

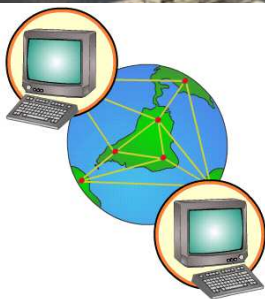
# FAST: Theory and practice of acceleration

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6 mars 2006

# Verification of reactive systems



## Reactive systems

- Software and/or hardware
- Autonomous
- **Critical**



Processors embedded in cars

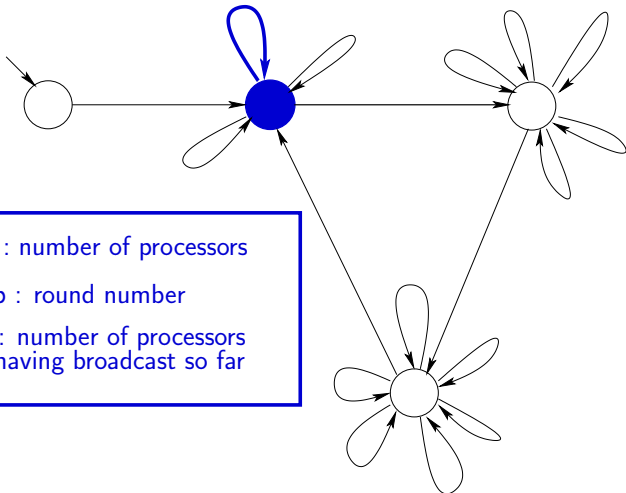
## The TTP protocol

- fault tolerance
- ensure no fault will propagate

TTP is supported by Audi, PSA, Renault, ...

# A model of the TTP [Bouajjani-Merceron 2002]

```
If  $d < N$  Do  
   $d := d + 1; C_p := C_p + 1$   
End Do
```

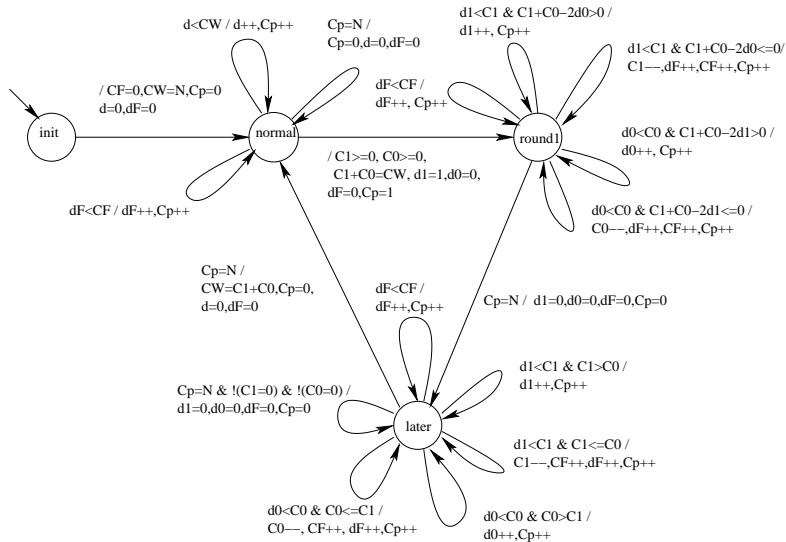


$N$  : number of processors

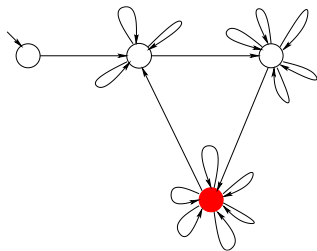
$C_p$  : round number

$d$  : number of processors  
having broadcast so far

# A model of the TTP [Bouajjani-Merceron 2002]



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## Question

In the **red location**, does

$$C_p = N \Rightarrow (C_0 = 0 \vee C_1 = 0)?$$

## Objective

Automatic verification for any value of  $N$

## Counter systems

- we study mathematical models of concrete systems
- automata extended with **unbounded integer variables**

## Properties to check

Reachability properties = properties of reachable configurations.

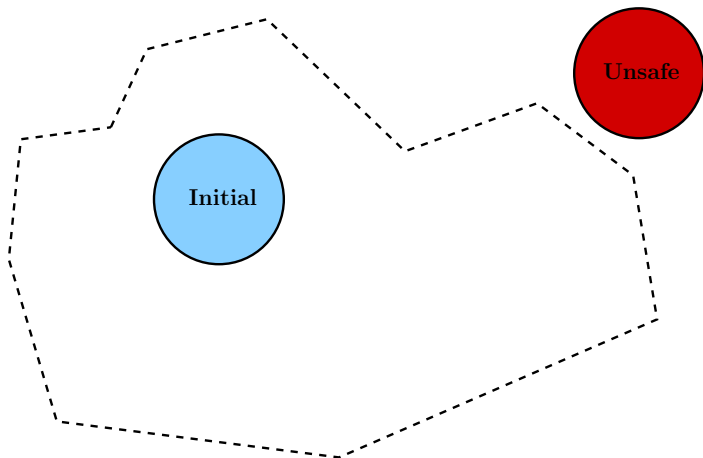
- useful: mutual exclusion, deadlock freedom, ...
- easy to check from the reachability set.

## Problems

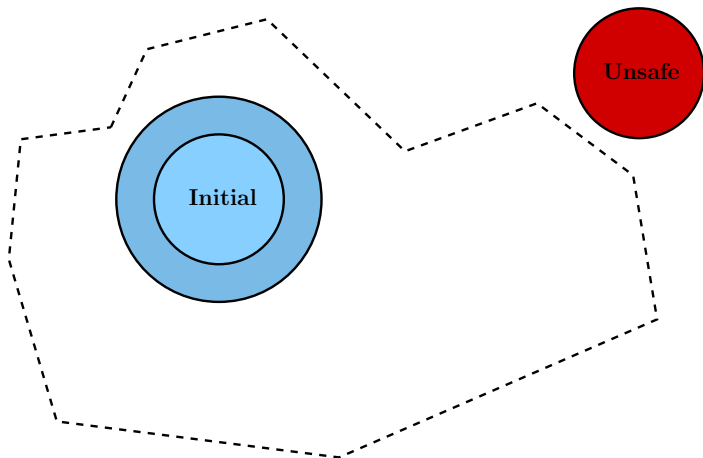
- Undecidable for two counters with  $(+1, -1, \stackrel{?}{=} 0)$
- One of the issues: **infinite** reachability set



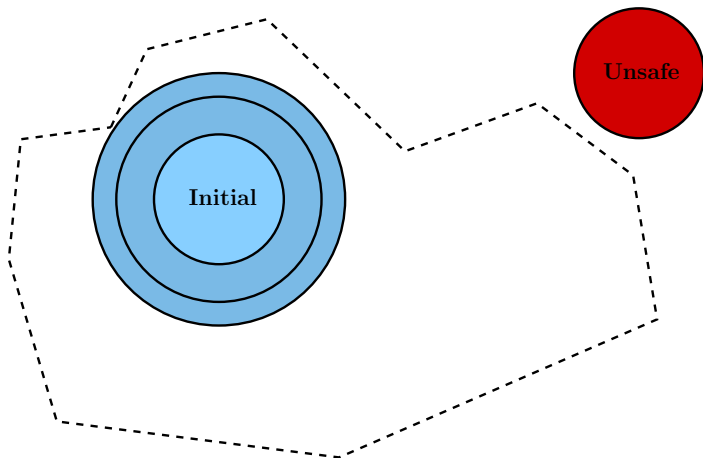
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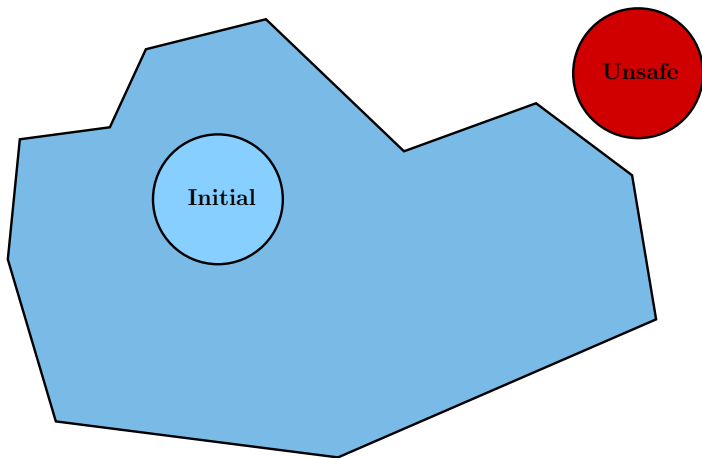
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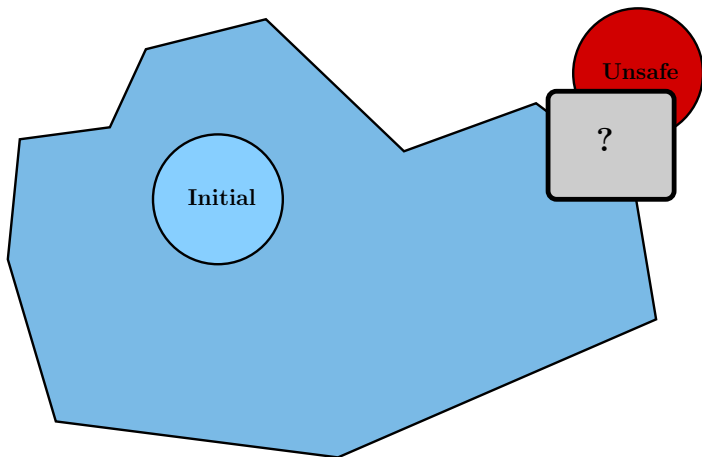
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# Back to the finite case



Enumerative methods do not work any more

## Algorithms for decidable subclasses

- Petri nets,
- timed automata, ...

or **Semi-algorithms to compute the reachability set**

- more expressive/realistic systems
- no guarantee of termination, we hope **practical termination**
- Extend iterative computation for infinite sets
- (**symbolic model-checking**)

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Issue 1: infinite set of reachable configurations.

Idea = manipulate infinite sets of configurations

- sets are represented **symbolically**.
- need basic symbolic operations  $\text{POST}$ ,  $\sqcup$ ,  $\sqsubseteq$ .

Example: intervals of integers

- Formula  $\phi_x : \{x > 5\}$  means that  $x$  ranges over all integers greater than 5
- After transition  $\xrightarrow{y:=x+1}$ , the possible values of  $y$  are exactly represented by  $\phi_y = \{y > 6\}$

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# First (and basic) symbolic procedure

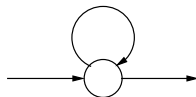
Iterative computation of  $\text{post}_S^*(X_0)$

- 1  $X \leftarrow X_0$
- 2 If  $\text{POST}(X) \subseteq X$  Goto 5
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- 4 Goto 2
- 5 Return  $X$

Issue 2: termination is scarce

because of circuits in the control graph ...

If  $x \geq 0$  Do  $x \leftarrow x + 2$



If  $X_0 = \{0\}$  then

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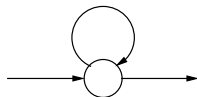
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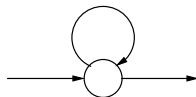
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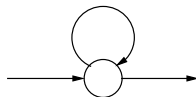
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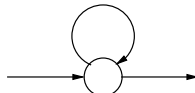


If  $X_0 = \{0\}$  then  $X = \{0, 2, \dots, 2k\}$

## Circuit acceleration

Enhance the convergence of the iterative symbolic procedure by computing in one step **the iteration** of a **sequence of transitions** (circuit).

If  $x \geq 0$  Do  $x \leftarrow x + 2$

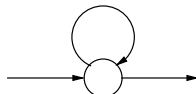


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If  $x \geq 0$  Do  $x \leftarrow x + 2$



If  $X_0 = \{0\}$  then  $\text{post}^*(X_0) = 2.\mathbb{N}$ , in one step.



## State-of-the-art in 2002

(*Karp-Miller 1969*)

(*Fribourg 1990*)

[Boigelot-Wolper 1994],

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## Remarks

- 1 very different techniques, no unifying view (comparison?)
- 2 still a gap between acceleration algorithm and fixpoint computation (how to select circuits?)

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Results : Unifying framework encompassing most of acceleration theorems [ATVA'05]

Issue : How to select circuits?

Results : Maximal heuristic, efficient in practice [CAV'03,ATVA'05]

Issue : Improve practical efficiency of acceleration

Results : “Convex acceleration” algorithm [TACAS'04]

Issue : Experimentations

Results : implementation of FAST [CAV'03],  
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## Presburger arithmetics

First order arithmetics without  $\times$ -operator

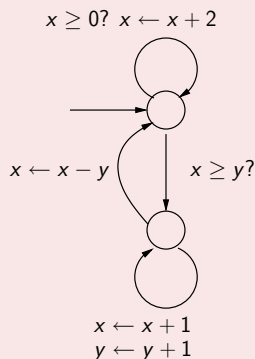
$$\phi ::= t \leq t \mid \neg\phi \mid \phi \vee \phi \mid \exists k.\phi \mid \text{true}$$

$$t ::= 0 \mid 1 \mid y \mid t - t \mid t + t.$$

Presburger set = set of solutions of a Presburger formula.

# Counter systems

- finite set of  $m$  variables  $x, y, z, \dots$  over  $\mathbb{N}$
- finite set of P-affine functions  $f = (M, v, G)$ 
  - $G \subseteq \mathbb{N}^m$  Presburger guard
  - $M$  square matrix
  - $v$  vector
- $\vec{var}' = f(\vec{var})$  iff
  - $\vec{var} \in G$
  - and  $\vec{var}' = M \cdot \vec{var} + v$



## Automata to recognize sets of integer vectors

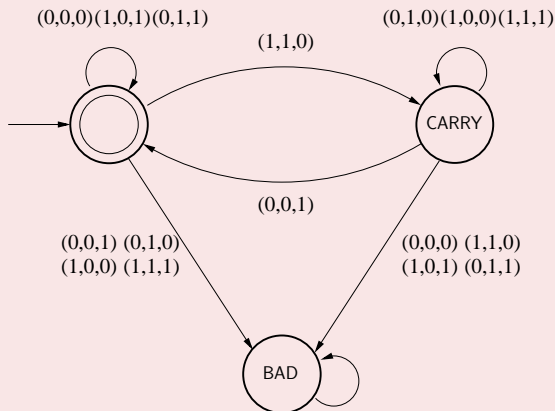
- a non-negative integer in basis 2 is a word over  $\{0, 1\}$
- automata recognize sets of words
- extensions
  - any integer: 2-complement encoding
  - vectors: tuples of letters, or variables entanglement

Presburger sets (and a little bit more) are recognized by automata.

Common operations on sets  $\longrightarrow$  standard operations on automata



# Symbolic representations: Automata



- DFA [Boudet-Comon 1996],
- NDD [Wolper-Boigelot 2000]).

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**Monoid of a function**  $f = (M, v, G): \{1, M, M^2, \dots, M^n, \dots\}$

Theorem [Finkel-Leroux 2002]

Let  $f = (M, v, G)$  a P-affine function with **finite monoid**. Then  $f^*$  is effectively defined by a Presburger formula

$$f^* = \{(x, x') \mid x \in G \wedge \exists k \geq 0. x' = \bar{f}^k(x) \wedge \forall i. 0 \leq i < k, \bar{f}^i(x) \in G\}$$

Building the formula is 3-EXP in  $|\mathcal{A}(G)|$ ,  $|v|_{\max}$ ,  $|M|_{\max}$  et  $m$ .

# Idea of the algorithm

- $f = (M, v, G)$  with finite monoid  $\langle M \rangle$ .
- $\bar{f} : \mathbb{Z}^m \rightarrow \mathbb{Z}^m, \forall x \in \mathbb{Z}^m, \bar{f}(x) = M.x + v$

- $\langle M \rangle$  finite, then  $\exists (a, b) \in \mathbb{N} \times \mathbb{N}$  such that  $M^{a+b} = M^a$
- We deduce that  $\forall n \in \mathbb{N}, \forall x \in \mathbb{Z}^m, \bar{f}^{a+n.b} = \bar{f}^a(x) + n.M^a.\bar{f}^b(0)$
- It comes that  $\bar{F} = \{(i, x, x') \in \mathbb{N} \times \mathbb{Z}^m \times \mathbb{Z}^m, x' = \bar{f}^i(x)\} \iff \bigvee_{r=0}^{a-1} \{(i, x, x') \mid x' = \bar{f}^r(x) \wedge i = r\} \bigvee_{r=0}^{b-1} \{(i, x, x') \mid \exists n \geq 0 (x' = \bar{f}^{a+r}(x) + n.M^{a+r}.\bar{f}^b(0)) \wedge (i = a + r + n.b)\}$

$$f^* = \{(x, x'), \exists i \geq 0, x' = f^i(x)\} \iff \{(x, x'), \exists i \geq 0 [(i, x, x') \in \bar{F} \wedge (\forall k (0 \leq k < i), \exists x'' \in G, (k, x, x'') \in \bar{F})]\}$$

## Convex translations [TACAS'04]

$f = (I_m, v, G)$  where  $I_m$  is the **identity matrix** and  $G$  **convex**

- No need to test if the predecessors are in the guard.
- The construction can be simplified.

## Theorem [TACAS'04]

Convex acceleration is quadratic in  $|\mathcal{A}(G)|$ .

## Faster acceleration ... complexity

parameter	magnitude	standard algorithm	convex algorithm
$ A(G) $	100.000	3-EXP	quadratic
$m$	5-50	3-EXP	EXP
$ V _{max}$	$\leq 10$	3-EXP	poly. in $m$
$ M _{max}$	$\leq 10$	3-EXP	= 1

$\mathcal{A}(f^*)$  = automaton representing  $f^*$  (transductor)

$ \mathcal{A}(f^*) $	Time (seconds)		Memory (MB)	
	Standard	Convex	Standard	Convex
16,766	10	7	31	13
26,409	5	2	17	18
41,950	18	10	52	30
190,986 (TTP)	50	9	400	140
380,332 (TTP)	↑↑↑	34	↑↑↑	534
?	↑↑↑	>900	↑↑↑	>500

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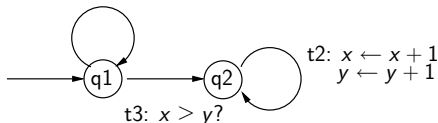


# What is computable with circuit acceleration?

## First answer

Flat system = at most 1 elementary circuit on each control node

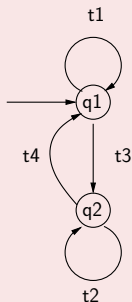
t1:  $x \geq 0?$   $x \leftarrow x + 2$



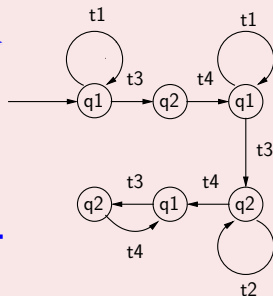
Circuit acceleration + flat  $S$  = computation of  $\text{post}_S^*(X_0)$

# Flattenings of non-flat systems

Système  $S$



Système  $S'$



$S'$  is a **flattening** of  $S$  iff  
 $S'$  is flat and  $S$  simulates  $S'$

$(S, X_0)$  is flattable iff  $\exists S'$  such that

- $S'$  is a flattening of  $S$
- $S$  and  $S'$  are equivalent for reachability.

Theorem 1 [ATVA'05]

$\text{post}^*(X_0)$  is computable by circuit acceleration iff  $(S, X_0)$  is flattable.

A restricted linear regular expression (rlre) over  $T$

$$\rho = w_1^* w_2^* \dots w_n^*, \text{ where } w_i \in T^*.$$

$\text{post}(\rho, X) =$  configurations reachable following transitions in  $\rho$

Theorem 2 [ATVA'05]

$\text{post}^*(X_0)$  is computable by circuit acceleration iff  $\exists$  a rlre  $\rho$  over  $T$  such that  $\text{post}^*(X_0) = \text{post}(\rho, X_0)$ .

Remark:  $\text{post}(\rho, X)$  is computable with circuit acceleration

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Remark:  $\text{post}(\rho, X)$  is computable with circuit acceleration

# Selection Heuristic (I)

Input:  $(S, X_0)$

- 1  $X \leftarrow X_0; k \leftarrow 0$
- 2  $k \leftarrow k + 1$
- 3 **Lunch**
- 4 If  $\text{post}(X) \subseteq X$  Goto 10
- 5 Enumerate the next  $\rho$  rlre over  $T$
- 6  $X \leftarrow \text{post}(\rho, X)$
- 7 Goto 4
- 8 **In parallel with**
- 9 When *Watchdog* stops Goto 2
- 10 Return  $X$

**maximal procedure:** terminates iff  $(S, X_0)$  is flattable

PB1(time) find quickly a good rlre

PB2(space) avoid as much as possible unnecessary expensive steps of computations

# Selection Heuristic (I)

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- 2  $k \leftarrow k + 1$
- 3 **Lunch**
- 4 If  $\text{post}(X) \subseteq X$  Goto 10
- 5 *Choose fairly*  $w \in T^{\leq k}$
- 6  $X \leftarrow \text{post}(w^*, X)$
- 7 Goto 4
- 8 **In parallel with**
- 9 **When** *Watchdog* stops **Goto** 2
- 10 Return  $X$

The procedure is still maximal

PB1(time) partitioning + *Watchdog*

PB2(space) *Choose*



## Results

The selection heuristic design is reduced to designing

- *Choose* (a standard solution is given)
- *Watchdog* (a standard solution is given)

The obtained procedure is then

- **maximal**
- **efficient** : good results on counter systems (cf. FAST)

$|T|^{\leq k}$  may be exponential in  $k$ .

Idea = reduce  $|T|^{\leq k}$  by removing redundant functions.

Three reductions:

- **union-reduction** [Finkel-Leroux 2002]
  - if  $f = (M, v, G_1)$  and  $g = (M, v, G_2)$ ,
  - let  $h = (M, v, G_1 \vee G_2)$
  - then  $(f + g)^* = h^*$
- **commutation-reduction** [CAV'03]
  - if  $f$  and  $g$  commute then  $f^*g^* = (f \cdot g)^* = (g \cdot f)^*$
- **conjugacy-reduction** [ATVA'05]
  - $(f_2 \cdot f_3 \cdot f_1)^* = I_d + f_2 \cdot f_3 \cdot (f_1 \cdot f_2 \cdot f_3)^* \cdot f_1$

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# Practical results

system	$ T $	$k$	$ C^{\leq k} $	U	Cm	Cj	U+Cm
csm	13	1	14	14	14	14	14
	<b>13</b>	<b>2</b>	<b>183</b>	<b>103</b>	<b>57</b>	<b>99</b>	<b>35</b>
consistency	8	1	9	9	9	9	9
	8	2	68	45	44	39	30
	<b>8</b>	<b>3</b>	<b>484</b>	<b>172</b>	<b>299</b>	<b>178</b>	<b>98</b>
swimming pool	6	1	7	7	7	7	7
	6	2	43	21	24	25	16
	6	3	259	56	114	97	28
	<b>6</b>	<b>4</b>	<b>1555</b>	<b>126</b>	<b>614</b>	<b>421</b>	<b>47</b>
	6	5	9331	252	3591	1977	86

U, Cm ,Cj : reductions (union, commutation, conjugacy)

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- 5 **The tool FAST**
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The previous results are implemented in FAST

### FAST works well in practice

- successfully verify 80% of 40 infinite systems [CAV'03].
- first automatic verification of TTP [TACAS'04]
- first automatic verification of CES



# Technological comparison

	ALV	LASH	FAST	TREX
system	relational	affine		restricted
symb. rep	automata			arith. + pdbm ( <i>undec.</i> $\sqsubseteq$ )
acceleration	no	circuits		<b>circuits</b> ( <i>partial.rec.</i> )
circuit selection		no	yes	yes, $\leq k$

# Practical comparison

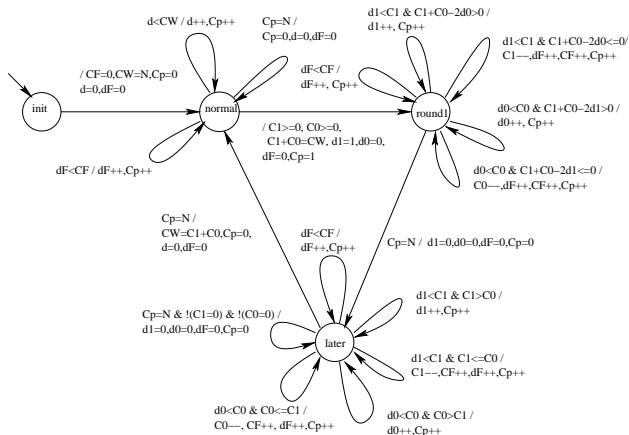
System	ALV	LASH	FAST	$k$	TREX
RTP (bounded)	T	T	T	1	T
Lamport (bounded)	T	T	T	1	T
Dekker (bounded)	T	T	T	1	T
ticket 2	T	T	T	1	T
kanban	↑	T	T	1	T
multipoll	↑	T	T	1	↑
prod/cons (2)	↑	T	T	1	-
ttp	↑	T	T	1	-
prod/cons (N)	↑	↑	T	2	-
lift control, N	↑	↑	T	2	T
train	↑	↑	T	2	T
csm, N	↑	↑	T	2	↑
consistency	↑	↑	T	3	-
swimming pool	↑	↑	T	4	↑
pncsa	↑	↑	↑	?	↑
incdec	↑	↑	↑	?	↑
bigjava	↑	↑	↑	?	↑

T: success within 20 minutes  
↑: no success within 20 minutes

$k$ : circuit length for FAST  
-: not an input of TREX

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# Verification of TTP by FAST



1 error [TACAS'04]

16 transitions, 9 variables, complex guards

- automatic verification
- Pentium 4 2.4 GHz, 1 Gbyte RAM : 940 sec. and 73 Mbytes.

Other tools:

- ALV does not terminate
- LASH terminates when good circuits are provided
- TTP does not fit T<sub>REX</sub> input model.

2 errors [TACAS'04]

20 transitions, 18 variables, even more complex guards

- standard acceleration does not work
- convex acceleration + overapproximation.

## 1 error [TACAS'04]

16 transitions, 9 variables, complex guards

- automatic verification
- Pentium 4 2.4 GHz, 1 Gbyte RAM : 940 sec. and 73 Mbytes.

Other tools:

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- TTP does not fit T<sub>REX</sub> input model.

## 2 errors [TACAS'04]

20 transitions, 18 variables, even more complex guards

- standard acceleration does not work
- convex acceleration + overapproximation.

# The CES protocol - presentation

- Supported by Philips
- multimedia streaming
- ensures reliable communications over lossy channels

Jonathan Billington and Lin Liu [Billington-Liu 2002]

- Colored Petri net modeling of the CES,
- infinite system, counters and **queues of parameterized length**
- (complex) proofs of many properties of the CES (ex: size of the reachability set w.r.t. the buffer lengths)

## Modeling issues

FAST does not handle queues.

- queues **simulated** by counters,
- **correctness of the simulation** is expressed as a reachability property of the counter system, and it is **checked by FAST** automatically.

## Results

Properties proved in [Billington-Liu 2002] are checked easily.



# Verification of pointer systems (work in progress)

## Manual management of memory resources (language C)

- memory heap = collection of memory cells
- a cell contains: data or address
- addresses  $\in \{\text{valid, invalid, NULL}\}$
- primitives: `new`, `free`, successor

## Common errors

- memory violation
- memory leak

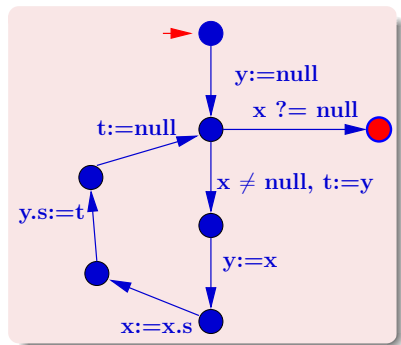
Work supported by EDF (2002-2004),  
and by RNTL AVERILES (2005-2008)

# Pointer systems

Programs:

- only one successor (lists, no trees)
- no data, only pointers

```
List reverse(List x) {  
  List y,t;  
  y =NULL;  
  while (x!=NULL) {  
    t=y;  
    y=x;  
    x=x->n;  
    y->n=t;  
    t=NULL;  
  }  
  return y;  
}
```



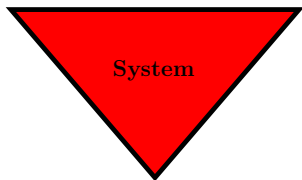
## Verification of pointer systems [AVIS'06,AVIS'04]

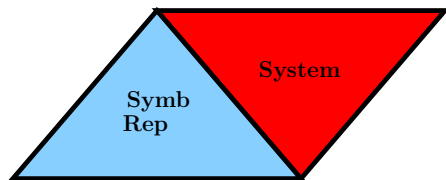
- Encode infinite sets of memory graphs by Presburger sets
- bisimulation between the pointer system and a counter system
- verification by FAST

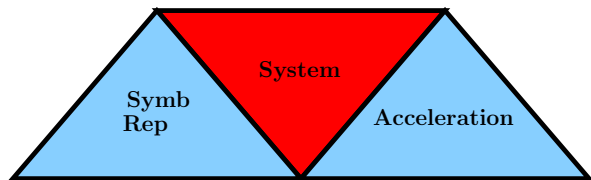
## A prototype is in progress (with A. Sangnier and É. Lozes)

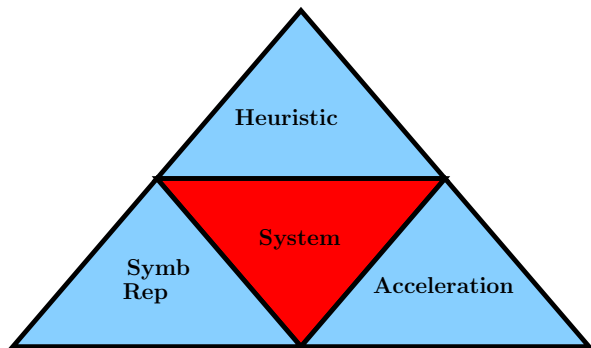
- Works well for  $\approx 10$  small standard examples
- Both qualitative and quantitative properties
- Allows to check programs with counters + pointers

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1. Generic methodology [ATVA'05]
  - unified acceleration framework
  - power and limits (flattable systems)
  - maximal circuit selection
  - generic optimizations (reductions)
2. Instantiation to counter systems
  - two acceleration algorithms
    - [Finkel-Leroux 2002]
    - [TACAS'04]
  - a reduction fit to counters [Finkel-Leroux 2002]
  - The tool FAST

## 3. Many experimentations

### ● Counter systems

- 40 infinite systems [CAV'03]
- TTP [TACAS'04]

### ● Counters + queues

- CES (in my PhD thesis, work with Laure Petrucci)
- *Stop and Wait Protocol [Billington-Gallasch-Petrucci 2005]*

### ● Pointer systems

- translation into counter systems [AVIS'06, AVIS'04]
- prototype, works on 10 standard examples (work with Étienne Lozes and Arnaud Sangnier)

## Short term

A new version `FASTER` is released [submission CAV'06]

- acceleration engine totally independent of the Presburger library
- a Presburger package based on shared automata and cache computation (Couvreur)
- a richer user language

## Future works

Scale-up our methods: abstract-refine and checks methods

Timed counter systems? (TPN, TA + counters, ...)