Proposition de stage de Master Recherche

Lieu :

Laboratoire Spécification et Vérification
École Normale Supérieure de Paris-Saclay, Cachan

Titre : WQO Finitary Powersets : Complexity and Algorithms

Description du stage :

WQOs and well-structured systems are a well-known tool used in particular for the verification of infinite-state models. A recent development in the area is the introduction by Schnoebelen, Schmitz, *et al.* of tools, concepts, and techniques for the complexity analysis of WQO-based algorithms (see "Mathematical foundations of the theory of infinite transition systems", Course M2-9-1 at MPRI). One of the outcomes of these developments is an analysis of the maximal length of bad sequences over various WQOs, packaged into so-called Length Function Theorems [CS08, FFSS11, SS11]. These results are finitary versions of the maximal order type results described, e.g., in [DSS17].

It is well-known that $(\mathcal{P}_f(A), \sqsubseteq_H)$ is a WQO when A is, where $\mathcal{P}_f$ is the *finitary powerset* operator that collects all the finite subsets of a set.[1] The WQO $\mathcal{P}_f(A)$ is "smaller" than the larger WQO $\mathcal{M}_f(A)$ that collects all finite *multisets* over A. It is smaller, but it is less well understood. For example one cannot derive the order type of $\mathcal{P}_f(A)$ from the order type of A the way it can be done for $\mathcal{M}_f(A)$. And there does not yet exists a specialized Length Function Theorem for WQOs of the form $\mathcal{P}_f(A)$ that would support the complexity analysis of programs relying on $\mathcal{P}_f$. One expects such an analysis to provide tighter complexity upper bounds than naive analyses based on overapproximating with e.g. $\mathcal{M}_f(A)$ or even A^*.

The goal of this Master Internship is to understand the structure of $(\mathcal{P}_f(A), \sqsubseteq_H)$ as a function of $(A, \leq)$ with the goal of developing Length Function Theorems for (cases of) $\mathcal{P}_f(A)$.

Some intermediary objectives along the way are :

1. Understanding the maximal length of controlled bad sequences over $\mathcal{P}_f(\mathbb{N}^k)$ when $\mathbb{N}^k$ is ordered by $\leq_\times$ and when it is ordered by $\leq_{\text{lexico}}$.
2. Understanding the interaction between $\mathcal{P}_f$ and other constructions, e.g., by showing isomorphisms and reflections like $\mathcal{P}_f(A \sqcup B) \equiv \mathcal{P}_f(A) \times \mathcal{P}_f(B)$.
3. Developing robust encodings based on powerset constructs for Hardy sequences, e.g., by adapting the techniques used in [Sch10, HSS12, HSS13].

This research project should suit a theoretically-minded student with some taste for abstract algorithmic constructions like what is encountered in basic courses on recursion theory and computational complexity. It is is an ideal opportunity for starting a PhD thesis in the INFINI group at LSV.

1. See exercise 1.10.(5) in the lecture notes for the M-2-9-1 course.

Contact :

Ph. Schnoebelen
Tél : (+33/0) 147 407 530
Email : phs@lsv.ens-cachan.fr

Références

[CS08] P. Chambart and Ph. Schnoebelen. The ordinal recursive complexity of lossy channel systems. In LICS 2008, pages 205–216. IEEE Comp. Soc. Press, 2008.

[DSS17] M. Džamonja, S. Schmitz, and Ph. Schnoebelen. On ordinal invariants in well quasi orders and finite antichain orders. arXiv :1711.00428 [math.LO], November 2017.

[FFSS11] D. Figueira, S. Figueira, S. Schmitz, and Ph. Schnoebelen. Ackermannian and primitive-recursive bounds with Dickson's lemma. In LICS 2011, pages 269–278. IEEE Comp. Soc. Press, 2011.

[HSS12] S. Haddad, S. Schmitz, and Ph. Schnoebelen. The ordinal-recursive complexity of timed-arc Petri nets, data nets, and other enriched nets. In LICS 2012, pages 355–364. IEEE Comp. Soc. Press, 2012.

[HSS13] Ch. Haase, S. Schmitz, and Ph. Schnoebelen. The power of priority channel systems. In CONCUR 2013, volume 8052 of LNCS, pages 319–333. Springer, 2013.

[Sch10] Ph. Schnoebelen. Revisiting Ackermann-hardness for lossy counter machines and reset Petri nets. In MFCS 2010, volume 6281 of LNCS, pages 616–628. Springer, 2010.

[Sch16a] S. Schmitz. Complexity hierarchies beyond Elementary. *ACM Trans. Computation Theory*, 8(1), 2016.

[SS11] S. Schmitz and Ph. Schnoebelen. Multiply-recursive upper bounds with Higman's lemma. In ICALP 2011, volume 6756 of LNCS, pages 441–452. Springer, 2011.