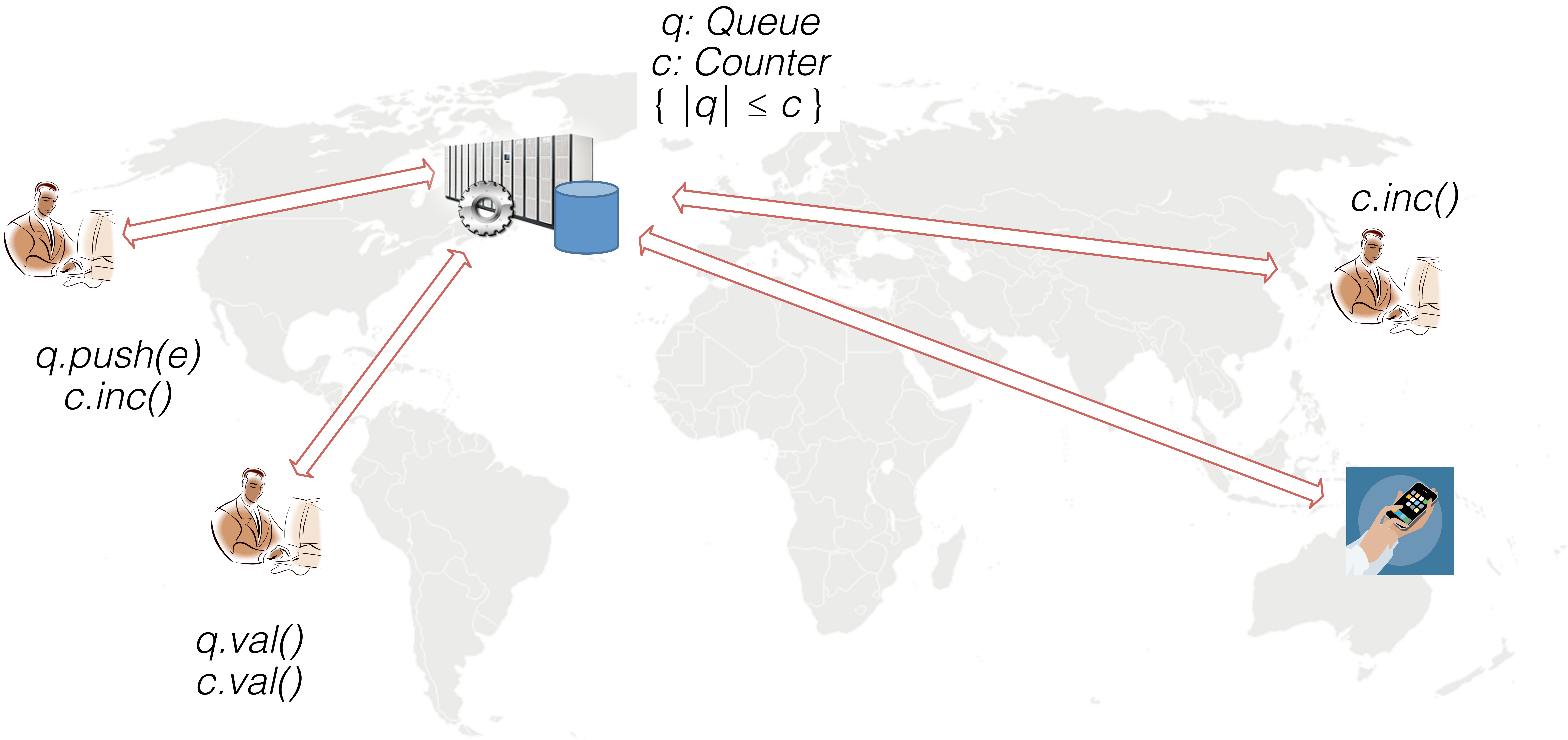
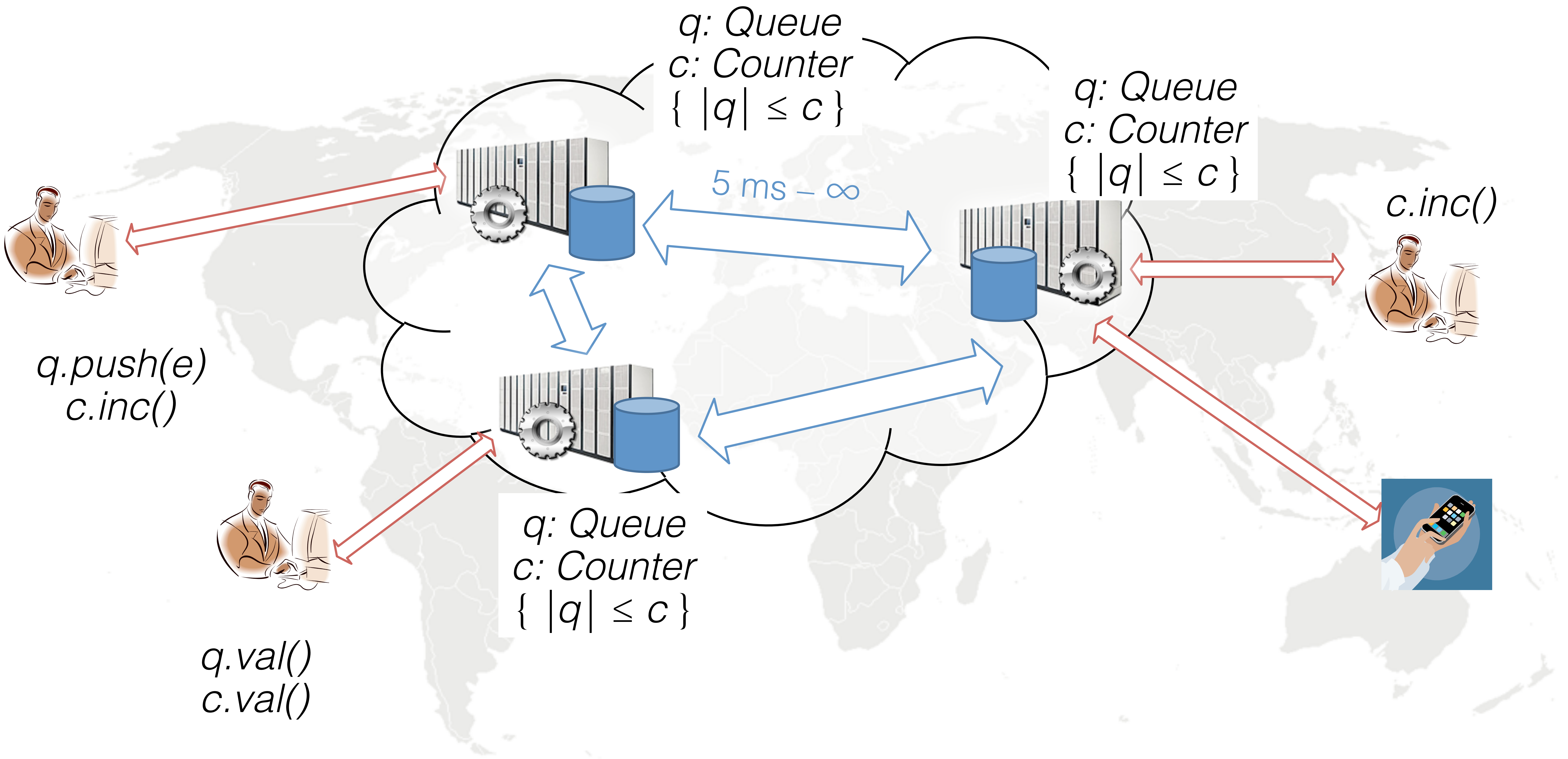
VERIFYING IMPLEMENTATIONS OF CRDTS

SHARED DATA TYPE



GEO-REPLICATED DATA TYPE



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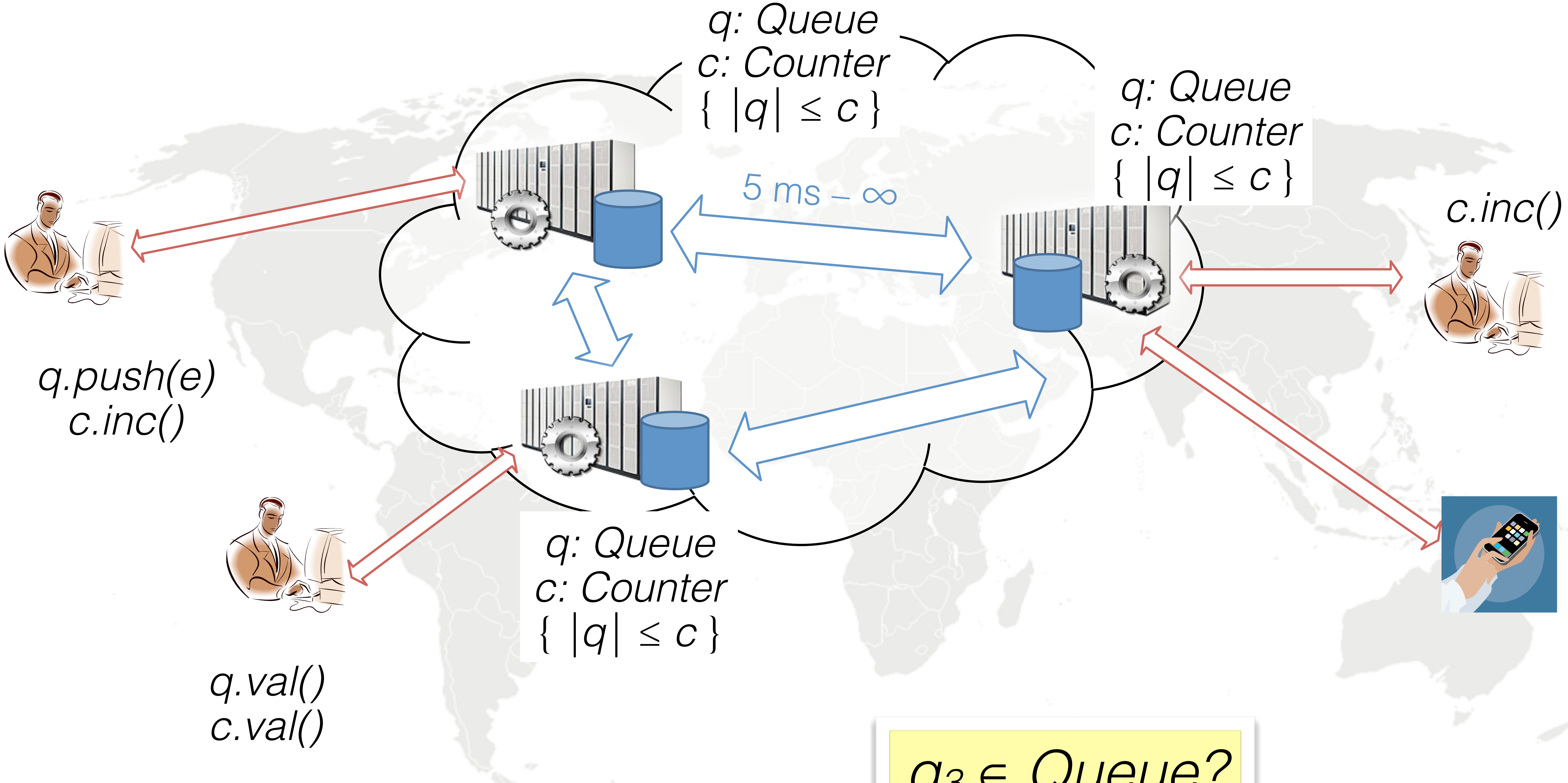
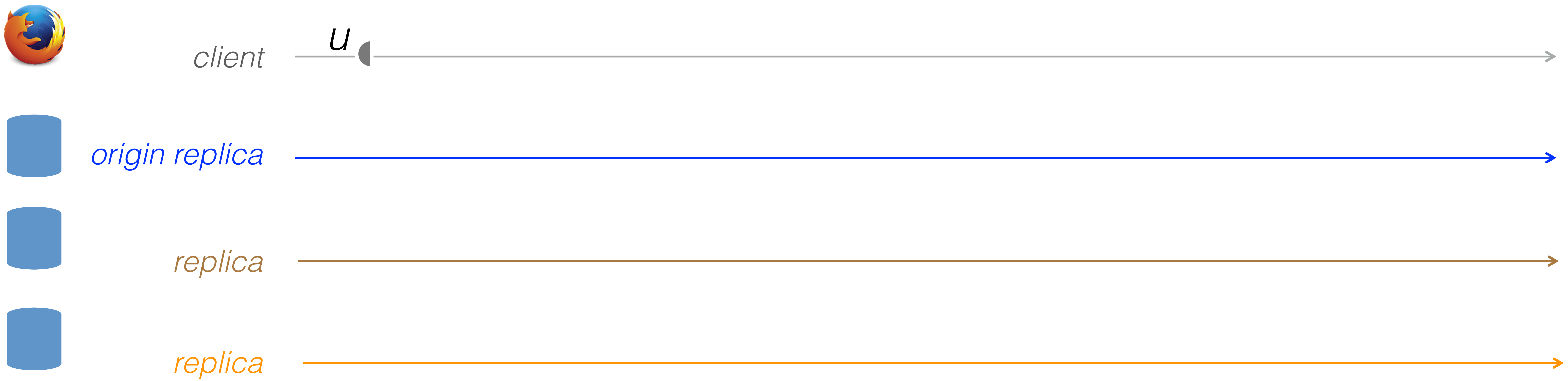


$q_3 \in$ Queue?

$q_1 = q_2$?

$|q_1| \leq c_4$?

Program Model (operation)



- $u: \mathit{state} \curvearrowright (\mathit{retval}, (\mathit{state} \curvearrowright \mathit{state}))$
- Prepare (@origin) $u_?$; deliver $u_!$
- Read One, Write All
- Deferred-Update Replication

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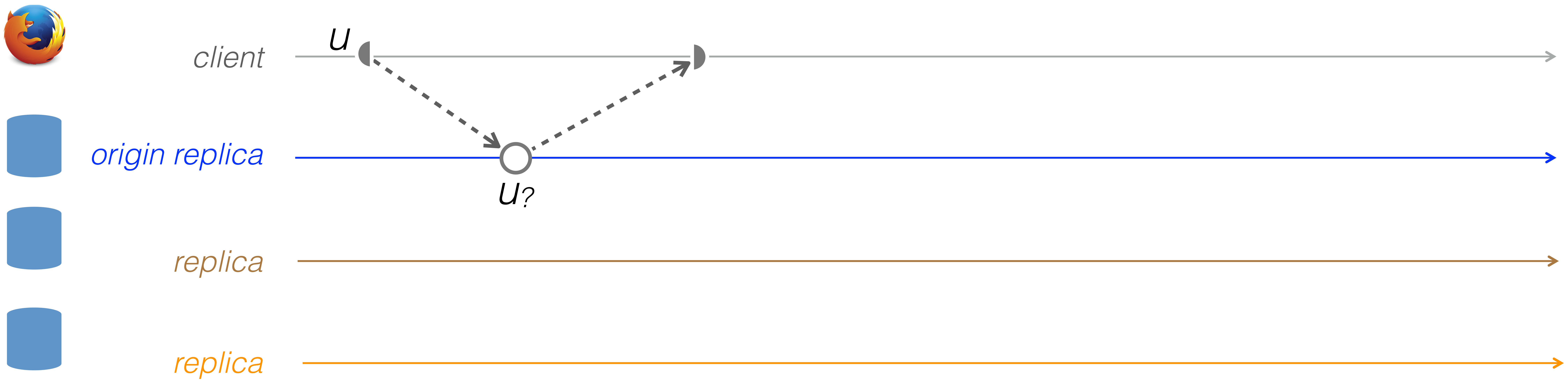

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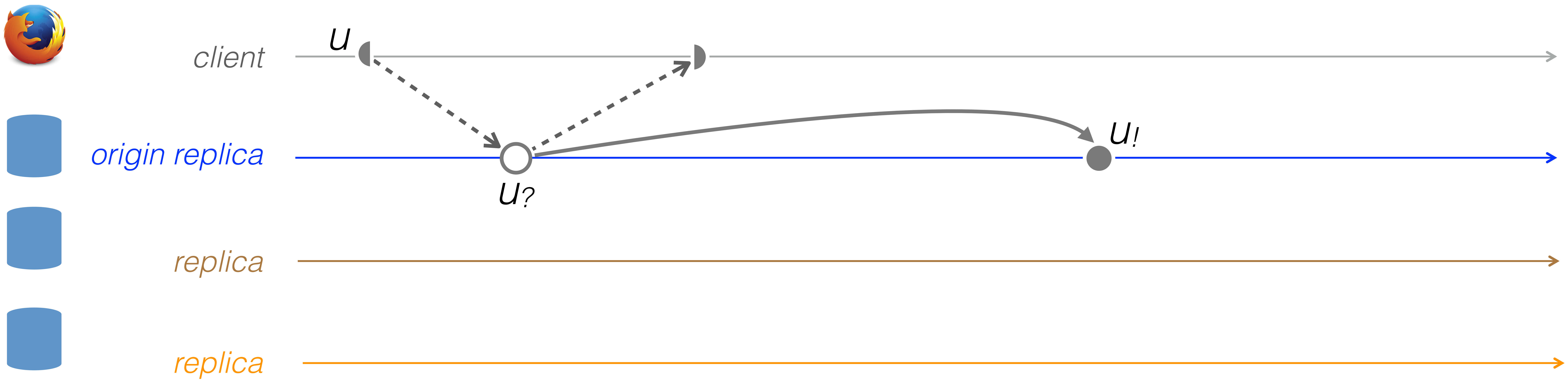

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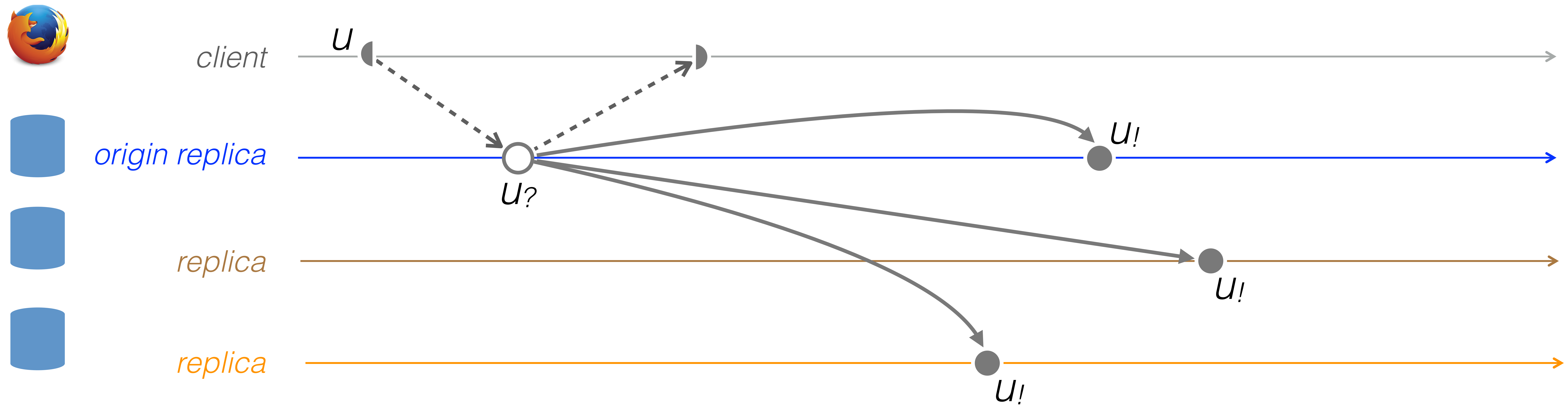

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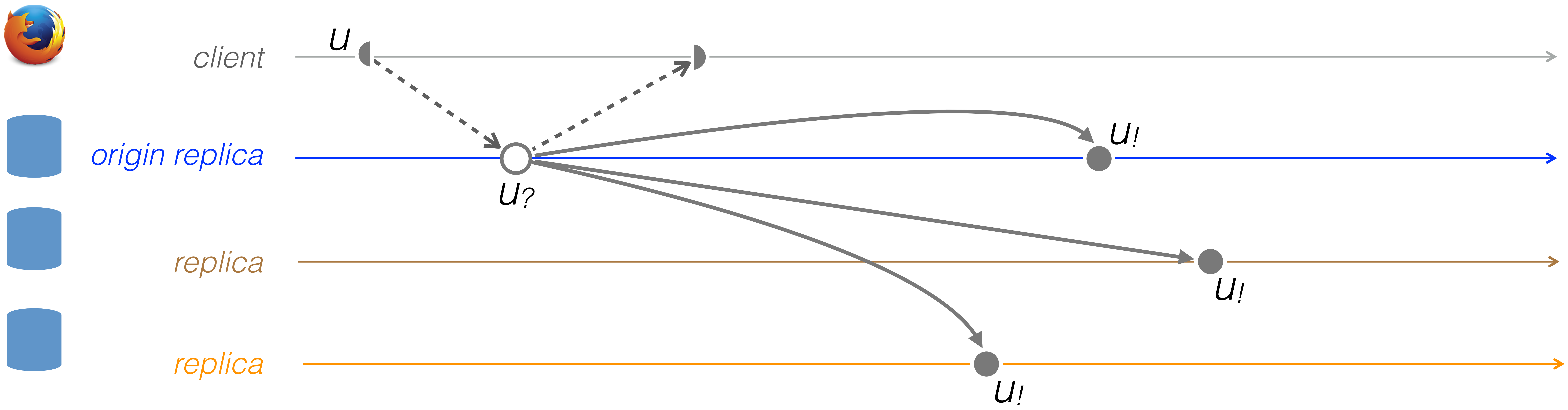

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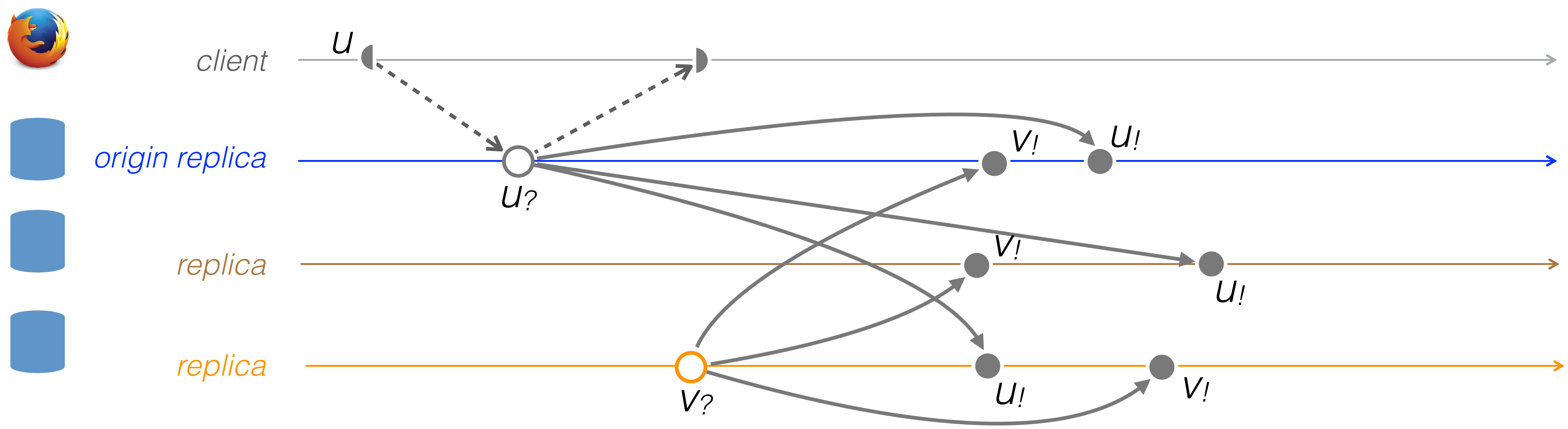

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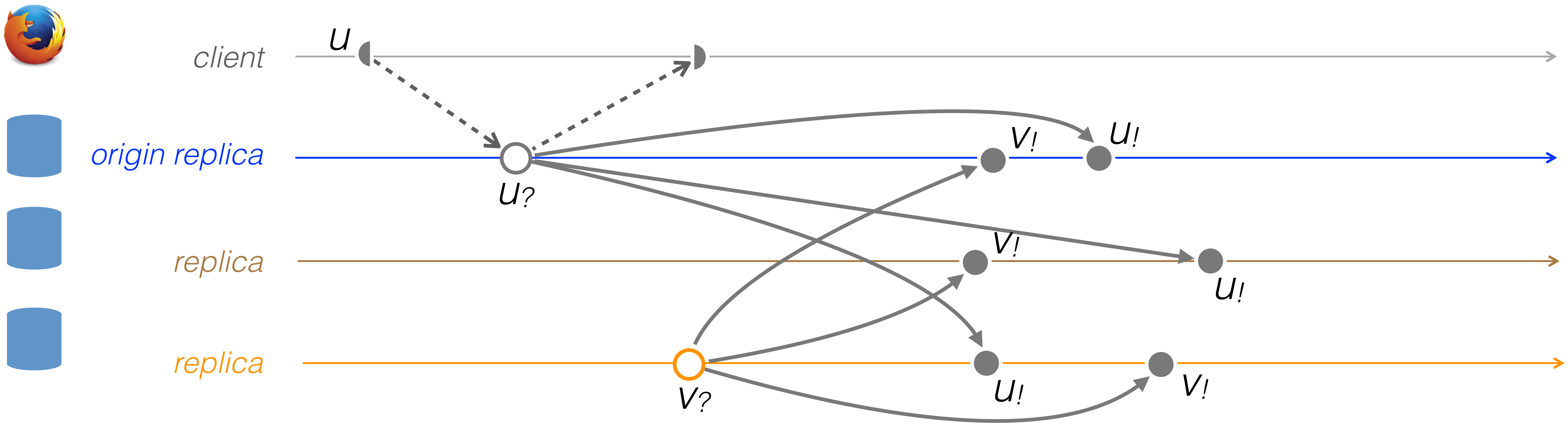

- Concurrent, Multi-master
- Strong: total order, identical state
- Weak: concurrent, interleaving, no global state

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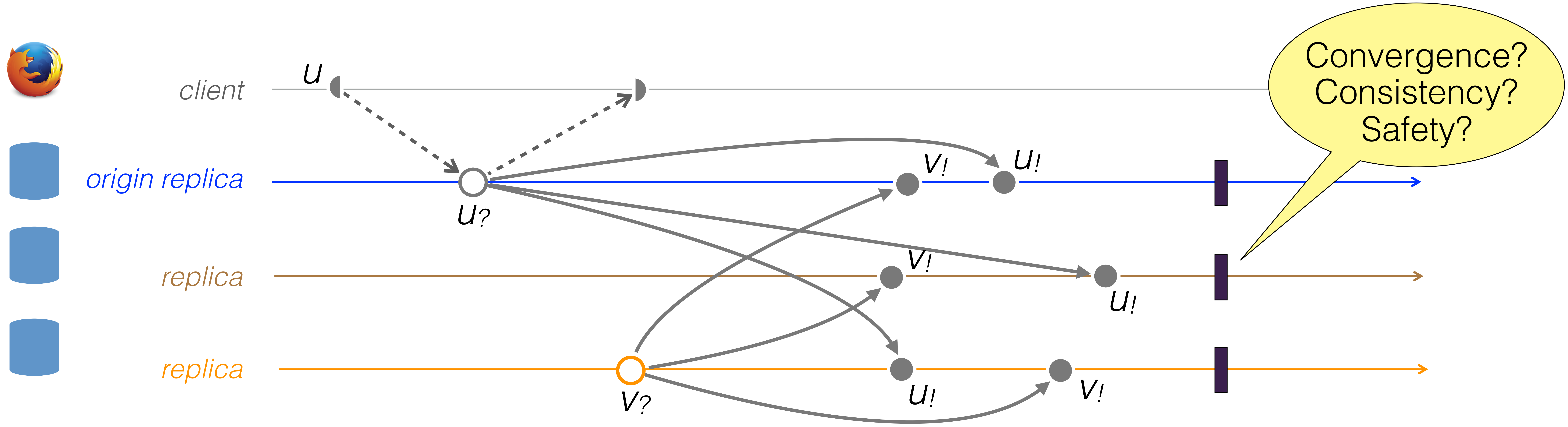

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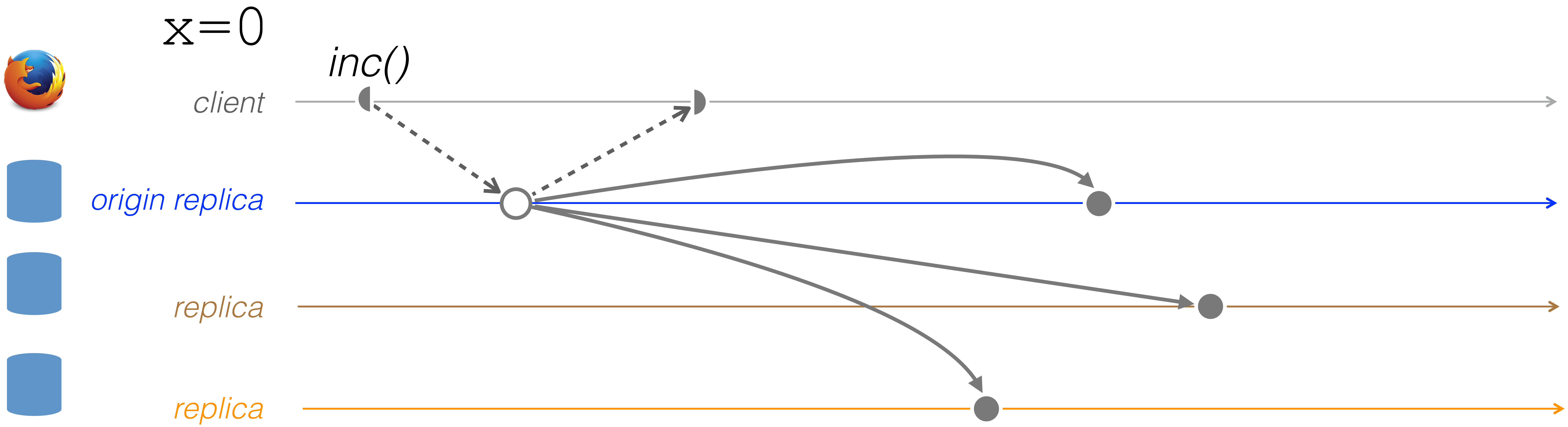
CRDT Examples

Example: Grow-Only Counter



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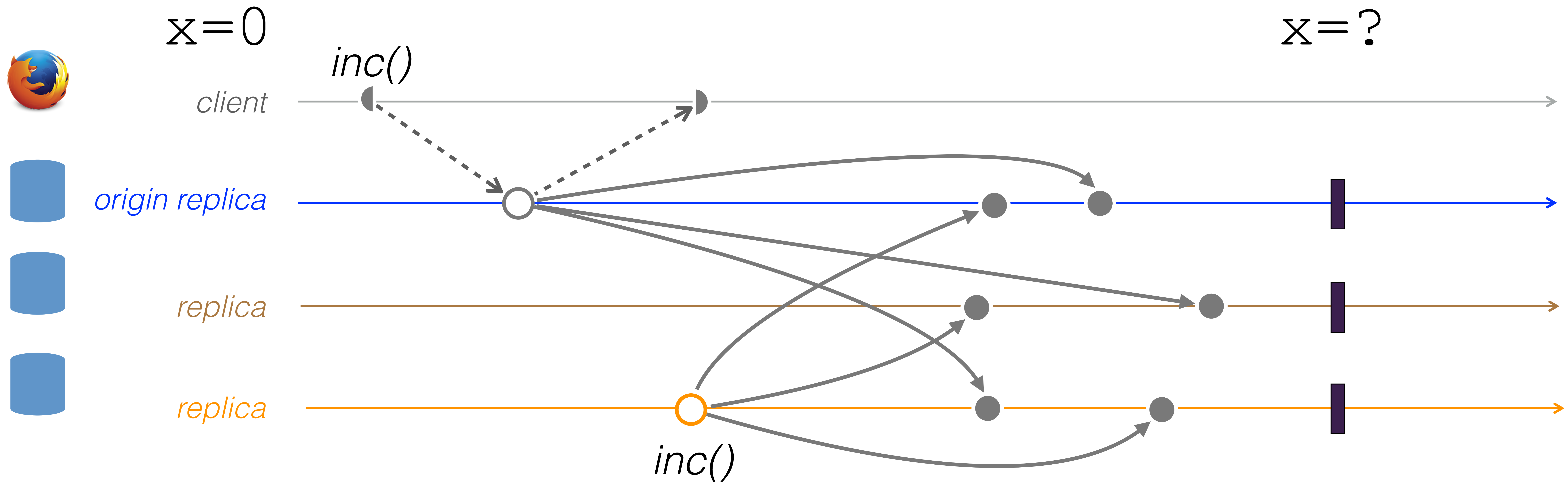

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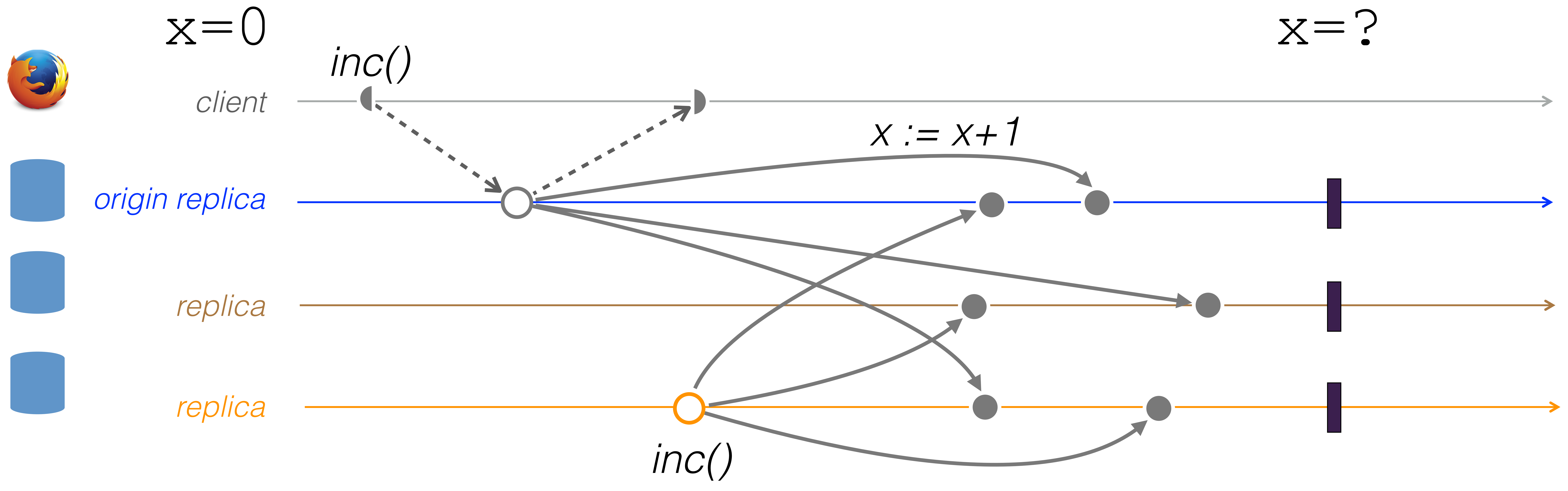

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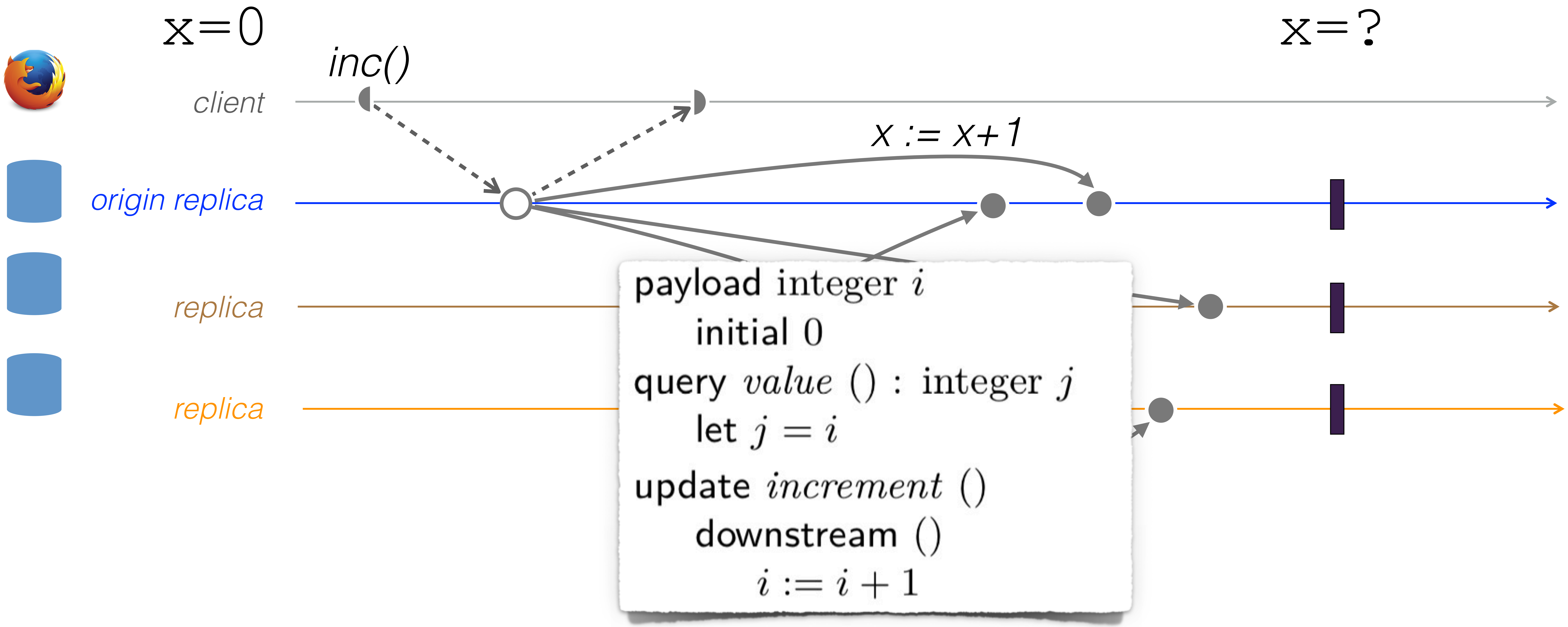
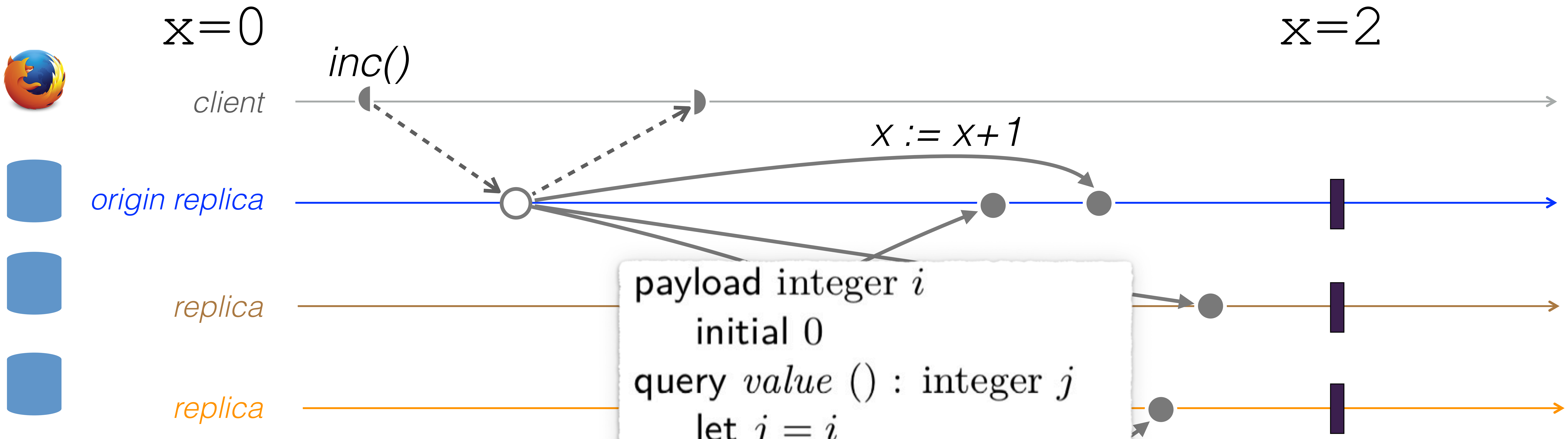


[A Comprehensive Study of CRDT's (Shapiro et al. 2011)]

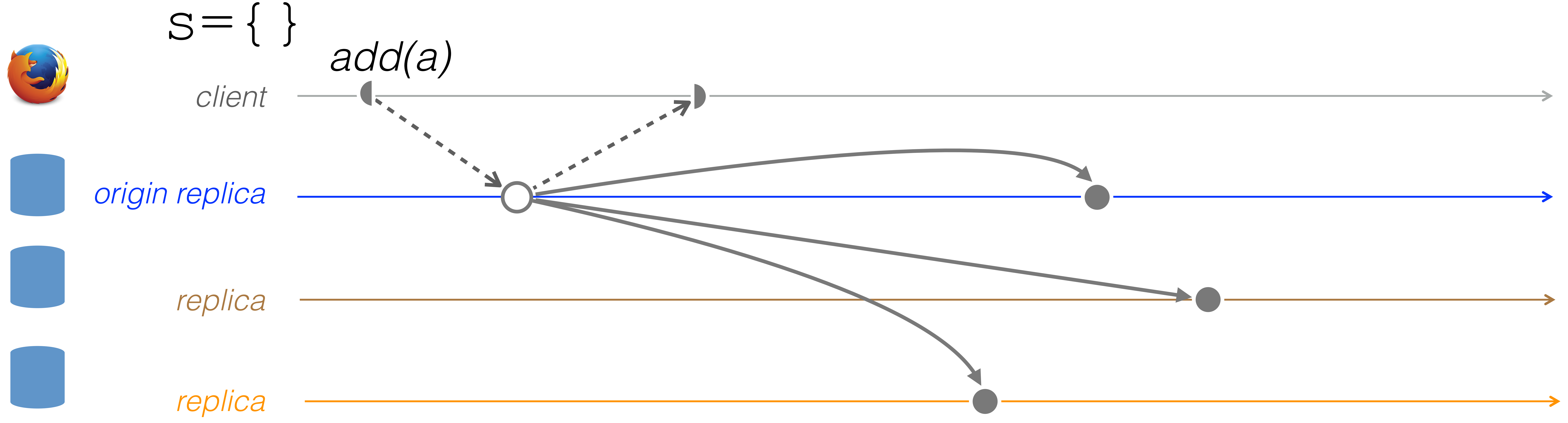
EXAMPLE: GROW-ONLY COUNTER



payload integer i
  initial 0
query *value* $()$: integer j
  let $j = i$
update *increment* $()$
  downstream $()$
    $i := i + 1$

[A Comprehensive Study of CRDT's (Shapiro et al. 2011)]

EXAMPLE: OBSERVED-REMOVE SET



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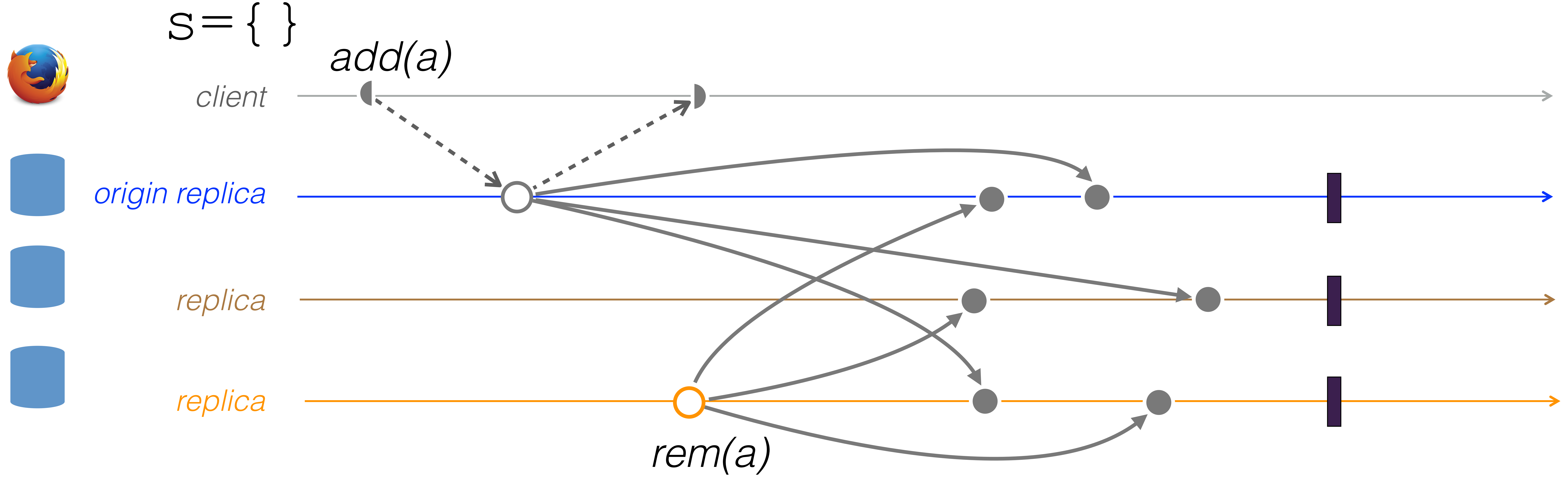

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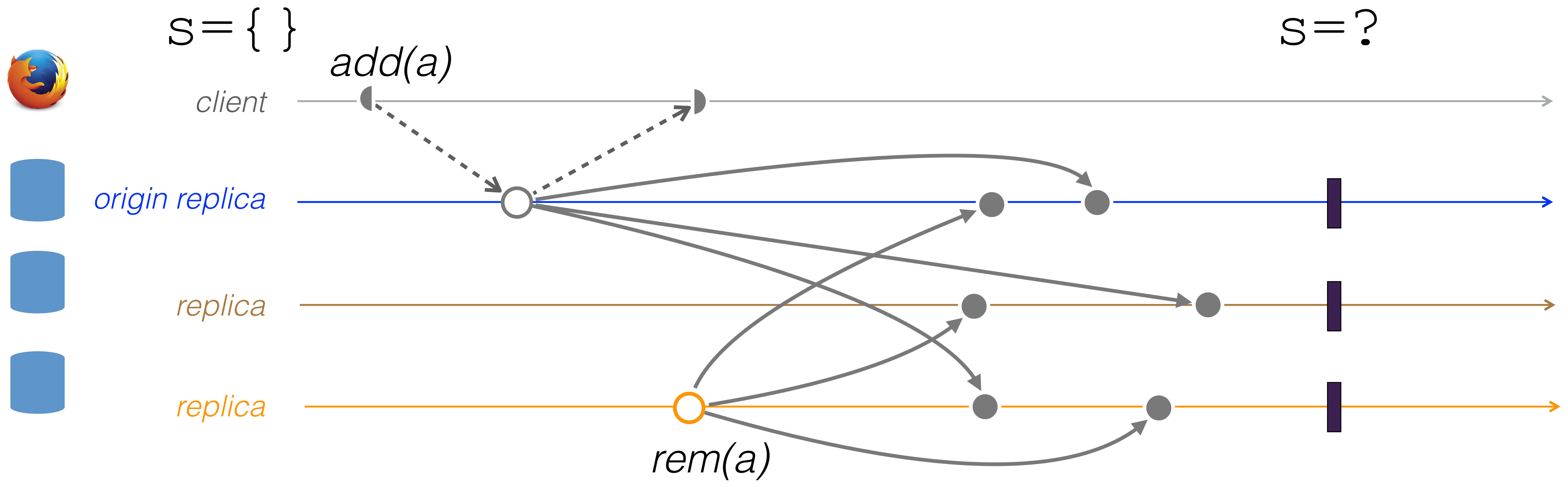

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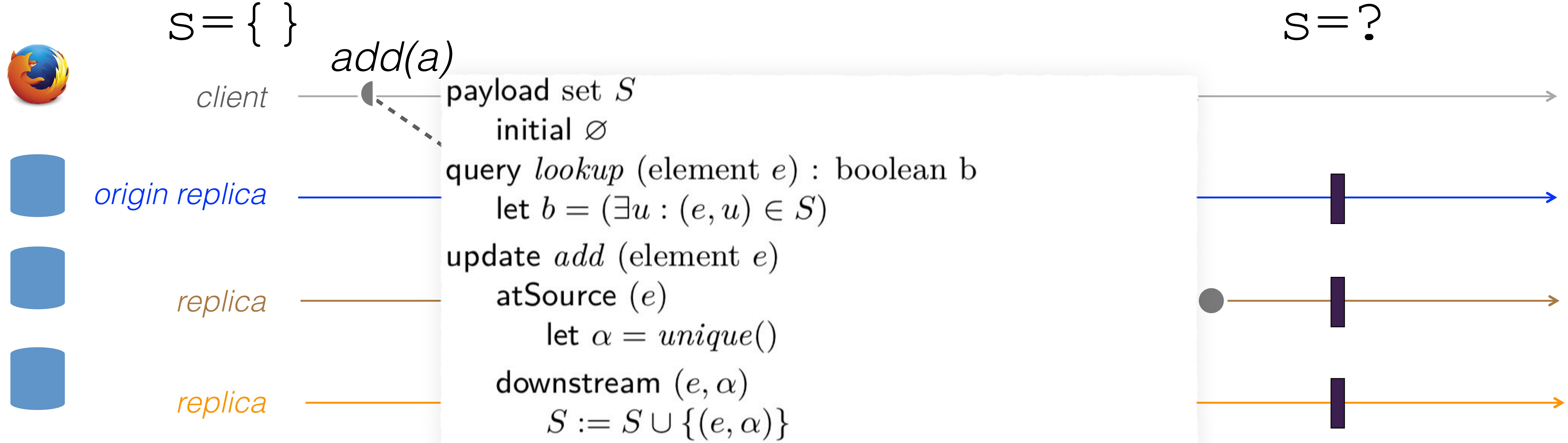

payload set S
 initial $\varnothing$
query $lookup$ (element e) : boolean b
 let $b = (\exists u : (e, u) \in S)$
update add (element e)
 atSource (e)
  let $\alpha = unique()$
 downstream (e, α)
  $S := S \cup \{(e, \alpha)\}$
update $remove$ (element e)
 atSource (e)
  pre $lookup(e)$
  let $R = \{(e, u) | \exists u : (e, u) \in S\}$
 downstream (R)
  pre $\forall (e, u) \in R : add(e, u)$ has been delivered
  $S := S \setminus R$

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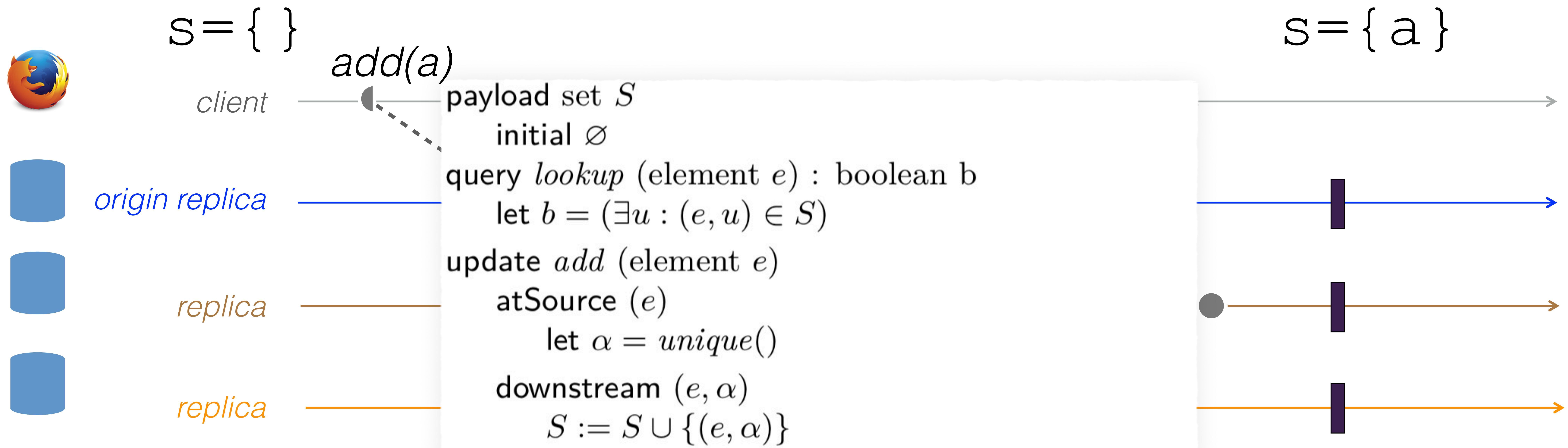

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Anomalies of concurrent updates

- Bank:
  - σ_{init} = 100€
  - Alice: *withdraw(20)* = { $\sigma \coloneqq 120$ }
  - Bob: *debit (60)* = { $\sigma \coloneqq 40$ }
  - σ = ???

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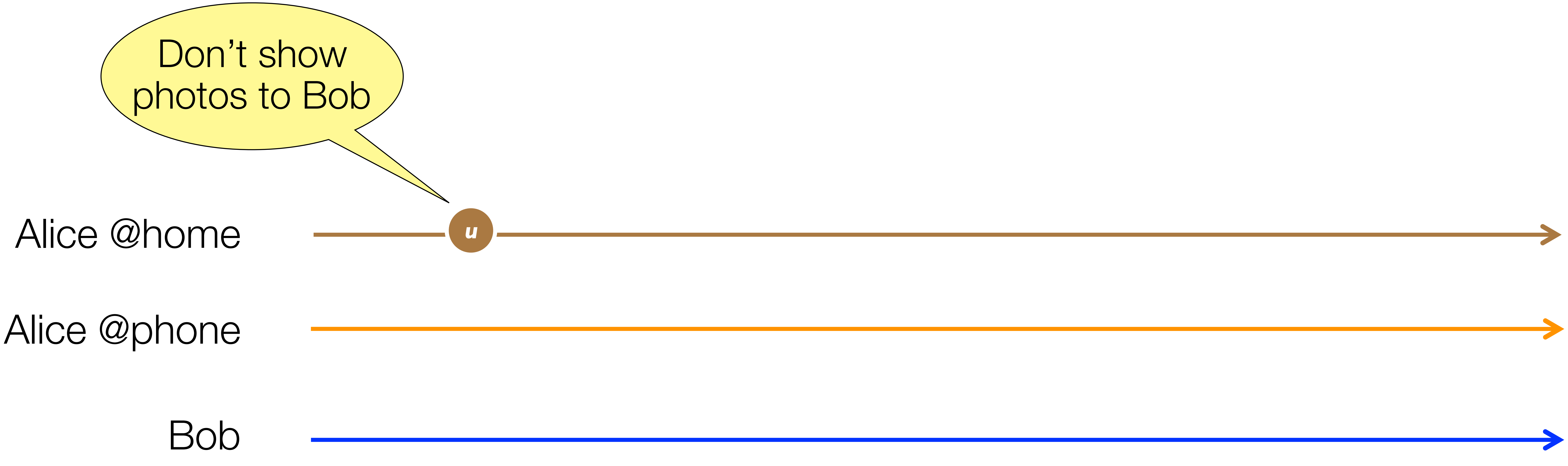
  - Bob: *debit (60)* = { $\sigma \coloneqq$ 40 }
  - σ = ???

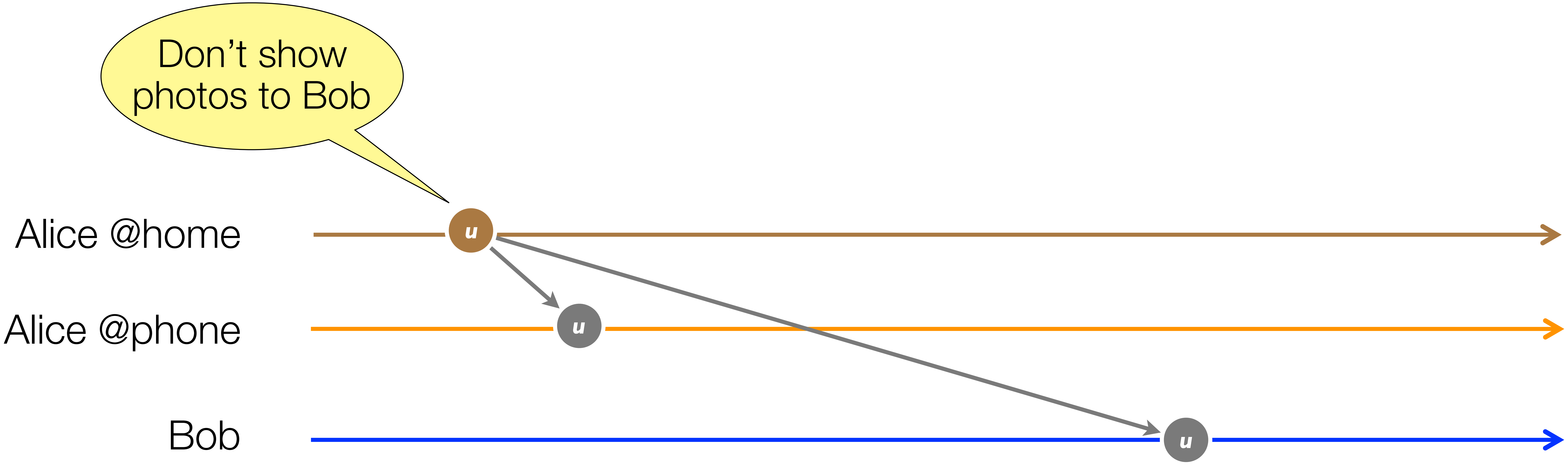
- File system:
  - σ_{init} = “/“
  - Alice: *mkdir (“/foo”); mkdir (“/foo/bar”)*
  - Bob: receives *mkdir (“/foo/bar”)*
  - σ = ???

Eventual Consistency



- access (Bob, photo) $\Longrightarrow$ ACL (Bob, photo)
- v observed effects of u $\Longrightarrow$ v should be delivered after u
- Available: doesn't slow down sender

Eventual Consistency



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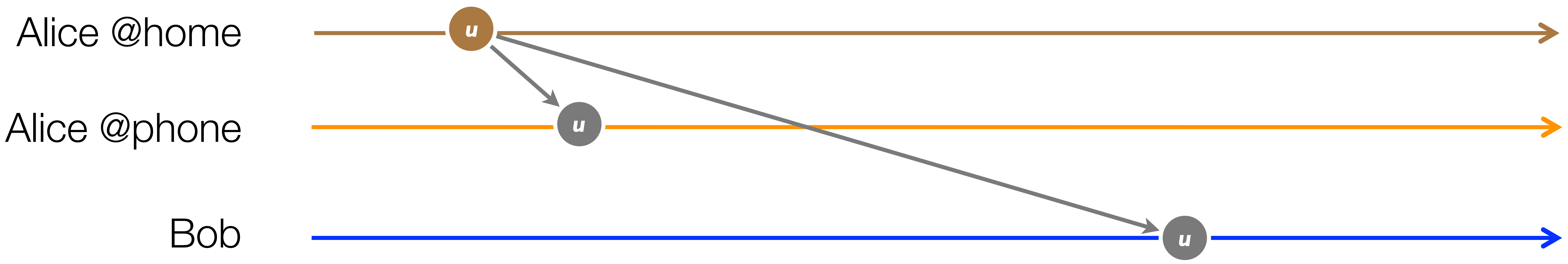

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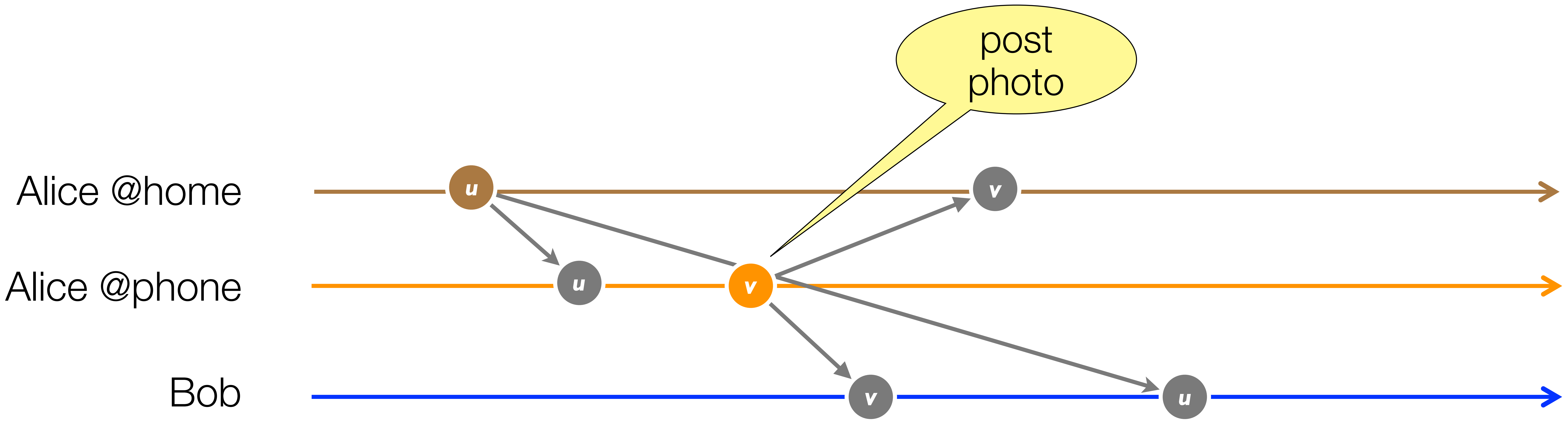

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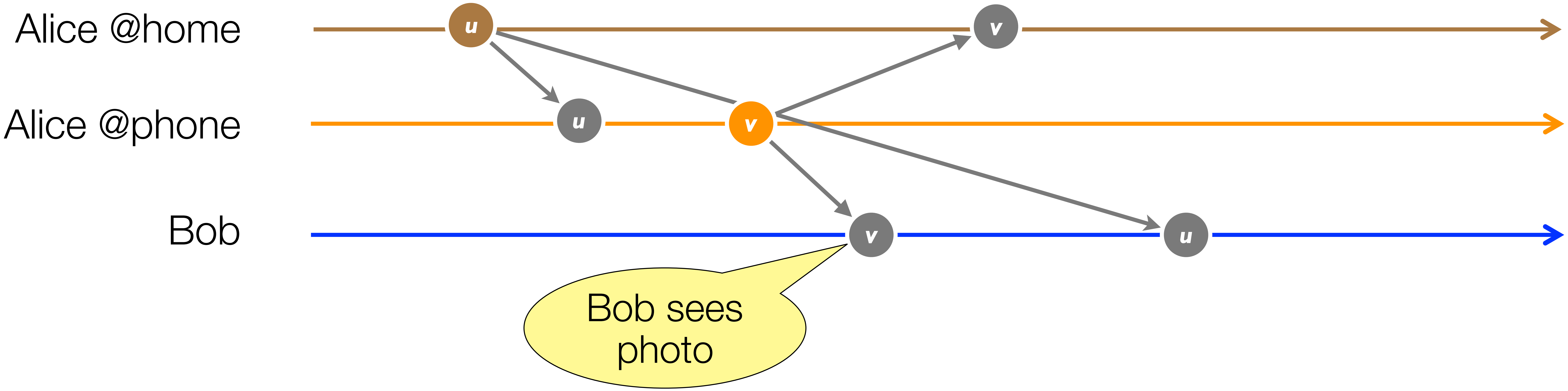

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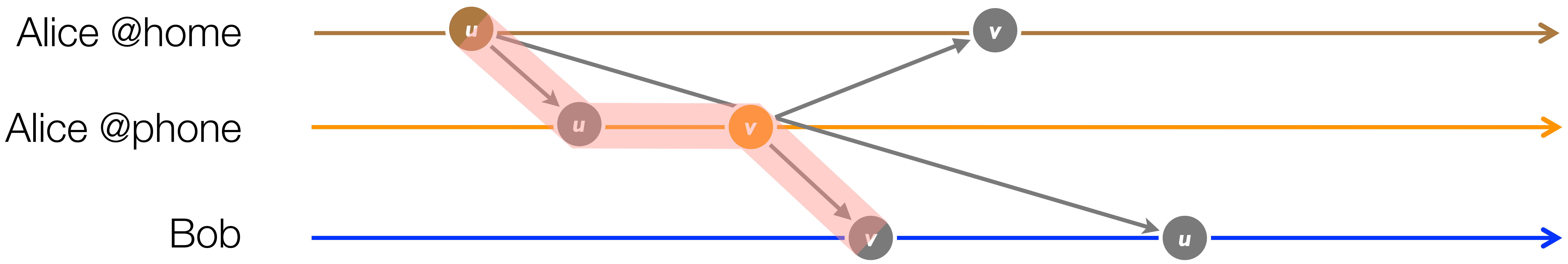

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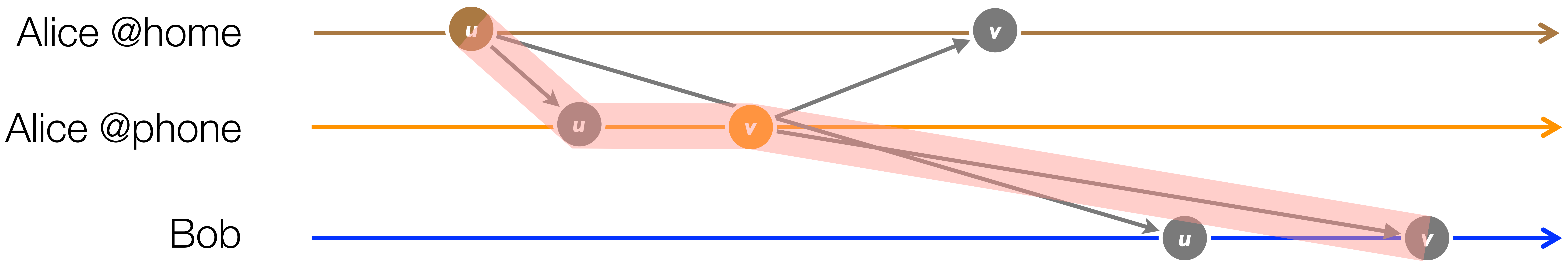

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INRIA
ROCQUENCOURT

A comprehensive study of
Convergent and Commutative Replicated Data Types *

Marc Shapiro, INRIA & LIP6, Paris, France
Nuno Preguiça, CITI, Universidade Nova de Lisboa, Portugal
Carlos Baquero, Universidade do Minho, Portugal
Marek Zawirski, INRIA & UPMC, Paris, France

Thème COM — Systèmes communicants
Projet Regal

http://bit.ly/1PBC4zc

Operation-based CRDTs

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  - Each operation is delivered to each replica

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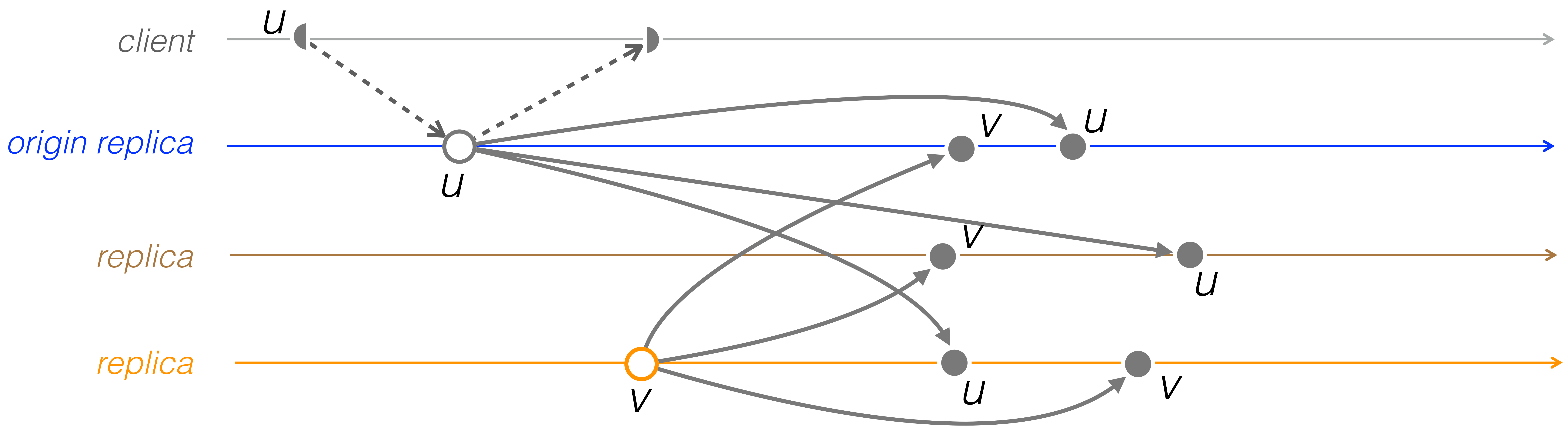
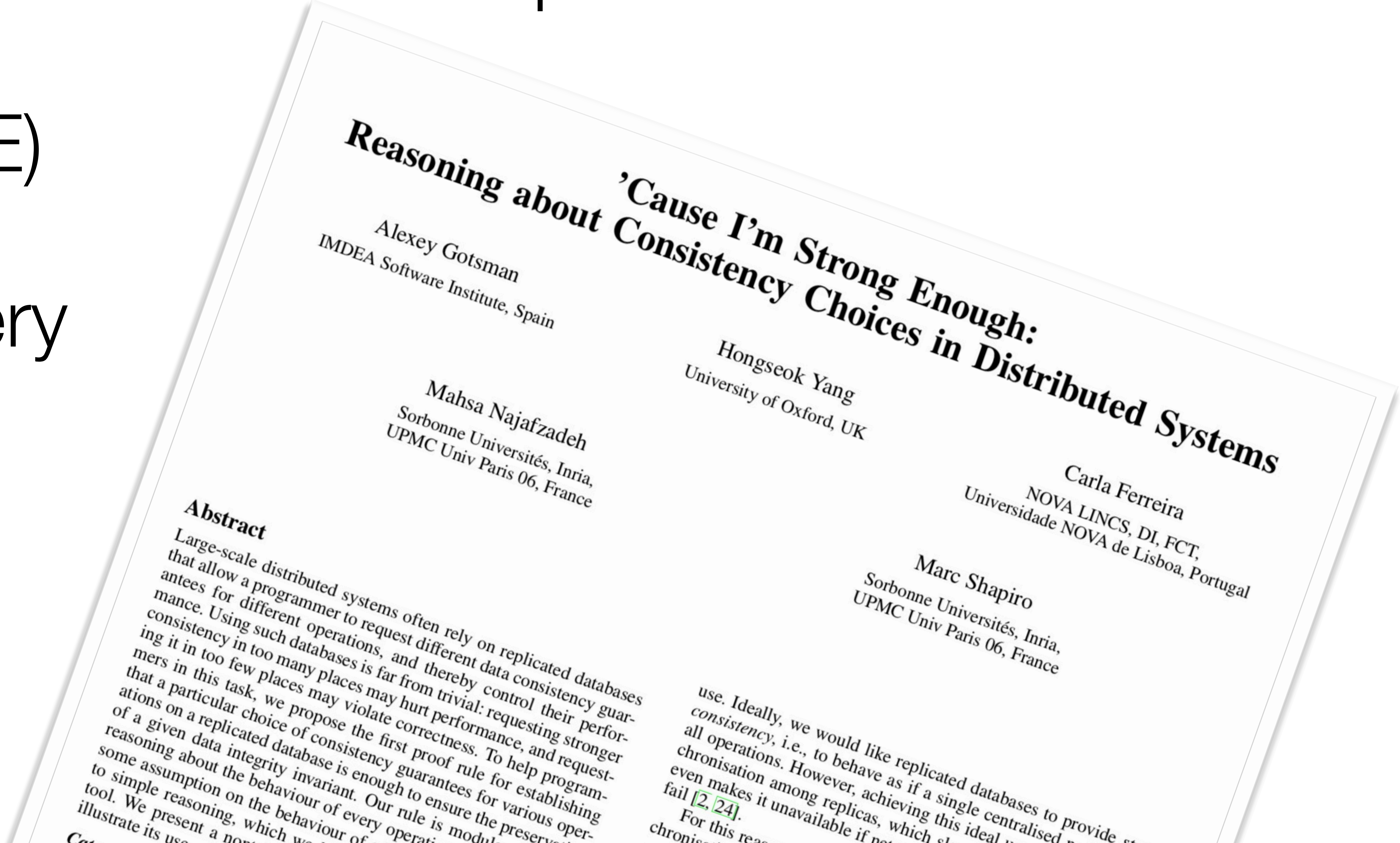


OPERATION-BASED CRDTs

- Operation-based CRDTs
  - Each operation is delivered to each replica
- Invariant Checking (CISE)
  - Requires causal delivery



'Cause I'm Strong Enough:
Reasoning about Consistency Choices in Distributed Systems

Alexey Gotsman
IMDEA Software Institute, Spain

Hongseok Yang
University of Oxford, UK

Mahsa Najafzadeh
Sorbonne Universités, Inria,
UPMC Univ Paris 06, France

Carla Ferreira
NOVA LINCS, DI, FCT,
Universidade NOVA de Lisboa, Portugal

Marc Shapiro
Sorbonne Universités, Inria,
UPMC Univ Paris 06, France

Abstract

Large-scale distributed systems often rely on replicated databases that allow a programmer to request different data consistency guarantees for different operations, and thereby control their performance. Using such databases is far from trivial: requesting stronger consistency in too many places may hurt performance, and requesting it in too few places may violate correctness. To help programmers in this task, we propose the first proof rule for establishing that a particular choice of consistency guarantees for various operations on a replicated database is enough to ensure the preservation of a given data integrity invariant. Our rule is modular ... reasoning about the behaviour of every operation ... some assumption on the behaviour of ... to simple reasoning, which ... tool. We present a ... illustrate its use ...

use. Ideally, we would like replicated databases to provide ... *consistency*, i.e., to behave as if a single centralised ... all operations. However, achieving this ideal ... chronisation among replicas, which ... even makes it unavailable if ... fail [2, 24].

For this reas... chronis...

State-based CRDTs

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  - Propagation of states (instead of operations)

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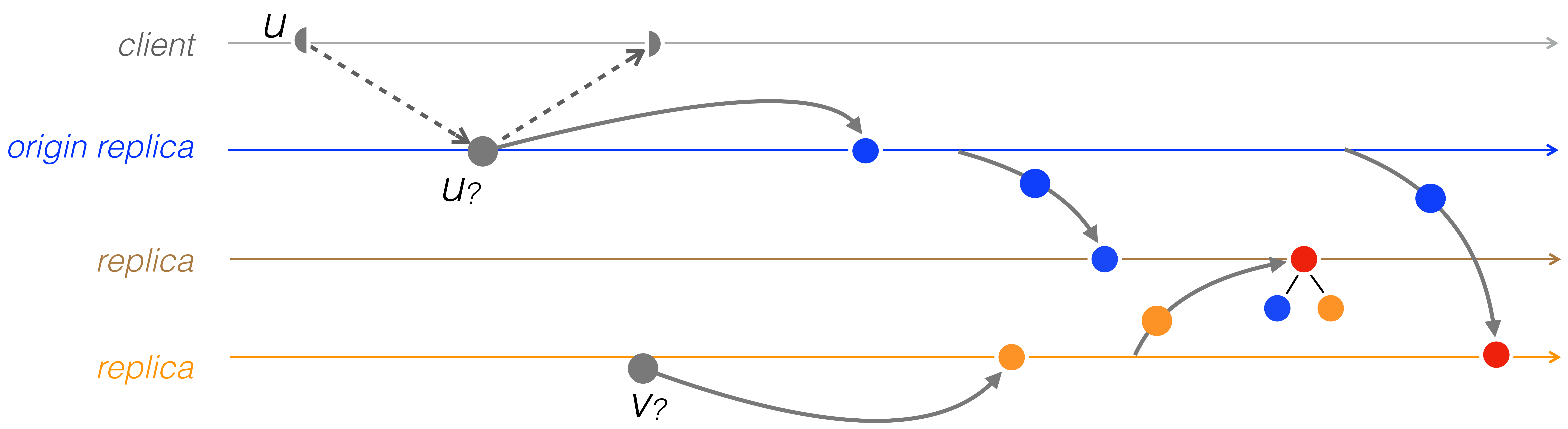
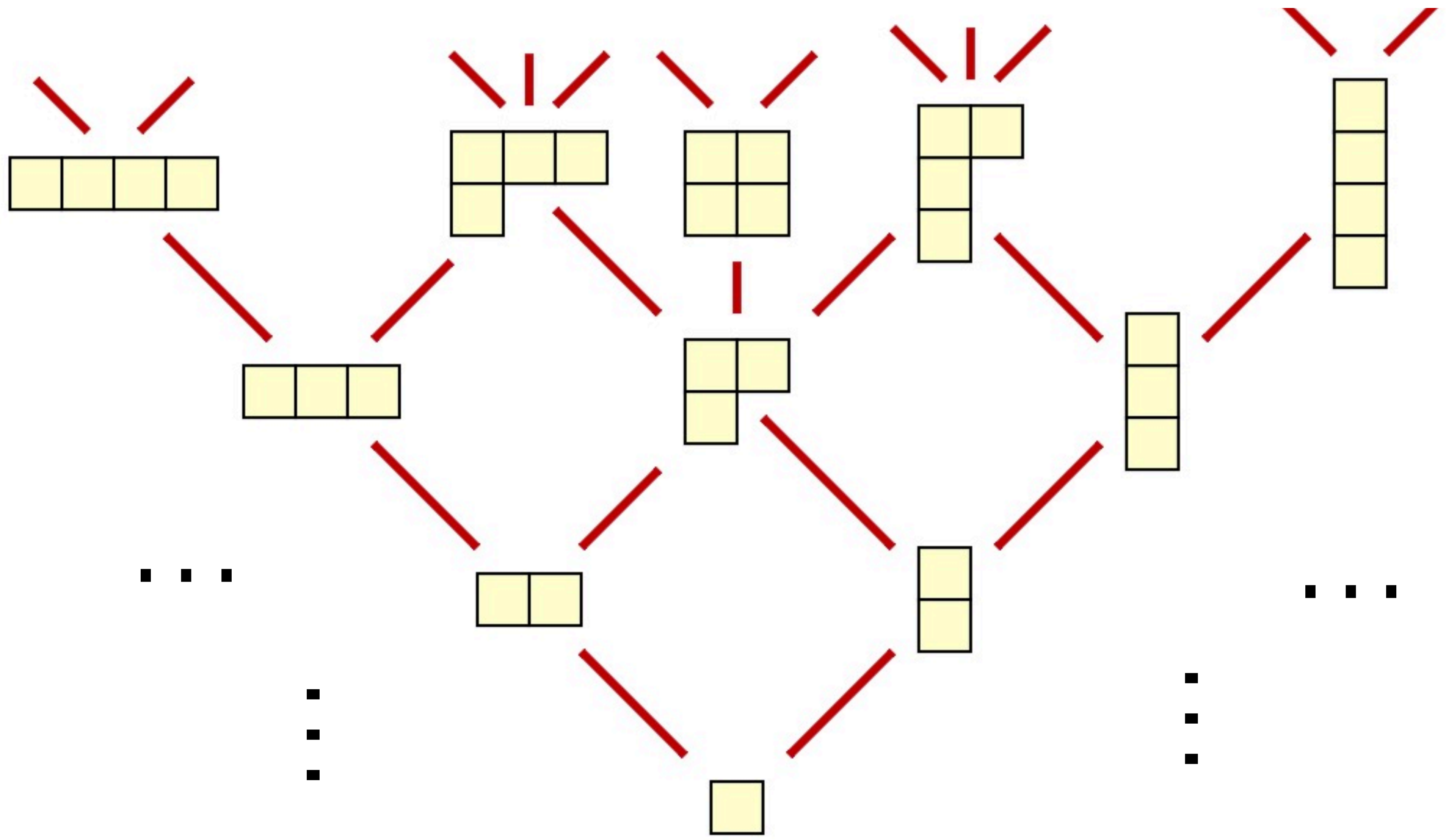


STATE-BASED CRDTS

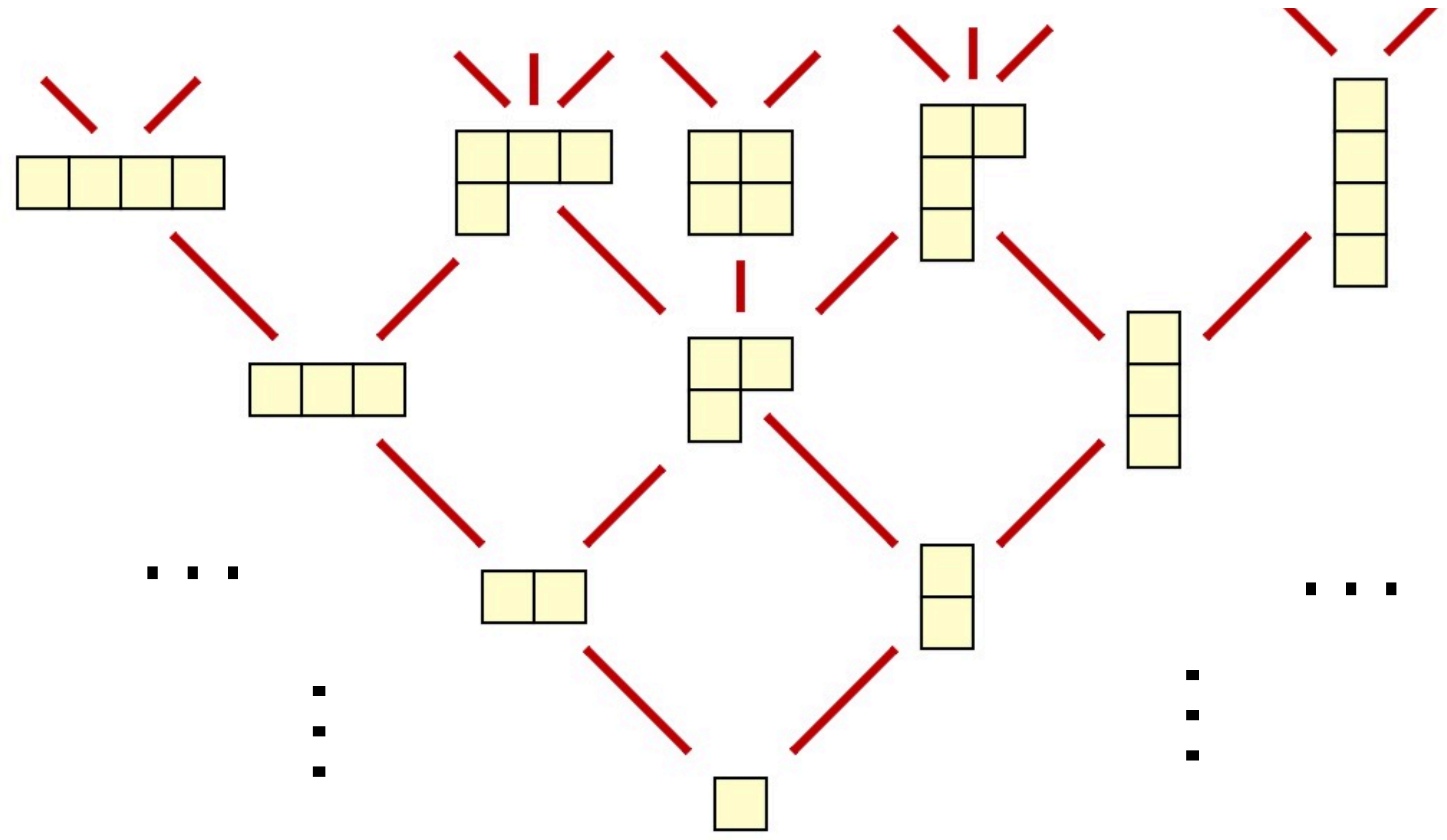
- State-based CRDTs
  - Propagation of states (instead of operations)
- States are **`merged`** on receive
  - Convergence: states resulting from *concurrent* operations result deterministically on a single state
  - No delivery assumptions

STATE-BASED CRDTS



STATE-BASED CRDTS

- State is a (join semi-)Lattice



STATE-BASED CRDTS

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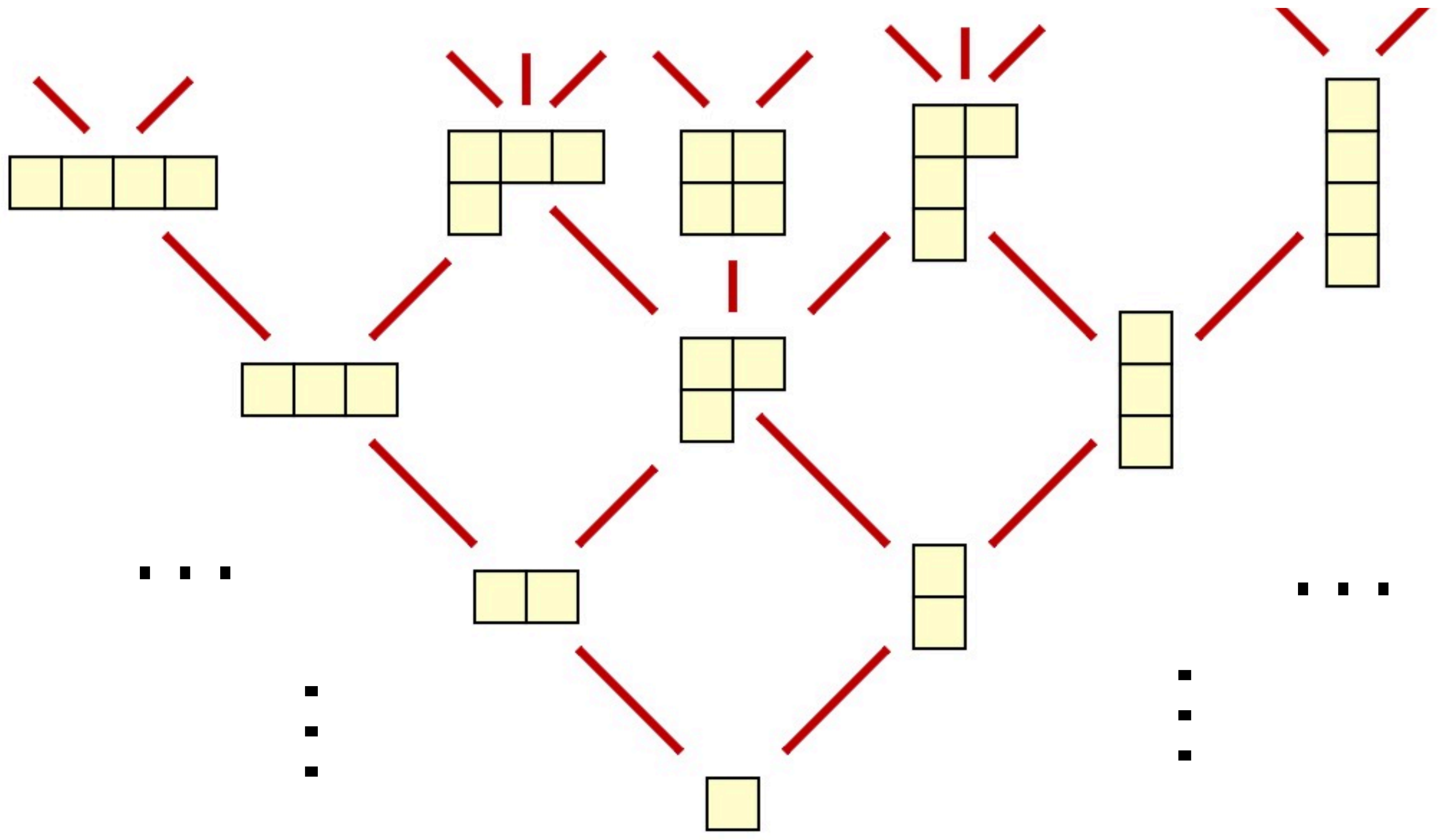
- Effectors send the state at the origin
  - Lazy update propagation



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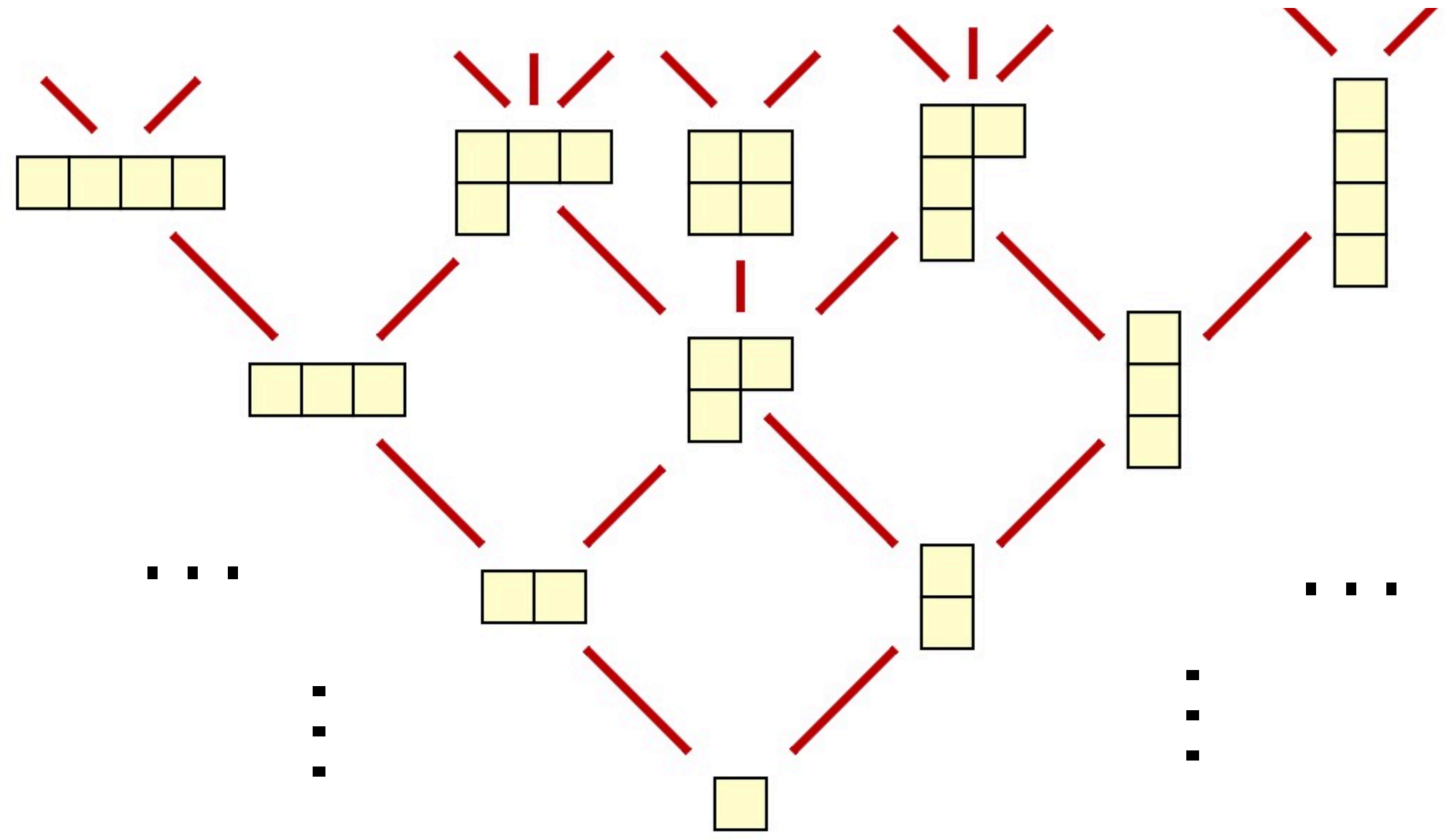

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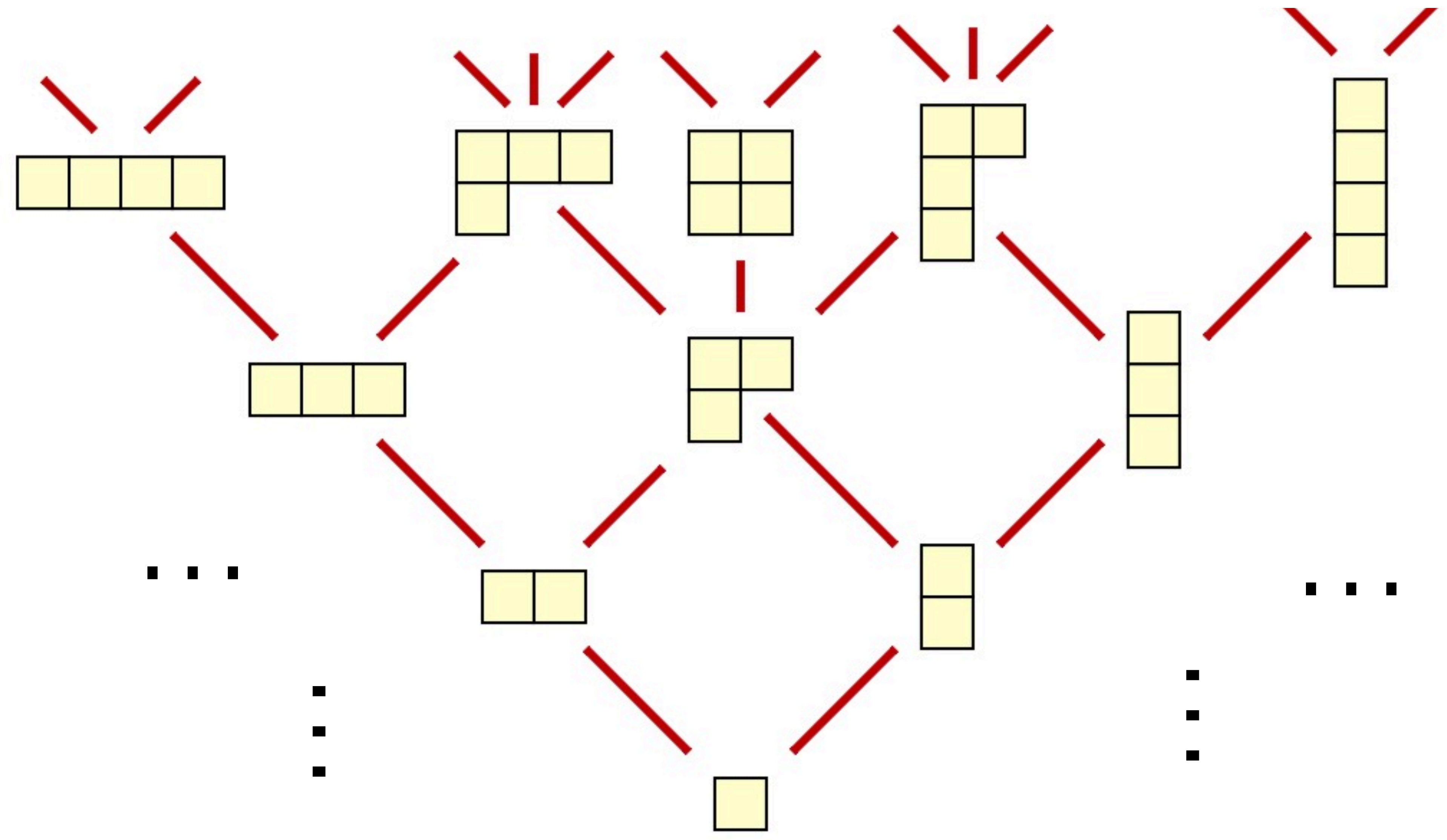
State-based CRDTs



- State is a (join semi-)Lattice
- Effectors send the state at the origin
  - Lazy update propagation
- `merge` function joins the state of two replicas
  - Join of the lattice
- Each operation is an inflation in the lattice

BOUNDED COUNTER

- N Replicas
- R matrix of positive counts
- U vector of negative counts

10	2	2	1
3	6	1	0
5	0	4	2
1	0	2	1

5
2
1
0

[Valter Balegas, 2016]

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Total: $\sum_i R[i][i] - \sum_i U[i]$

[Valter Balegas, 2016]

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BOUNDED COUNTER

- N Replicas
- R matrix of positive counts
- U vector of negative counts
- Invariant: 0 ≤ Total
- **`increment`** @i

		i		j		
	10	2	2	1		5
i	3	6	1	0		2
	5	0	4	2		1
j	1	0	2	1		0

Total: $\sum_i R[i][i] - \sum_i U[i]$

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Bounded Counter

- N Replicas
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Total: $\sum_i R[i][i] - \sum_i U[i]$

Rights @i: $R[i][i] + \sum_j R[j][i] - \sum_{j \neq i} R[i][j] - U[i]$

[Valter Balegas, 2016]

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		i		j		U
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Total: $\sum_i R[i][i] - \sum_i U[i]$ 13

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[Valter Balegas, 2016]

Bounded Counter

- N Replicas
- R matrix of positive counts
- U vector of negative counts
- Invariant: $0 \leq$ Total
- **`increment`** @i
- **`decrement`** @i
- **`transfer`** @i → j

		i		j		
	10	2	2	1		5
i	3	6	1	0		2
	5	0	4	2		1
j	1	0	2	1		0

Total: $\sum_i R[i][i] - \sum_i U[i]$ 13

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- **`increment`** @i
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		i		j		
	10	2	2	1		5
i	3	6	1	0		2
	5	0	4	2		1
j	1	0	2	1		0

$$\texttt{merge}((M_0, V_0), (M_1, V_1)) = (\texttt{max}(M_0, M_1), \texttt{max}(V_0, V_1))$$

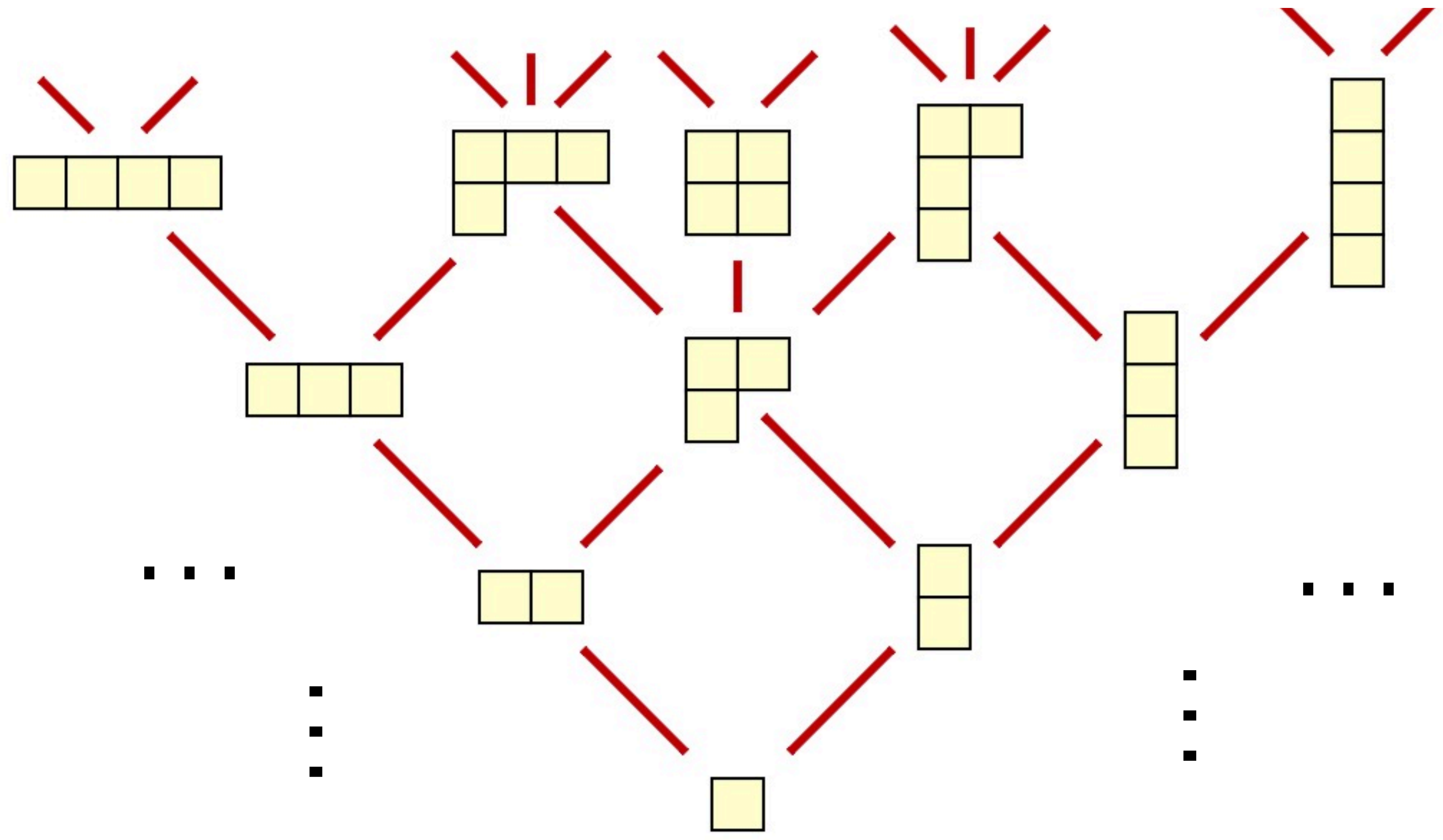
[Valter Balegas, 2016]

Checking Invariants

State-Based CRDTs

INVARIANTS FOR SB-CRDTS

- CRDT (lattice) constraints



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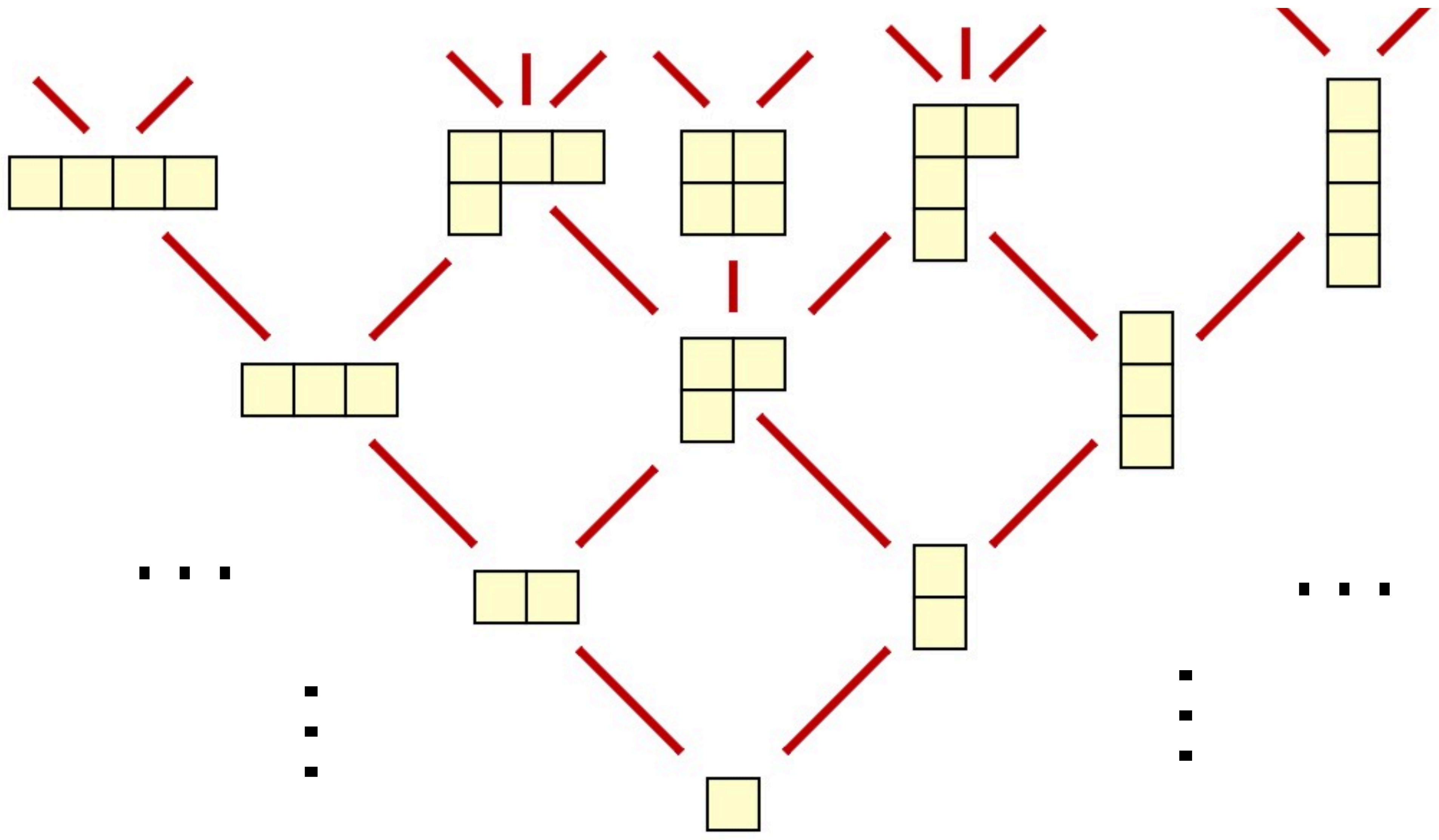

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  - Operations are inflations

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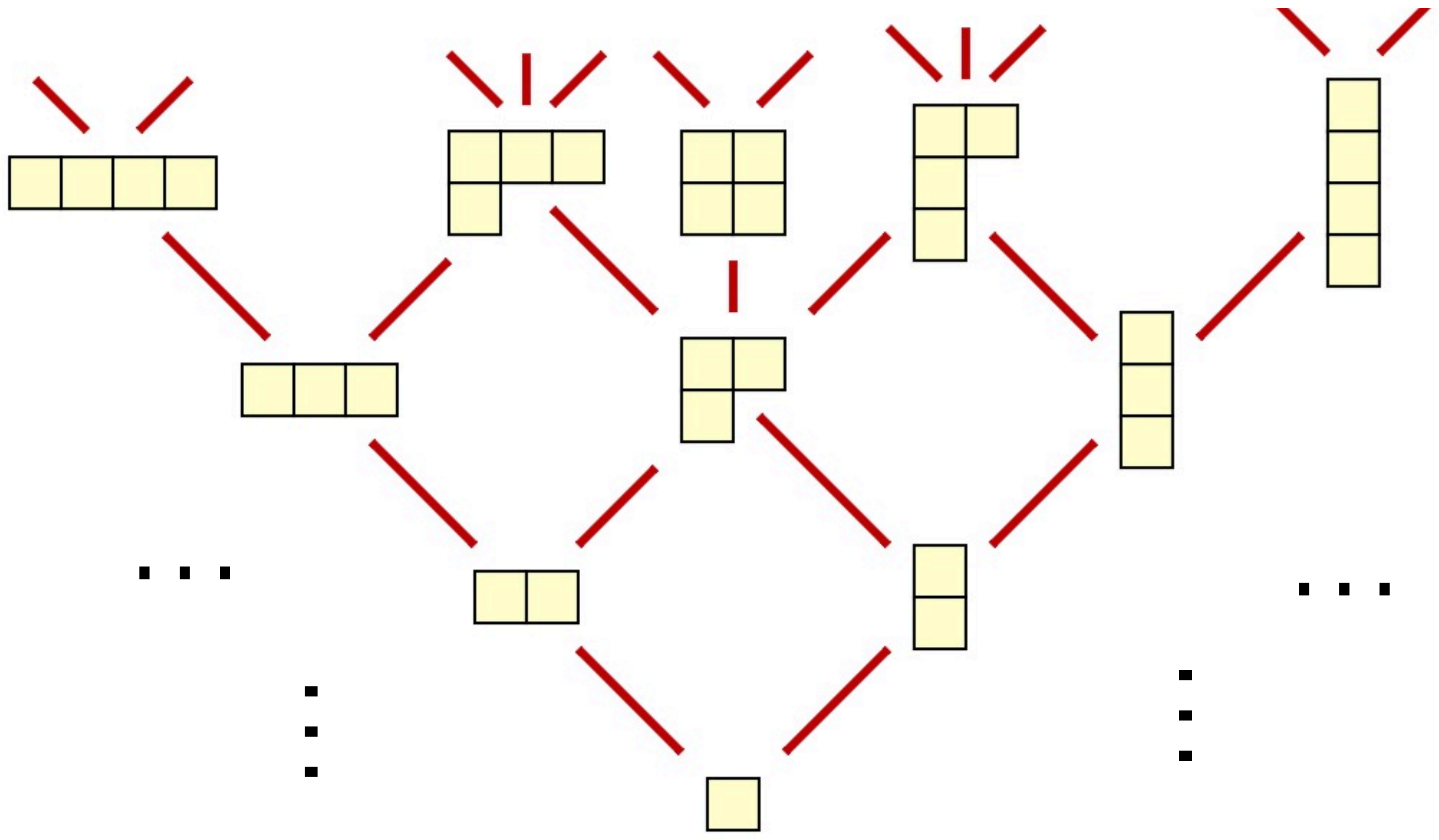

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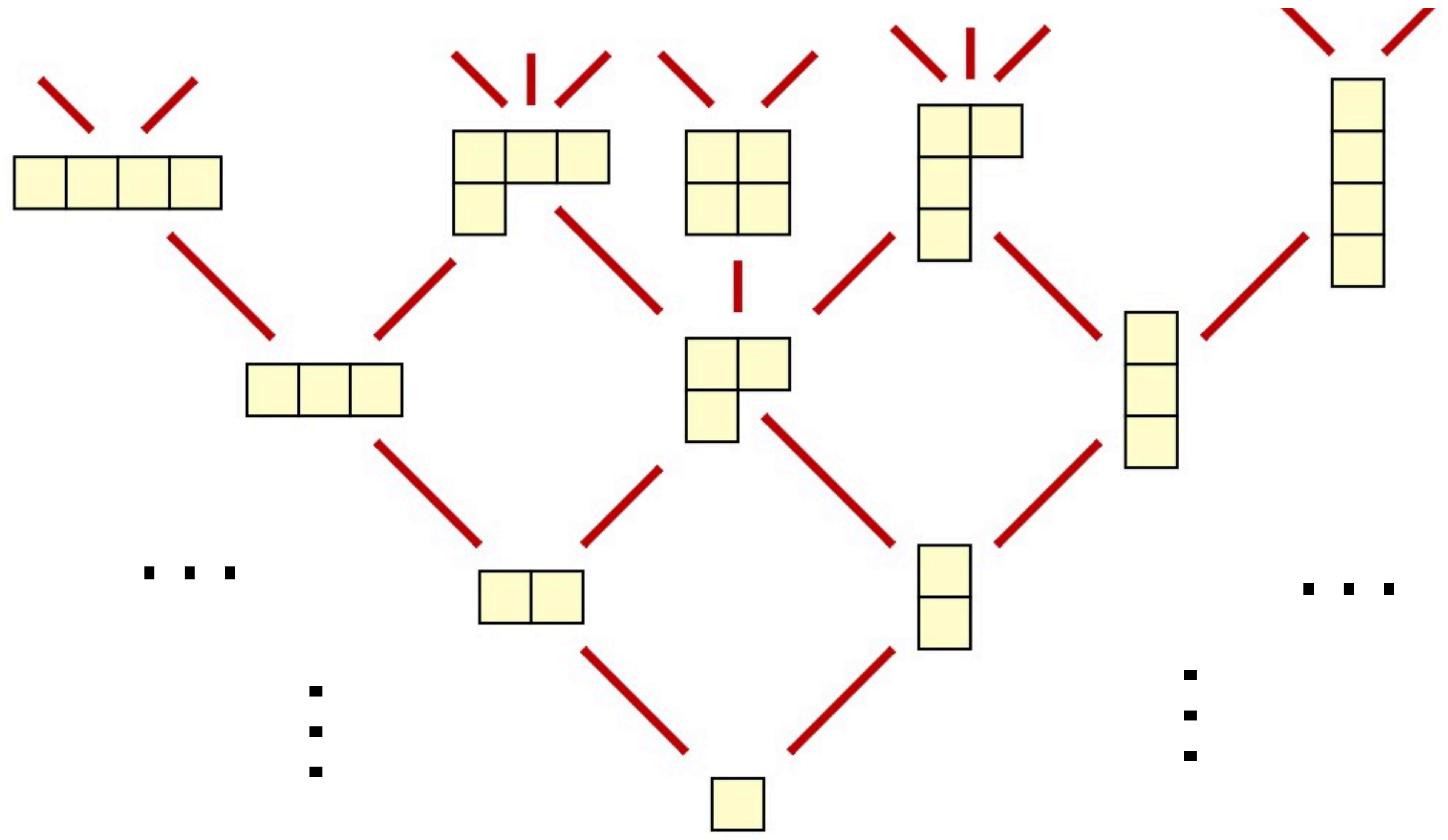
  - **`merge`** is join (LUB)

$$\forall \sigma, \sigma',\ \mathtt{merge}(\sigma, \sigma') = \sigma'' \Rightarrow \sigma'' = \mathsf{LUB}_{\sqsubseteq}(\sigma, \sigma')$$

Invariants for SB-CRDTs

- Invariant constraints



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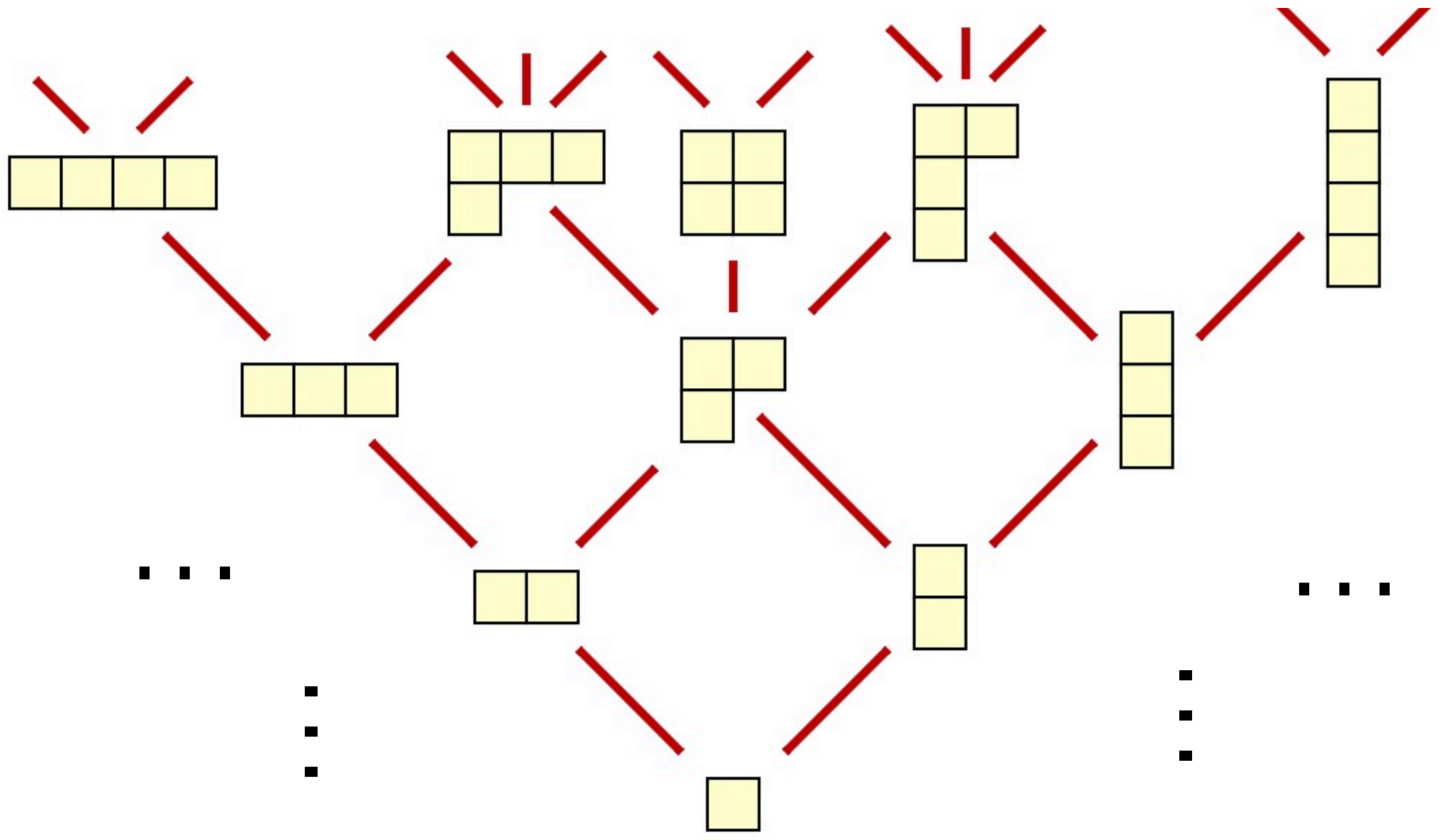
- Invariant constraints
  - Operations preserve the invariant



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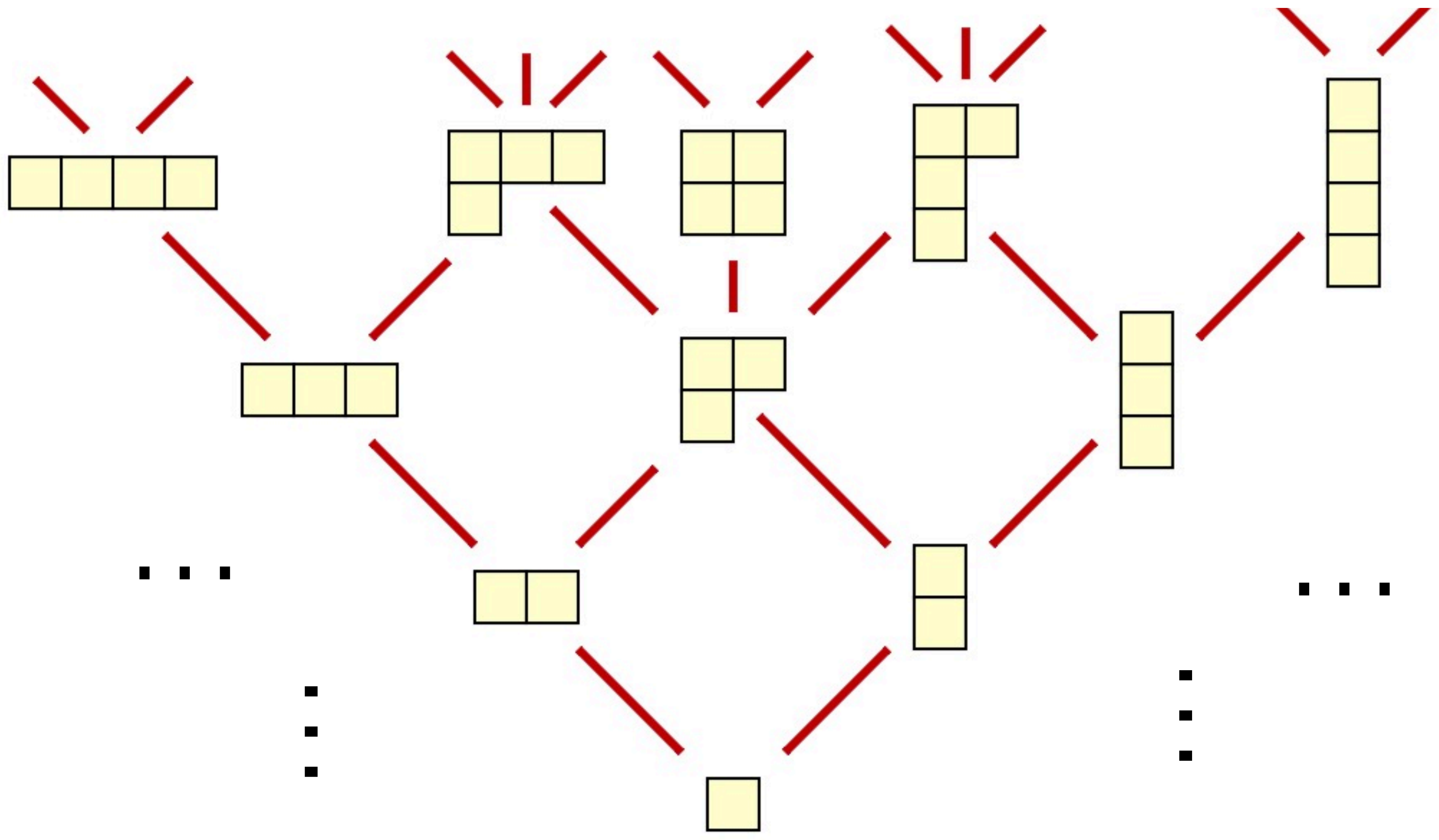

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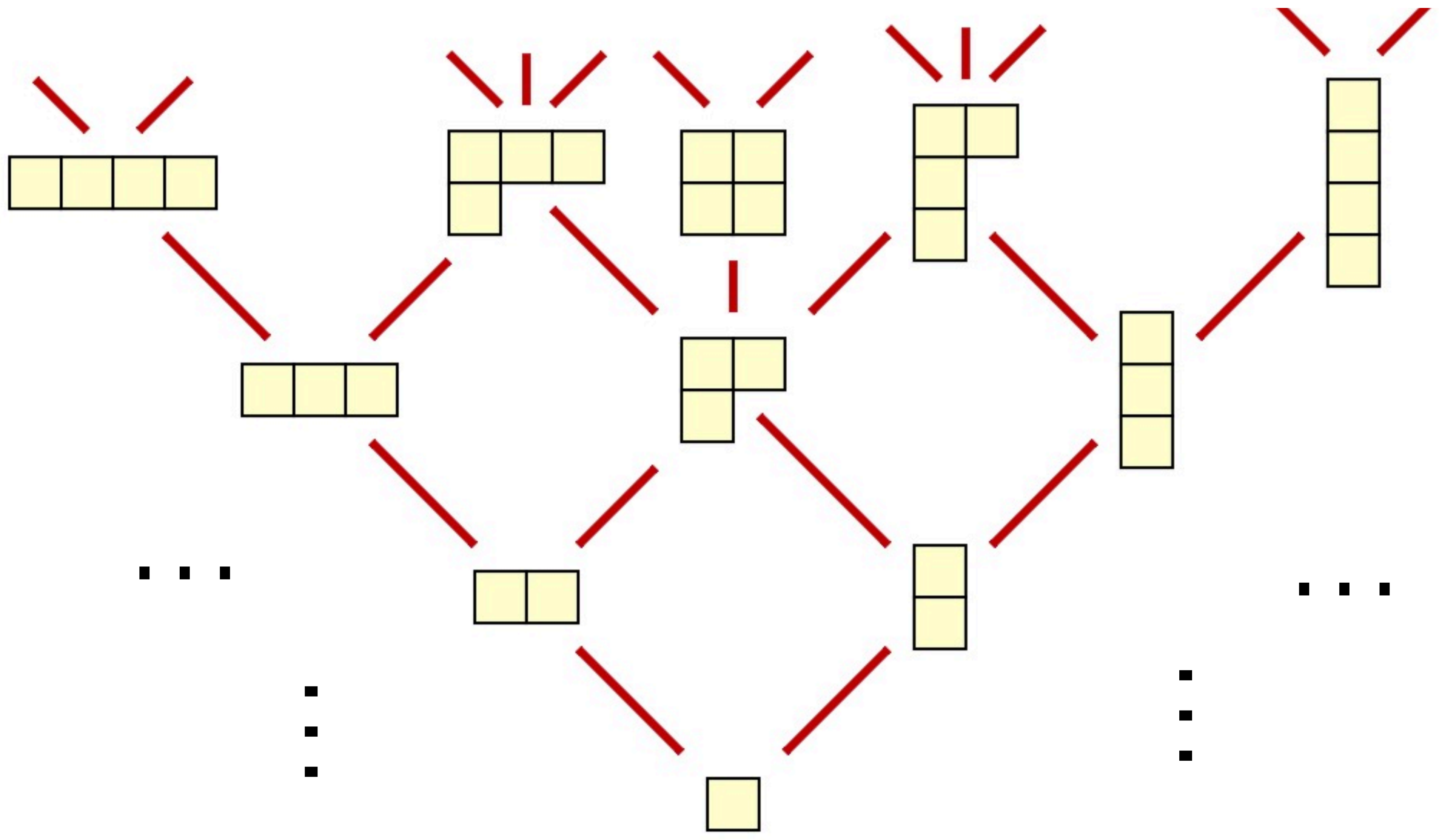

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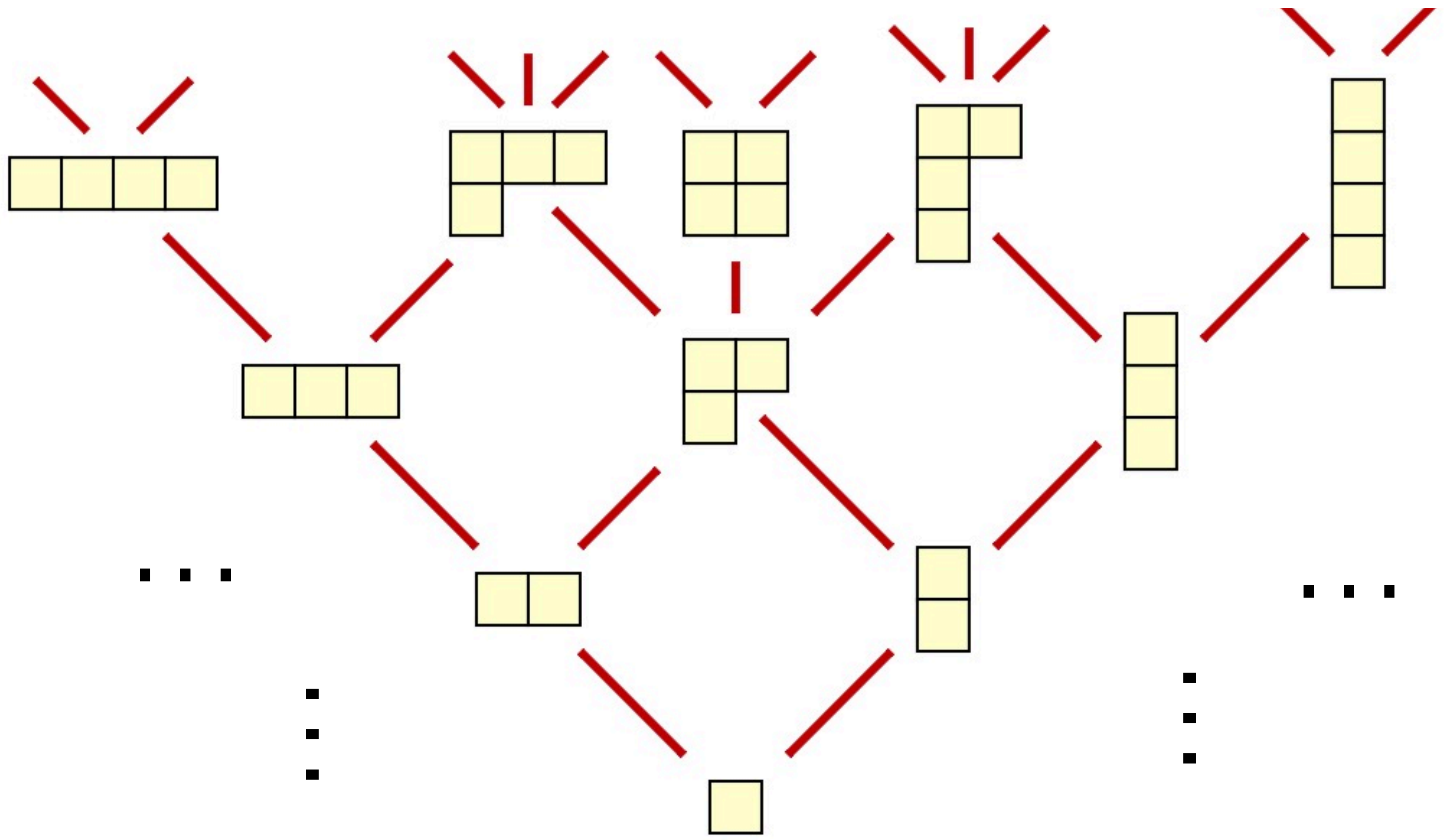

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Eg: Bounded Counter

$$\mathsf{Total}(\sigma) \geq 0 \wedge \mathsf{Total}(\sigma') \geq 0$$

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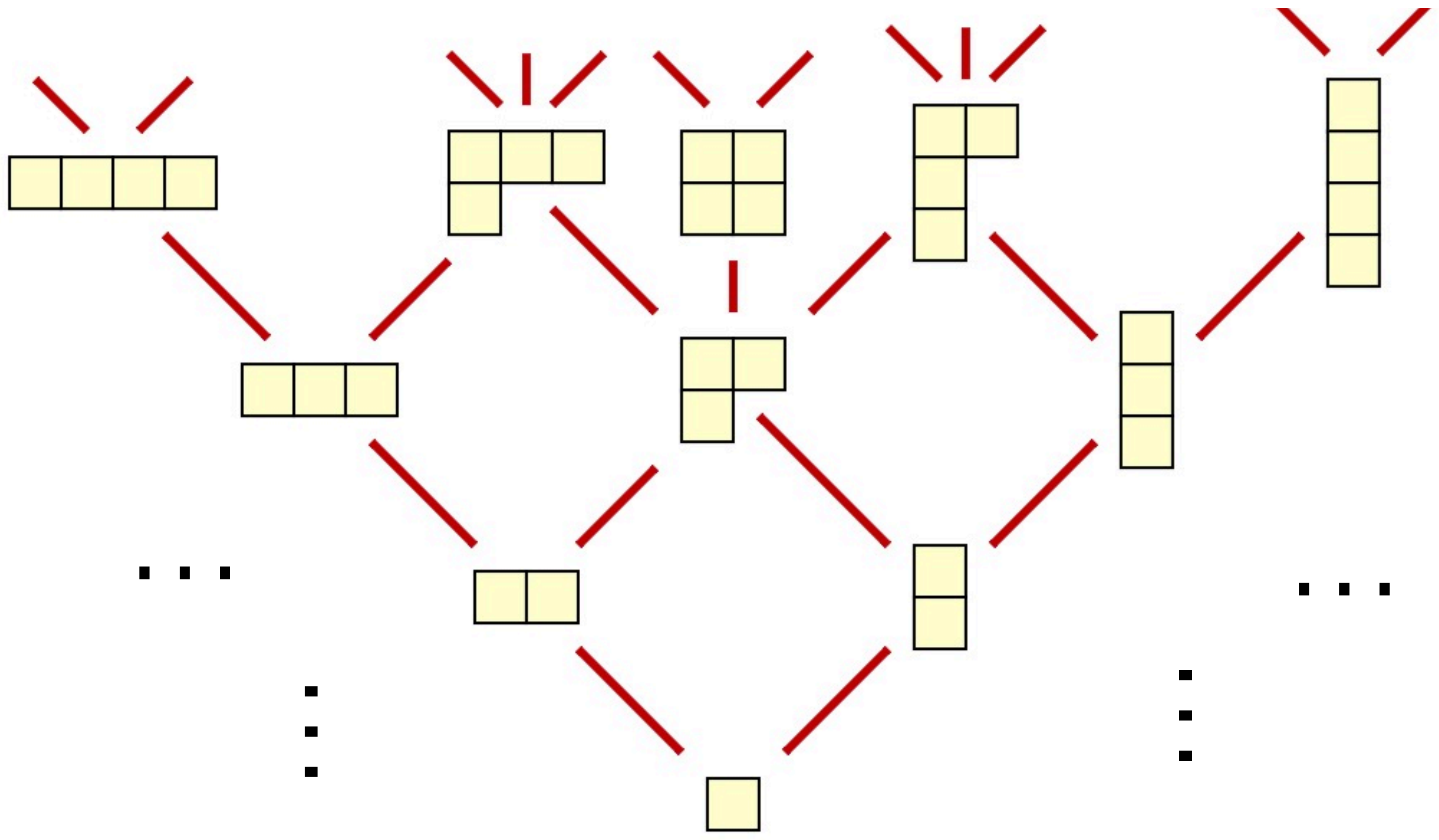

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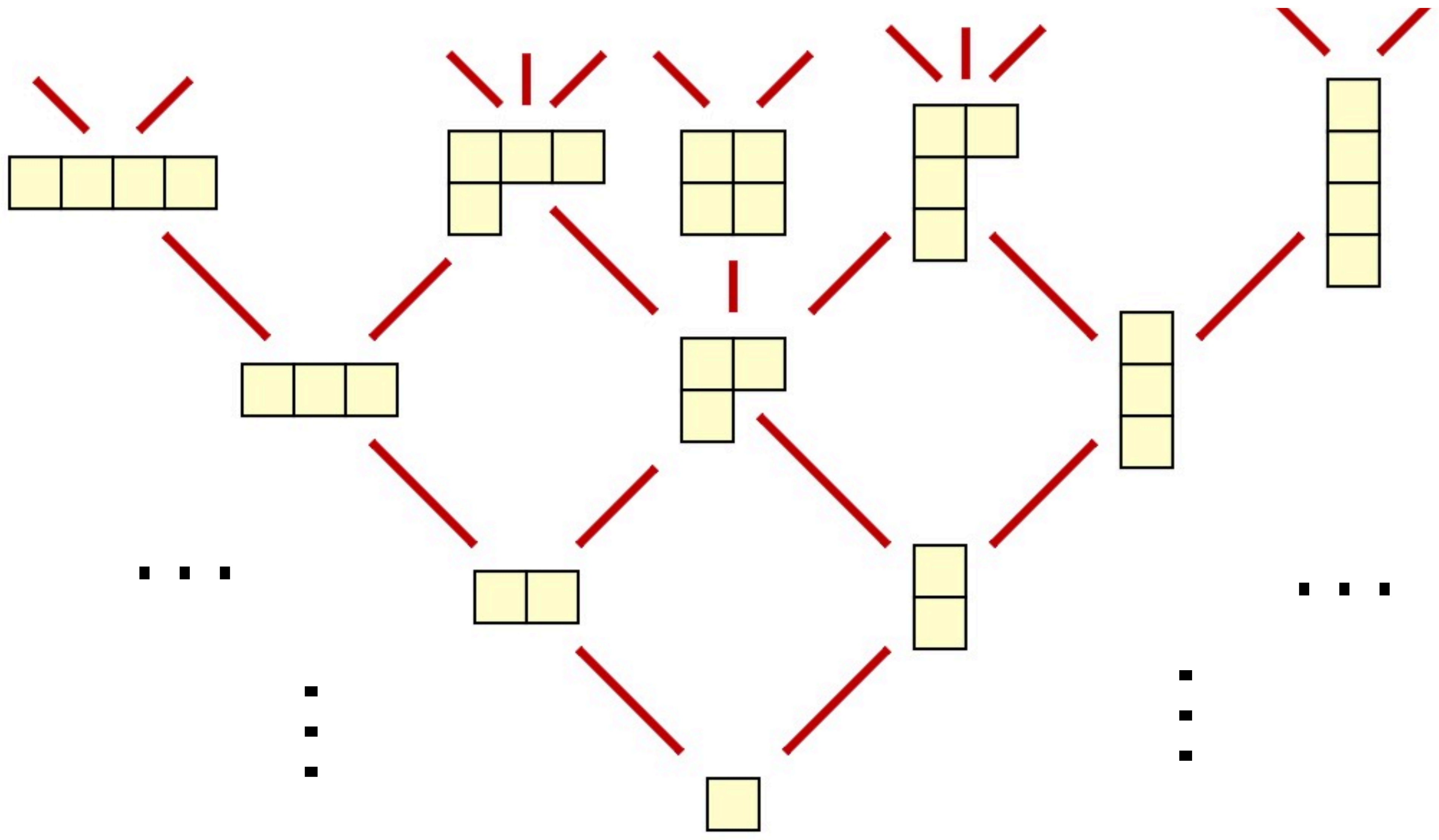

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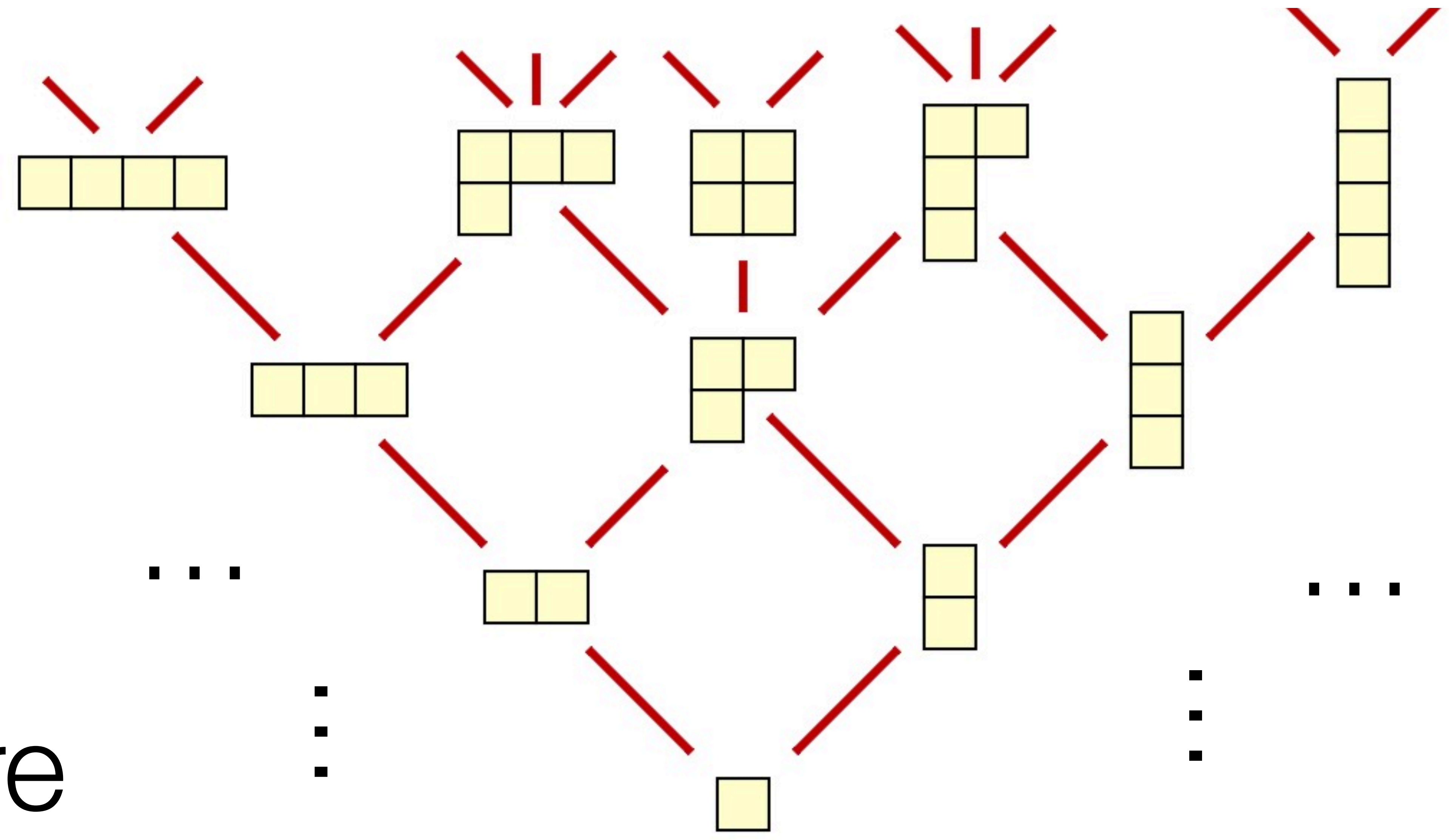

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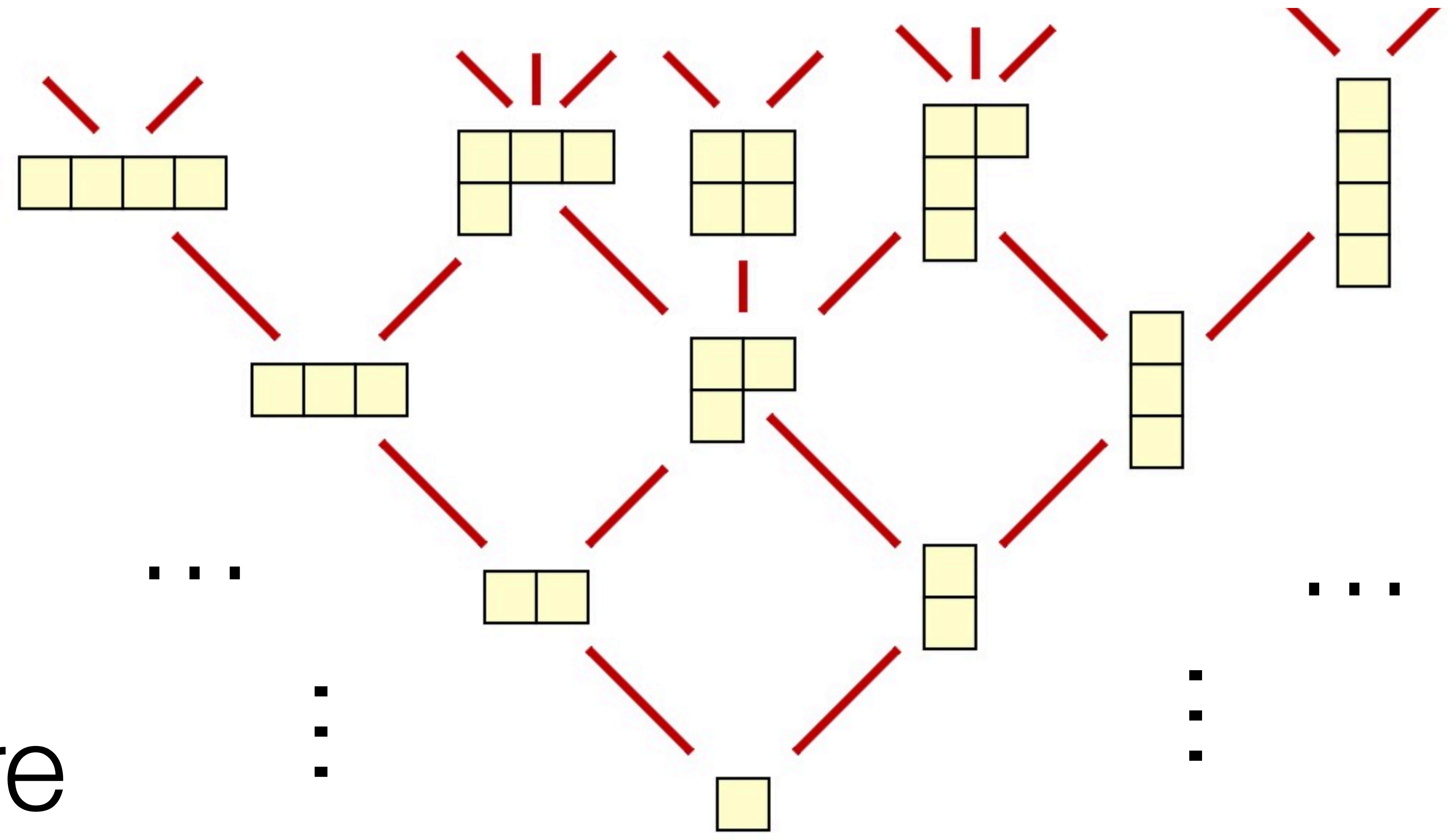

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  - **`merge`** Pre is an invariant

Example: Token Implementation

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```
transfer((t,V),r_o):
  assert(V[r] = 1 ∧ (∀t_o ≠ t, t≥t_o))
  t = t+1
  V[r_s] = 0 # self
  V[r_o] = 1 # other
```

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merge((t,V),(t_o,V_o)):
  t = max(t,t_o)
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- Merge precondition: $\mathsf{Pre}_{\mathtt{merge}}((\mathtt{t}_s, \mathtt{V}_s), (\mathtt{t}_o, \mathtt{V}_o)) = \mathsf{Inv}(\mathtt{t}_s, \mathtt{V}_s) \wedge \mathsf{Inv}(\mathtt{t}_o, \mathtt{V}_o) \wedge$
  $(\mathtt{t}_s = \mathtt{t}_o \Rightarrow \mathtt{V}_s = \mathtt{V}_o) \wedge$
  $(\mathtt{V}[\mathtt{r_s}] = 1 \Rightarrow \mathtt{t}_s \geq \mathtt{t}_o)$

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- Input
  - Definition of Order
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Tool Support

- Input

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~/r/c/c/s/s/soteria (origin/rewriting±) ▶ python3 soteria.py specs/token.spec
INFO     : ************ token ***************
INFO     : Checking the syntax
INFO     : Parsing the specification
INFO     : Checking the well-formedness of the specification
INFO     : Checking convergence
INFO     : Checking safety
INFO     : The specification is safe!!!
```

  - Implemented on top of Boogie
  - Performs the lattice and invariant checks
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Research Opportunities

- Beyond Simple Invariants
  - Pre/Post conditions of client programs using (1+) CRDTs
- Transactions + CRDTs
- Consistency Models: Eventual, Causal, Strong, …
- Synchronization?
- …