases} (n_1, n_2) & \text{if } n_2 \leq R \\ (n_1 + n_2 - R, R) & \text{else} \end{cases}$$
How to guarantee variance reduction?

Goal: a modified Benoulli law for $W_{s_0}$

- $V_{s_0} \sim \text{Bernoulli}(\{0, 1\}, \mu(s_0))$
- $W_{s_0} \sim \text{Bernoulli}(\{0, \mu\cdot(f(s_0))\}, \frac{\mu(s_0)}{\mu\cdot(f(s_0))})$
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**Theorem (necessary and sufficient condition)**

$$\forall s \in S, \, \mu^{\bullet}(f(s)) \geq \sum_{s' \in S} P(s, s') \cdot \mu^{\bullet}(f(s'))$$

*Is a necessary and sufficient condition for $W_{s_0}$ to follow a Bernoulli law.*
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Intuition: $\forall s \in S, \mu(s) = \sum_{s' \in S} P(s, s') \cdot \mu(s)$
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Results

- Variance reduction is at least $\mu^\bullet(f(s_0))$.
- A true confidence interval can be computed.
How to check the property in a structural way?

**Theorem**

Assume there exists a family of functions \((g_s)_{s \in S}\), 
\(g_s : \{t \mid P(s, t) > 0\} \rightarrow S^\bullet\) such that:

1. \(\forall s \in S, \forall t^\bullet \in S^\bullet, P^\bullet(f(s), t^\bullet) = \sum_{s' | g(s') = t^\bullet} P(s, s')\)
2. \(\forall s, t \in S \text{ such that } P(s, t) > 0, \mu^\bullet(f(t)) \leq \mu^\bullet(g_s(t))\)

Then \(C^\bullet\) is a reduction of \(C\) with guaranteed variance.

**Interest**

- Condition 1 checked by examination of \(M\) and \(M^\bullet\).
- Condition 2 only involves comparison of items of \(\mu^\bullet\).
Illustration of the local conditions

1. \( \forall s \in S, \forall t^* \in S^*, \quad P^*(f(s), t^*) = \sum_{s' \mid g(s') = t^*} P(s, s') \)

2. \( \forall s, t \in S \) such that \( P(s, t) > 0 \), \( \mu^*(f(t)) \leq \mu^*(g_s(t)) \)

\[
\begin{align*}
\mu^*(t_1^*) & \leq \mu^*(f(t_1)) \\
\mu^*(t_2^*) & \leq \mu^*(f(t_2)) \\
\mu^*(t_3^*) & \leq \mu^*(f(t_3))
\end{align*}
\]
Illustration of the local conditions

1. \( \forall s \in S, \forall t^\bullet \in S^\bullet, P^\bullet(f(s), t^\bullet) = \sum_{s'|g(s')=t^\bullet} P(s, s') \)

\[ \begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array} \quad \begin{array}{c}
s \\
t_1 \\
t_2 \\
t_3
\end{array} \quad \begin{array}{c}
f(s) \\
t_1^\bullet \\
t_2^\bullet \\
t_3^\bullet
\end{array} \quad \begin{array}{c}
\alpha \\
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\end{array} \]

2. \( \forall s, t \in S \) such that \( P(s, t) > 0 \), \( \mu^\bullet(f(t)) \leq \mu^\bullet(g_s(t)) \)

\[ \begin{align*}
\mu^\bullet(t_1^\bullet) & \leq \mu^\bullet(f(t_1)) \\
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\mu^\bullet(t_2^\bullet) & \leq \mu^\bullet(f(t_3))
\end{align*} \]

A coupling theorem

Let \( S^\otimes \) be a coupling relation of \( C^\bullet \) with itself by respect to \( s_- \) and \( s_+ \), then for all \( (s, s') \in S^\otimes \), we have \( \mu^\bullet(s) \geq \mu^\bullet(s') \).
Methodology with guaranteed variance reduction

1. Specify a reduced model $\mathcal{M}^\bullet$ with associated Markov chain $\mathcal{C}^\bullet$ and a function $f$.

2. Establish using analysis of $\mathcal{C}$ and $\mathcal{C}^\bullet$ and using a coupling $\mathcal{C}^\bullet$ that the reduction guarantees the variance reduction.

3. Compute numerically $\mu^\bullet$.

4. Compute statistically $\mu(s_0)$ using the importance sampling induced by $\mu^\bullet$. 
Handling Time Bounded Reachability

Time bounded reachability is strongly related to reactivity.

Difficulties

Observation 1

The rarity of an event can be triggered by the time bound.

\[
\begin{array}{cccccc}
q & p & q & p & q & p \\
\circlearrowleft & a & \circlearrowright & a & \circlearrowleft & a \\
& p & & & & b
\end{array}
\]
Handling Time Bounded Reachability

Time bounded reachability is strongly related to reactivity.

Difficulties

Observation 1
The rarity of an event can be triggered by the time bound.

Observation 2
For finite horizon discrete and continuous time Markov chains behave differently.
From bounded reachability to unbounded reachability

\[ S_u = S_{ab} \times [1, u] \cup \{ s_-, s_+ \} \]

Requires a stronger coupling theorem.
Principle of the method

Apply guaranteed importance sampling to $C_u$

Let $\mu^\bullet_v$ be the time bounded reachability probability with horizon $v$. $\mu^\bullet_v$ can be computed using equalities

$$
\begin{align*}
\mu^\bullet_v &= P^\bullet \cdot \mu^\bullet_{v-1} \\
\mu^\bullet_0(s_+) &= 1 \\
\mu^\bullet_0(s) &= 0 \quad \forall s \neq s_+
\end{align*}
$$

Problem

- $\mu^\bullet_v$ is computed by increasing values of $v$.
- During the simulation $\mu^\bullet_v$ are used by decreasing values of $v$. 

Space consumption problem

Store all vectors

\[ u \]

\[ \mu^u \]

\[ (P^\bullet)^u \mu_0 \]

\[ \bullet \bullet \bullet \]

\[ P^\bullet \mu_0 \]

simulation order

computation order

1 0

\[ \mu_1 \mu_0 \]

\[ \mu^\bullet \mu_0^\bullet \]

\[ \mu^\bullet \mu_0^\bullet \]

\[ \mu^\bullet \mu_0^\bullet \]

Notations:

- \( m \) is the number of states of \( C \).
- \( d \) is the maximal number of outgoing transitions of a state of \( C \).

Complexity

- Time complexity: \( \Theta(m d u) \)
- Space complexity: \( \Theta(m u) \)
Space consumption problem

Notations:

- $m$ is the number of states of $C^\bullet$.
- $d$ is the maximal number of outgoing transitions of a state of $C^\bullet$.

Complexity

- Time complexity: $\Theta(mdu)$
- Space complexity: $\Theta(mu)$
Comparison

Three algorithms

- The naive method
- Static and dynamic storage for $\mu_v$
- Fully dynamic storage for $\mu_v$

<table>
<thead>
<tr>
<th>Complexity</th>
<th>Algo 1</th>
<th>Algo 2</th>
<th>Algo 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td>$\Theta(mu)$</td>
<td>$\Theta(m\sqrt{u})$</td>
<td>$\Theta(m \log u)$</td>
</tr>
<tr>
<td>Time for the precomputation</td>
<td>$\Theta(mdu)$</td>
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<tr>
<td>Additional time for the simulation</td>
<td>0</td>
<td>$\Theta(mdu)$</td>
<td>$\Theta(mdu \log(u))$</td>
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</table>
Bounded reachability in CTMC

Uniformization

- Every CTMC is equivalent to a *uniform* CTMC, i.e. where all sojourn time is state are equal.
- Transient behavior of a uniform CTMC can be efficiently computed from the transient behavior of the associated DTMC.

Application to rare event handling

- Estimation of the time bounded reachability probabilities in the DTMC.
- Computation of the time bounded reachability probabilities in the CTMC via the uniformization formula.
- Elaborated tuning for the confidence interval.
1 Introduction

2 Theoretical framework

3 Experimentation
   - Implementation
   - Examples

4 Conclusion and Perspectives
Adaptation of COSMOS

Modifications related to rare event

- Implementation of the importance sampling.
- Numerical computation of the transient behaviors.
- Implementation of the three algorithms.
- Implementation of the uniformization method.

General purpose improvements

- Parallelization of the simulation.
- Integration of COSMOS into the platform CosyVerif
An example

\[ \lambda \rightarrow n_1 \xrightarrow{\rho_1} n_2 \xrightarrow{\rho_2} \]

\[ \mathcal{M} \]

\[ \lambda \rightarrow n_1 \xrightarrow{\rho_1} n_2 \xrightarrow{\rho_2} \rightarrow R \]

\[ \mathcal{M}^* \]

- Parameters: \( \lambda = 0.1, \rho_1 = \rho_2 = 0.45 \),
- Formula: They are at least \( N \) clients between two idle periods.
- Generation of 20000 trajectories
- Numerical result: \( \mu(s_0) = 3.80122 \cdot 10^{-31} \)
Example of the tandem ($N = 50$)

We perform experimentation with different values of $R$.

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<th>size of $C$</th>
<th>size of $C^*$</th>
<th>$\mu^*(s_0)$</th>
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$C^*$ is much smaller than $C$. 

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The estimated value is always close to the true value of \( \mu(s_0) \).
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The confidence interval is tight even for small $R$. 

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Example of the tandem with large values of $N$
Other examples

- **Tandem (the second queue is full before the system is empty)**
  - Infinite system (the first queue is unbounded)
  - Finite reduced system

- **Tandem (the second queue is full before the first one)**
  - Theoretical guarantee
  - Experimentally the acceleration is sufficient.

- **Parallel ruin**
  - Concurrent system
  - The reduced system is build by removing synchronization between process

- **Dining philosopher problem**
  - Extension of the method but no theoretical guarantee.
  - The distribution of $W_{s_0}$ is heavy tailed.
Conclusion and Perspectives

- **Contributions**
  - Design of an importance sampling method with variance reduction and true confidence interval
  - Integration in a tool
  - Several conclusive case studies

- **Perspectives**
  - Handling more general infinite systems
  - Search of Petri net classes with automatic computation of the reduced model.
  - Automated or assisted proofs of coupling
Publications

  Échantillonnage préférentiel pour le model checking statistique.

  Coupling and Importance Sampling for Statistical Model Checking.

  Importance Sampling for Model Checking of Time-Bounded Until.
  Research Report LSV-12-04, February 2012. 14 pages.

  Importance Sampling for Model Checking of Continuous-Time Markov Chains.
  Research Report LSV-12-08, May 2012. 15 pages.

- Submission to “Discrete Event Dynamic Systems”