Robot games

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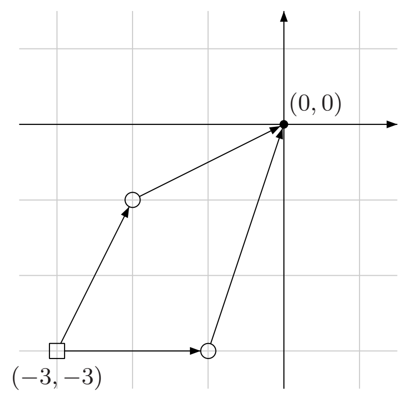
We introduce robot games, and we give the simplest definition for which decidability is open.

1. Definition. Let $U, V \subseteq \mathbb{Z}^2$ be two finite sets of two-dimensional integer vectors. A *robot game* is played in rounds from an initial configuration $x_0 \in \mathbb{Z}^2$ as follows. In each round, player 2 chooses a vector $v \in V$, then player 1 chooses a vector $u \in U$, and the configuration in the next round is $x + v + u$ where x is the configuration in the current round. The objective of player 1 is to reach the configuration $(0, 0)$. A *strategy* for player 1 is a function $\sigma : \mathbb{Z}^2 \to U$ and a *strategy* for player 2 is a function $\pi : \mathbb{Z}^2 \to V$. The *play* according to σ and π from initial configuration x_0 is the infinite sequence $x_0 x_1 \dots$ such that for all $i \geq 0$, we have $x_{i+1} = x_i + v + u$ where $v = \pi(x_i)$ and $u = \sigma(x_i + v)$.

A configuration x_0 is *winning* for player 1 if there exists a strategy σ such that for all strategies π, in the resulting play from x_0 there exists $i \geq 0$ such that $x_i = (0, 0)$.



2. Example. Let $U = \{(1, 3), (2, 1)\}$ and $V = \{(2, 0), (1, 2)\}$. The initial configuration $(-3, -3)$ is winning for player 1. The set of winning configurations for player 1 is $\{(-3k, -3k) \mid k \geq 0\}$. Note that the set of winning configurations is closed under sum (i.e., $x + y$ is a winning configuration if x and y are winning configurations).

3. Decision problem. Given an initial configuration $x_0 \in \mathbb{Z}^2$ and two finite sets $U, V \subseteq \mathbb{Z}^2$, the problem is to decide whether x_0 is a winning configuration in the robot game defined by U, V. Whether this problem is decidable and what is its complexity are open questions.

4. Extension. Extensions can be considered in several directions:

- Robot games in dimension $d \geq 3$.
- Reachability objectives can be defined by a (possibly upward-closed) set of target configurations.
- Players have internal states (e.g., for player 1 the set U of available moves may change as the game is played, according to some finite-state machine).

5. Partial results. The decision problem is undecidable if the game is played on a graph *with states* of player 1 and states of player 2, with $\mathbb{Z}^2$ or $\mathbb{N}^2$ as the vector space (as in games on VASS, vector-addition systems with states) [1, 3]. The one-player version of robot games (i.e., where $V = \{(0, 0)\}$) is decidable by a reduction to linear programming. The robot games defined in one dimension (with $U, V \subseteq \mathbb{Z}$ and $x_0 \in \mathbb{Z}$) are also decidable [2]. The problem is undecidable in dimension $d \geq 9$, and in dimension $d \geq 3$ if player 1 has internal states [4]. In general, robot games in dimension d and internal states for player 1 can be reduced to games in dimension $d + 6$ and no states [4].

References

[1] P. A. Abdulla, A. Bouajjani, and J. d'Orso. Deciding monotonic games. In *Proc. of CSL: Computer Science Logic*, LNCS 2803, pages 1–14. Springer, 2003.

[2] A. Arul and J. Reichert. Personal communication, January 2013.

[3] T. Brázdil, P. Jancar, and A. Kucera. Reachability games on extended vector addition systems with states. In *Proc. of ICALP*, LNCS 6199, pages 478–489. Springer, 2010.

[4] Y. Velner. Personal communication, June 2011.