Sujet de stage de recherche / M2

Titre

FAC spaces.

Encadrants

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Description du sujet

A Noetherian space is a topological space in which every open is compact [Gou13, Section 9.7]. This is the blunt mathematical definition, and if you have not yet run away in panic, then you are perhaps the right person for the job.

If you *have* run away in panic, come back ! This is not just mathematics, this is computer science, and it has applications. The author of these lines realized a few years ago that they could be used in the context of verification of infinite-state systems [Gou10], extending the already rich toolkit of *well structured transition systems* (WSTS), based on well quasi-orders. WSTS are infinite state transition systems for which many properties of interest in verification are decidable, and if we replace well quasi-orders by the more general notion of a Noetherian space, we obtain larger classes of programs that can be verified automatically.

Even more recently, Alain Finkel realized that one can extend the class of WSTS in yet another direction, the so-called *well-behaved transition systems* (WBTS) (`https://arxiv.org/abs/1608.02636`). The change consists in replacing well quasi-orders by the larger class of *FAC orders*, that is, orders with no infinite antichain.

FAC orders and well quasi-orders are order-theoretic notions, while Noetherian spaces are a topological notion. A natural question which the interested student should answer is :

> What is the proper common generalization of FAC orders and Noetherian spaces ?

The key property of FAC orders is the following result, attributed to Erdős and Tarski [ET43] : a poset $(X, \leq)$ if FAC if and only if the downwards-closed subsets are finite unions of ideals. This is the property that makes A. Finkel's algorithm work.

Hence a good attempt at the notion of FAC space should be, following a standard glossary between order theory and topology : a *FAC space* is a topological space in which the closed subsets are exactly the finite unions of irreducible closed subsets. However, contrarily to FAC orders, no other characterization is known. That would be needed to understand which spaces (of configurations, in the view of verifying infinite transition systems) are FAC and which are not.

One possibility would be to imitate Kabil and Pouzet's proof of the Erdős-Tarski theorem [KP92, Lemma 5.3] and try to reproduce it in a topological setting.

There are many other proofs of that theorem in the literature, and the student should read them and compare them [ET43, Bon75, Pou79, PZ85, Fra86, LMP87], in the light of a possible extension to the topological case.

This subject naturally leads to a PhD thesis on the verification of new computer systems, not amenable to previous methods.

Remarques

This internship requires some familiarity in well-quasi-orderings and topology, with a view toward verification. Proficiency is not required, but a desire to learn is.

Références

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