A Hybrid-Dynamical Model for Passenger-flow in Transportation Systems ⋆

Stefan Haar * **and Simon Theissing** *

* *MExICo team, INRIA and LSV, CNRS & ENS de Cachan, France (e-mail: {stefan.haar, simon.theissing}@inria.fr).*

Abstract: In a network with different transportation modes, or multimodal public transportation system (MPTS), modes are linked among one another not by resources or infrastructure elements - which are not shared, e.g., between different metro lines - , but by the flow of passengers between them. Now, the movements of passengers are steered by the destinations that individual passengers have, and by which they can be grouped into trip profiles. To use the strength of fluid dynamics, we therefore introduce a multiphase hybrid Petri net model, in which the vehicle dynamics is rendered by individual tokens moving in an infrastructure net, while passenger quantities are given as vectors - whose components correspond to trip profiles - and evolve at stations according to fluid dynamics. This model is intended as a building block for obtaining supervisory control, via transport operator actions, to mitigate congestion.

Keywords: Modeling, Networks, Petri nets, Specification, Transportation systems

1. INTRODUCTION

In a multimodal public transportation system (MPTS), different lines with separate infrastructure and belonging to different operators offer fixed-route passenger transportation services. These different *modes* can be assumed not to share their infrastructures or any other resources that would couple their performances together; nonetheless, performance issues such as delays and congestion do propagate from one mode to another via passenger transfers between them. Thus, contrary to the situation in single-mode transportation control where vehicle movements are paramount, see e.g. Ding and Chien (2001), it is here the passenger transfers that have to play a central role in modelling and analyzing perturbations that spread across multimodal networks.

Passengers move according to their *trip profiles*, i.e. their destination and a pre-chosen path through the system toward that destination. At each stop of a vehicle, the movement of all passengers of the same profile will be governed by the same dynamic rule: either all board, or all alight, or all remain where they are, waiting for the right stop before alighting, or waiting for the right train etc before boarding. This may change in case of a traffic perturbation or disruption; imagine e.g. loudspeaker announcements in trains and on platforms advising passengers to prefer alternative routes. In such a situation, all *or part of* the passengers in a trip profile will switch to a different trip profile, and follow its dynamics henceforth until destination, or further changes.

In the literature, several approaches can be found, e.g. in

- Traffic assignment models as discussed in Fu et al. (2012), where network flow models are used to allocate traffic loads to routes: passengers travel according to efficient paths. These models are static, i.e. do not make vehicle movements explicit; in fact, only load capacities are considered, not the actual transportation performance, let alone its variations.
- the Max Plus-Algebra approach to transportation systems such as in Nait-Sidi-Moh et al. (2002), the focus is on synchronization of vehicle arrivals and departures at local points in the network, with the objective of minimizing, and improving robustness to, operation-related delays. The dynamics induced by passenger movements or congestion are not included, and there seems to be no easy way to add them.
- Multi-agent systems, which offer a fine-grain view of individual actions, are the basis, e.g. of Microsimulation platforms such as MATSim in Balmer (2007), in which agents are moved in a transport network in order to process individual activity plans that comes along with an iterative optimization of the agents' travel behaviours. There also exist discrete, Petri-style models of multi-agent systems such as nets-within-nets in Köhler et al. (2003) or Bednarczyk et al. (2005), and related models. In fact, the presence of passengers inside a moving vehicle is a case of nets within nets: every passenger is both a Petri net reflecting their trip profile and the current state within the intended trajectory, and a token inside the net representing the vehicle's state; whereas the vehicle at the same time moves as a token in the infrastructure net. However, the analysis methods developed thus far for Nets-within-nets-type models focus on reachability and other semantic issues. Our approach focusses on quantities of passengers of the same type, and introduces fluid approximations so as to account for uncertainties in network observation,

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while allowing faster computations of quantitative dynamics.

Our approach can be seen as an extension of (timed) hybrid Petri nets in the sense of David and Alla (2010) and as applied to urban traffic control in Di Febbraro et al. (2004); Dotoli et al. (2008); Júlvez and Boel (2010). State space explosion in such models can be overcome e.g. with integrality relaxation as discussed in Silva and Recalde (2002), and Silva and Recalde (2005). However, the model developed here extends the existing ones in that places are marked with multi-dimensional passenger vectors on places, rather than scalar "liquids"; one may think of these nets as of *coloured* fluid Petri nets. In contrast to Dotoli et al. (2008); Júlvez and Boel (2010) we do not employ a first order approximation of the continuous non-linear dynamics describing the passenger flows so as to obtain an overall piece-wise linear model dynamics. Instead, we comply with Di Febbraro et al. (2004) in that the non-linear transition flows are directly integrated into the firing semantics.

The article is structured as follows. In section 2 we introduce multiphase fluid Petri nets. We use them in section 3 as modelling blocks, in order to capture the passenger arrival and departure processes from / to the outside world of a MPTS; the passenger transfers in the stations; and the passenger flows between the stations and the stopped vehicles. Finally, we provide conclusions and an outlook on future work in section 4.

2. MULTIPHASE FLUID PETRI NETS

In the eyes of fluid dynamics, a place of a classical timed fluid Petri net holds a single phase fluid; the marking of that place defines a quantity of the fluid; and enabling and firing rules define flows of the single phase fluid between the places, i.e. single phase flows. Now, in a multiphase fluid Petri net (mFPN), some places, called multiphase reservoirs (mr) hold a multiphase fluid, i.e. are marked with a *vector* of non-negative real numbers, in which each number refers to the quantity of a particular phase. All other places, called simple reservoirs (sr), are marked with a single non-negative real number that abstracts away from the different phases of the fluid, and refers to a quantity of the multiphase fluid as a whole.

We will now define the structure of mFPNs and the markings of their places, together with balance equations that provide a continuous-time dynamics. Thereby, we relate the marking with the multiphase flows by means of flow transformation matrices. Finally, we take into account capacity-limitations of the network.

Definition 1. A multiphase fluid net (mFN) is a 4-tuple $\mathrm{N} := (P, T, F, \mathrm{c})$, with

- the finite set of places P,
- the finite set of transitions T, in which $P \cap T = \emptyset$,
- the flow relation $F \subseteq (P \times T) \cup (T \times P)$, and
- the colour function $\mathrm{c} : P \to \{sr, mr\}$ that specifies whether a given place is a simple or multiphase reservoir.

Remark 2. Throughout the rest of this article, denote as $P_v := \mathrm{c}^{-1}(\{mr\})$ the set of multiphase reservoirs, and as $P_s := \mathrm{c}^{-1}(\{sr\})$ all simple reservoirs of the considered mFN N. As usual, we note for any place or transition $u \in P \cup T$ the pre- and post-set of u as ${}^\bullet u := \{v \in P \cup T \text{ s.t. } (v, u) \in F\}$ and $u^\bullet := \{v \in P \cup T \text{ s.t. } (u, v) \in F\}$, respectively.

As shown in Fig. 1, we represent multiphase reservoirs as ordinary circles, simple reservoirs as dashed circles, and transitions as boxes. Moreover, we connect an arc from place $p \in P$ to transition $t \in T$ iff $p \in {}^\bullet t$, and from transition t to place p iff $p \in t^\bullet$.

Remark 3. Throughout the rest of this article, $\tau \in \mathbb{R}_{\geq 0}$ denotes a time instant that will be clear from the context, and $X := \{1, 2, \ldots, x\}$ the set of all different phases $x \in \mathbb{N}_{>0}$ of the fluid in the considered mFPN.

We store the *marking* of the simple and multiphase reservoirs of an mFN in two functions and obtain an mFPN.

Definition 4. An x-phased fluid Petri net, with $x \in \mathbb{N}_{>0}$, is a 3-tuple $\mathcal{N} := (\mathrm{N}, \mathrm{M}, \mathrm{m})$, where

- N is a multiphase fluid net,
- $\mathrm{M} : P_v \times \mathbb{R}_{\geq 0} \to (\mathbb{R}_{\geq 0})^x$ the multiphase marking, and
- $\mathrm{m} : P_s \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ the simple reservoirs' marking.

Dynamics. Now, we define how mFPN $\mathcal{N}$'s marking changes as a function of time starting from the initial marking at $\tau = 0$. Assign at $\tau \geq 0$ to every transition $t \in T$ the x-phased flow

$$\begin{aligned} \phi : T \times \mathbb{R}_{\geq 0} &\to (\mathbb{R}_{\geq 0})^x \\ (t, \tau) &\mapsto \phi(t, \tau), \end{aligned}$$

and the $(x \times x)$-dimensional flow transformation matrix

$$\begin{aligned} \mathrm{R} : T \times \mathbb{R}_{\geq 0} &\to (\mathbb{R}_{\geq 0})^{x \times x} \\ (t, \tau) &\mapsto \mathrm{R}(t, \tau). \end{aligned}$$

Next, we set up a balance equation at every multiphase and simple reservoir, in which we integrate both as indicated in Fig 1. Thus, with the above notation, at τ the marking of every multiphase reservoir $v' \in {}^\bullet t \cap P_v$ decreases according to ϕ, and the marking of every simple reservoir $s' \in {}^\bullet t \cap P_s$ according to $1^T \phi$. On the contrary, the marking of every multiphase reservoir $v'' \in t^\bullet \cap P_v$ increases according to $\mathrm{R}\,\phi$, and the marking of every simple reservoir $s'' \in t^\bullet \cap P_s$ according to $1^T \mathrm{R}\,\phi$. We then obtain for every multiphase reservoir $v \in P_v$ the balance equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{M}(v, \tau) := \sum_{t \in {}^\bullet v} \mathrm{R}(t, \tau)\ \phi(t, \tau) - \sum_{t \in v^\bullet} \phi(t, \tau), \qquad (1)$$

and for every simple reservoir $s \in P_s$ the balance equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{m}(s, \tau) := 1^T \sum_{t \in {}^\bullet s} \mathrm{R}(t, \tau)\ \phi(t, \tau) - 1^T \sum_{t \in s^\bullet} \phi(t, \tau). \qquad (2)$$

Here, we have used the following notations: Let M be an $m \times n$ matrix with $m, n \in \mathbb{N}_{>0}$, and u a column vector of length m. $M[i, \cdot]$ then denotes the i-th row of matrix M with $i \in \{1, 2, \ldots, m\}$, $M[\cdot, j]$ its j-th column with $j \in \{1, 2, \ldots, n\}$, and M^T its transpose. $u[i]$, on the other hand, denotes the element in the i-th row of vector u. Moreover, 0 denotes a matrix of zeros only, and 1 of ones only. The dimension of such a matrix will be clear from the context.

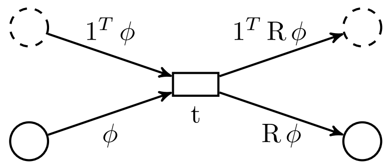
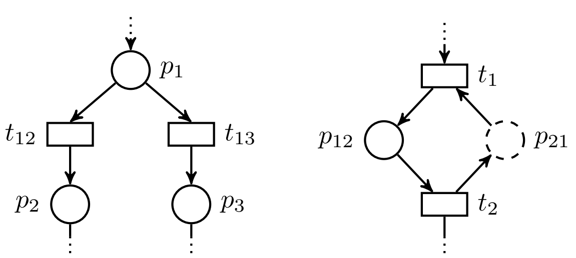


Fig. 1. The integration of a multiphase flow ϕ and a flow transformation matrix R into an mFPN

In general, enabling and firing rules specify the multiphase flows and flow transformation matrices in (1) and (2). Obviously, their choice depends on the concrete use case. However, independently of that, the non-negativity of the marking must be ensured at any τ. Thus, $\mathrm{M}\,(v,\tau)\,[i] = 0$ must imply $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{M}\,(v,\tau)\,[i] \geq 0$ for every phase $i \in X$, and $\mathrm{m}\,(s,\tau) = 0$ must imply $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{m}\,(s,\tau) \geq 0$. One easy approach might be to define several sets of constant multiphase flows and flow transformation matrices, with each set being applied as a function of the marking so as to adopt e.g. the constant speed approach after David and Alla (2010).

Capacities. Having our use case in mind, we now address capacity-limited networks, in which (i) the different phases of a fluid are routed along different paths, and (ii) the flow can be described as a continuous function of time. In fact, we can map all paths in such a network to the flow transformation matrices of an mFPN. To see how this works, we consider Fig. 2a, and assume that it depicts an extract from an mFN that in turn captures the structure of a network, in which the two phases of a flow are routed along two different paths: Phase 1 is routed from p_1 to p_2 via t_{12}, and phase 2 from p_1 to p_3 via t_{13}. For the moment being, we further assume that there is no transformation of the fluid between its two phases. In other words, the part of the flow that is leaving p_1 at τ according to phase 1 via t_{12}, namely $\phi\,(t_{12},\tau)\,[1]$, is identical to the corresponding flow that is joining p_2, namely $\mathrm{R}\,(t_{12},\tau)\,[1,1]\,\phi\,(t_{12},\tau)\,[1]$. Similarly, the part of the flow that is leaving p_1 according to phase 2 via t_{13}, namely $\phi\,(t_{13},\tau)\,[2]$, is identical to the corresponding flow that is joining p_3, namely $\mathrm{R}\,(t_{13},\tau)\,[2,2]\,\phi\,(t_{13},\tau)\,[2]$. However, independently of the actual choice of the multiphase flows, $\phi\,(t_{12},\tau)$ and $\phi\,(t_{13},\tau)$, this is equivalent to saying that the flow transformation matrix assigned to

- transition t_{12}, namely $\mathrm{R}\,(t_{12},\tau)$, is such that the element in its first row and first column is one, and all the other elements are zero, and
- transition t_{13}, namely $\mathrm{R}\,(t_{13},\tau)$, is such that the element in its last row and last column is one, and all the other elements are zero.



(a) A simple branch

(b) A multiphase and simple reservoir in cross connection

Fig. 2. Common subnets in an mFN

Thus, if we extrapolate this to any mFPN, then all flow transformation matrices take on the role of a local forwarding. They must have the shape of diagonal matrices with the diagonal elements being either zero or one, depending on the specification of the different phases. This property of the $\mathrm{R}\,(\bullet,\bullet)$-matrices does not guarantee the proper forwarding of the different phases of the fluid, though. First, we associate a flow transformation matrix with every transition, and not e.g. with every arc connecting a transition to a place in its postset. Thus, if we abstract away from all simple reservoirs, then the mFN must neither contain forks nor joins as depicted in Fig. 3a and in Fig. 3b, respectively. Second, the two flow transformation matrices assigned to the transitions t_{12} and t_{13} in the branch from Fig. 2a must not forward the part of the fluid that refers to one and the same phase at the same time. These considerations lead us to the following definition.

Definition 5. A multiphase fluid Petri net $\mathcal{N}$ is *routing-proper* at τ if

- $|{}^\bullet t \cap P_v| \leq 1$ and $|t^\bullet \cap P_v| \leq 1$ for every transition $t \in T$, and
- $1^T\,\mathrm{R}\,(t_1,\tau)\,[\cdot,i] > 0$ and $1^T\,\mathrm{R}\,(t_2,\tau)\,[\cdot,i] > 0$ together imply $t_1 = t_2$ for any two transitions $t_1, t_2 \in T$, with ${}^\bullet t_1 \cap {}^\bullet t_2 \cap P_v \neq \emptyset$, and any phase $i \in X$.

Note that the integration of the flow transformation matrices into Def. 5 leaves the door open for a re-routing, an attenuation, and an amplification of the multiphase flow. However, one thing after the other.

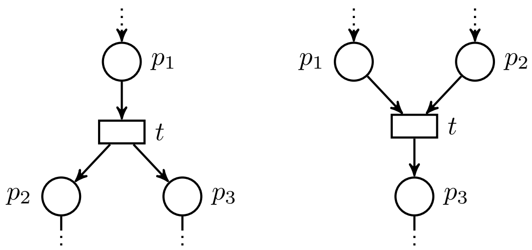
A *re-routing* situation is given in an mFPN if part of the fluid that refers to a particular phase is transformed into another phase between two multiphase reservoirs. For instance, the fact that one third of the fluid that refers to phase 1 at p_1 from Fig. 2a is transformed into phase 2 on the way to p_2 is reproduced in the flow transformation matrix that is assigned to t_{12} as follows: The element in its first row and first column is chosen to be two-thirds, and the element in its second row and first column one third. Thus, in case of a re-routing, the shape of at least one flow transformation matrix deviates from that of a diagonal matrix, which is not inconsistent with Def. 5.

In the previous examples we have assumed the conservation of all multiphase flows. Indeed, the single columns of all flow transformation matrices added together to either zero or one. However, in some use cases it might be necessary to account for *attenuations* or *amplifications* of the multiphase flows that are caused by effects such as leakages and feed-ins. Therefore, the column sums of the flow transformation matrices might have to be chosen bigger or less than one.

Definition 6. A multiphase fluid Petri net $\mathcal{N}$ is *conservative* at τ iff for every transition $t \in T$ and phase $i \in X$

- $1^T\,\mathrm{R}\,(t,\tau)\,[\cdot,i] \in \{0,1\}$, and
- $1^T\,\mathrm{R}\,(t,\tau)\,[\cdot,i] = 0$ implies $\phi\,(t,\tau)\,[i] = 0$.

Remark 7. The property of an mFPN to be conservative according to Def. 6 refers to the transformation of the different multiphase flows. The fact that some of them might be absorbed in form of sinks is not in contrast with it.



(a) A fork (b) A join

Fig. 3. Subnets that make an mFN routing-improper

We still have to show how the *capacity limits* in a network can be mapped to an mFPN. In doing so, we take advantage of the simple reservoirs in that we interconnect them with the multiphase reservoirs as shown in Fig. 2b so as to obtain marking invariants. Starting from some initial values, the sum of the scalar marking of p_{21} and the 1-norm of the vector marking of p_{12} remains constant independently of the vector flows and flow transformation matrices that are assigned to t_1 and t_2.

3. USE CASE: MULTIMODAL TRANSPORT

From a macroscopic point of view, passengers enter the *infrastructure* of an MPTS at an access to one of its stations. They then travel according to pre-chosen paths, given by their trip-profiles, that include transfers within the stations, and boarding and disembarking processes to and respectively from the stopped vehicles. Eventually, they leave the transportation system to the outside world at an exit from another station. Thus, the infrastructure of an MPTS is open w.r.t. the passengers; however, it is not w.r.t. the vehicles. The latter remain in the transportation grids of their mode that can be decomposed into a finite set of geographical positions, called waypoints, and route segments connecting them. To name a few, a particular transportation grid might refer to the street network that confines the movements of all buses along the different lines, or the rail network that confines the movements of all commuter trains. Based on that, we integrate all passenger flows and vehicle movements into one common hybrid-dynamical model. In that model, we map a particular station including all interacting passenger flows to a separate mFPN. Thereby, we think of the station as an accumulation of locations such as entrance areas and platforms that are interconnected by corridors as shown in the lower part of Fig. 4. Boarding and disembarking areas provide interfaces to the stopped vehicles, and accesses and exits to the outside world. We account for capacity limits such as the limited number of passengers at a platform in form of simple reservoirs (cf. Fig. 2b). We map each transportation grid, on the other hand, including the vehicles that are operated on it, to a separate net-within-nets. In doing so, we map each waypoint to a separate place that is represented by a double circle, and each route segment to a transition that is represented by a double box as shown in the upper part of Fig. 4.

W.r.t. the *vehicle movements*, we map each vehicle token to a separate token that resides within a waypoint, in which we assume that a waypoint can hold at maximum one vehicle token at a time. This vehicle token has an internal structure and state of its own. The structure is

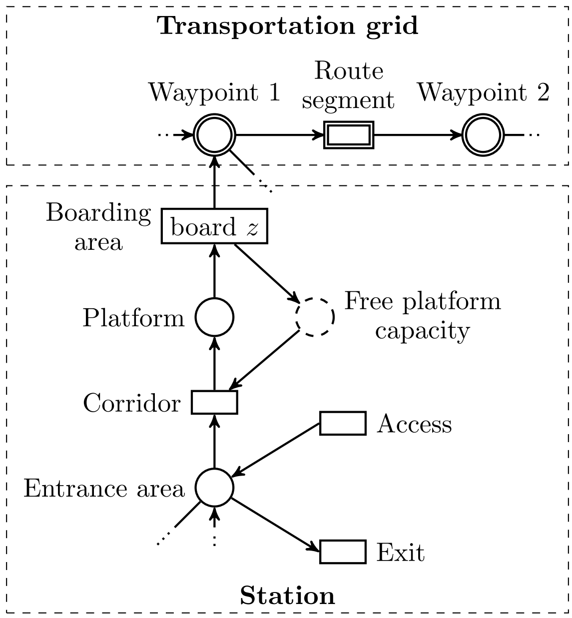


Fig. 4. An extract from the infrastructure of a MPTS

depicted in Fig. 5: One multiphase reservoir is dedicated to the passengers on-board, and one simple reservoir accounts for the remaining capacity. Having said that, the state of a vehicle token is defined by (i) the vector marking of its passenger compartment, i.e., how many passengers per trip-profile are on board, and its remaining capacity; (ii) its driving speed; and (iii) its mission identifier that specifies a service route, in the form of a path in a transportation grid together with a sequence of stops along that path. Whether or not a vehicle token is eligible to be moved from one waypoint to another is thus specified by its mission identifier, and has to be integrated into the enabling and firing rules that define the vehicles' discrete-event dynamical movements. However, we are not going into these easy but cumbersome details here, but focus on passenger flows instead.

To begin with, we note that an arc connects the station's boarding area from Fig. 4 to waypoint 1 in the transportation grid. This connection indicates the possibility of a synchronization of the passenger flows between the station and a vehicle token at waypoint 1. Thereby, a synchronization takes place if a vehicle token enters waypoint 1 and stops (indicated by a zero driving speed), whose passenger compartment comprises one transition that has the same inscription as the station's boarding area. Assuming this to be true, then the common inscription implies that the vector flows and flow transformation matrices that are assigned to the respective transitions are identical as long

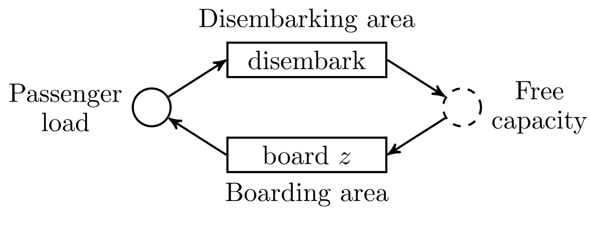


Fig. 5. The passenger compartment of a particular vehicle token

as the vehicle token remains stopped at waypoint 1. As opposed to this, the vector flow that is assigned to the station's boarding area must be zero if no suitable vehicle token has stopped at waypoint 1. Similarly, the vector flow that is assigned to a vehicle token's boarding or disembarking area must be zero as long as its position and driving speed do not permit a synchronization with the boarding or disembarking area of a station. In this way, we can connect the stations and the transportation grids. Thereby, we could also use simple edges to connect the boarding and disembarking areas to the waypoints. However, we go for arcs for the easy specification of the passengers' trip profiles. In fact, each *trip profile* defines a path that starts at an access to a station and ends at an exit from another station in the infrastructure of the concerned MPTS. We then demand that the markings of the multiphase reservoirs in all stations and passenger compartments have the same dimension and the same allocation: The dimension is defined by the total number of the different trip profiles, i.e., the different phases; and the allocation is such that phase 1 always refers to trip profile 1, phase 2 always to trip profile 2, and so forth. That way, we can specify all flow transformation matrices, in which we assume a proper routing of all trip profiles according to Def. 5 and the conservation of the passenger flows according to Def. 6.

Remark 8. In the following, any $t \in T$ can be in a station or in a passenger compartment; the meaning will be clear from the context. Moreover, in our setting, there is at most one place with a vector-marking in the preset of any transition $t \in T$; if that place exists, we denote it by $\mathrm{v}^-(t)$. Similarly, ${}^\bullet t$ contains at most one place with a scalar marking, which is then denoted $\mathrm{s}^-(t)$.

Limits on passenger flows. Before we now discuss the different types of passenger flows one by one, we first note that all of them are limited. Obviously, the maximum flow of passengers who are crossing a corridor is limited, as well as the flow of passengers who are boarding a vehicle through its doors or leaving the station from one of its exits. Thus, we associate with every transition t at time instant $\tau \in \mathbb{R}_{\geq 0}$ the maximum flow of $(t, \tau) \mapsto \phi_{\max}(t, \tau)$ passengers per second, with $\phi_{\max} : T \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, and require that

$$1^T \phi(t, \tau) \leq \phi_{\max}(t, \tau), \forall \tau \in \mathbb{R}_{\geq 0}. \quad (3)$$

Assuming that $\mathrm{s}^-(t)$ exists, we do further demand that $\phi_{\max}(t, \tau)$ approaches zero, when $\mathrm{m}(\mathrm{s}^-(t), \tau)$ is approaching zero. Hence, if t implements a corridor, then we demand that the flow through that corridor approaches zero when the location this corridor is leading to is becoming full.

Remark 9. The maximum throughput of an infrastructure such as a station's corridor or a vehicle's boarding area depends on its geometry on the one hand, and on the passengers including but not restricted to their physical conditions, their capacities of memory and orientation, and their interaction with the other passengers on the other hand. Estimated values can be obtained from experiments and rough calculations; see e.g. Daamen (2004); Fujiyama et al. (2014); Transportation Research Board (2013).

We integrate the *passenger arrival processes* by appropriately defining the vector flows that are assigned to the stations' accesses. One can use the respective flow transformation matrices as *gates*: Either they have the shape of identity matrices, or all of their elements are zero.

Closer look at the passenger flow's composition. The fact that the corridors and the exits have limited throughputs, obligates us to specify the composition of a particular flow of passengers who are *transferring* between two locations in a station or *departing* from an access to the outside world. Therefore, we look at the station from Fig. 4, and anticipate that three different trip profiles have been specified: The passengers at the entrance area of the first two trip profiles, namely trip profiles 1 and 2, want to cross the corridor in order to go to the platform, and the passengers of trip profile 3 want to take the exit in order to leave the station to the outside world. Thus, $1^T \mathrm{R}(\text{corridor}, \tau)\ \mathrm{M}(\text{entrance area}, \tau)$ out of $1^T \mathrm{M}(\text{entrance area}, \tau)$ passengers at the entrance area want to cross the corridor at τ, since by assumption $1^T \mathrm{R}(\text{corridor}, \tau)[\cdot, 1] = 1$, $1^T \mathrm{R}(\text{corridor}, \tau)[\cdot, 2] = 1$, and $1^T \mathrm{R}(\text{corridor}, \tau)[\cdot, 3] = 0$, respectively. Let us integrate now the ingredients of the dynamics at transition t:

Definition 10. Assuming that transition $t \in T$ is not an access, then it is enabled at τ iff

- $1^T \mathrm{R}(t, \tau)\ \mathrm{M}(\mathrm{v}^-(t), \tau) > 0$, and
- $\mathrm{m}(\mathrm{s}^-(t), \tau) > 0$.

Obtaining the passenger flow. Now we know how many passengers want to be part of a transferring or a departing passenger flow. However, we do not know which passengers can do so, since the marking of a location in a station does not contain any information about the arrangement of the passengers. We thus specify the vector flow for transition t and trip profile $i \in X$ at τ as

$$\phi(t, \tau)[i] := \frac{1^T \mathrm{R}(t, \tau)[\cdot, i]\, \mathrm{M}(\mathrm{v}^-(t), \tau)[i]}{1^T \mathrm{R}(t, \tau)\ \mathrm{M}(\mathrm{v}^-(t), \tau)} \phi_{\max}(t, \tau) \quad (4)$$

if t implements a corridor or an exit, and is enabled according to Def. 10. Therein, the passenger flow assigned to i is proportional to the number of passengers who want to cross t according to i, namely $1^T \mathrm{R}(t, \tau)[\cdot, i]\, \mathrm{M}(\mathrm{v}^-(t), \tau)$ passengers, w.r.t. the total number of passengers who want to cross t, namely $1^T \mathrm{R}(t, \tau)\ \mathrm{M}(\mathrm{v}^-(t), \tau)$ passengers. Its upper bound is specified by the maximum throughput of t that ensures the non-negative marking of $\mathrm{s}^-(t)$.

Finally, some words on the specification of the *boarding* and *disembarking* passenger flows. If a boarding area of a vehicle token's passenger compartment is in synchronization with that of a station, then, in the computation of the respective passenger flow we merge both in form of a single transition and thus dock the vehicle token's passenger compartment to the station as indicated in Fig. 6. We then compute the vector flow for this newly introduced transition according to (4) and Def. **??** and assign it to the original boarding areas. Thereby, the maximum throughput in (4) plays the role of a design parameter. For instance, we can couple a boarding and a disembarking area so as to account for simultaneous boarding and disembarking procedures. We can also use it to relate the time it takes to board a vehicle to its passenger load. The flow transformation matrices that are assigned to the boarding and disembarking areas, on the other hand, can be used so as to specify rules for the vehicle stops. For instance, we

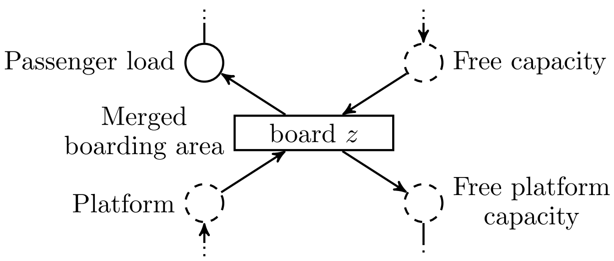


Fig. 6. The merged boarding areas of a vehicle token and a station

can require a vehicle to stop at a waypoint iff more than one passenger wants to board it, summing over all trip profiles at that platform; or similarly, that a vehicle token cannot be moved to another waypoint before all passengers who want to disembark from it have done so.

An interesting way to *exploit* the obtained model is to integrate it into an event-discrete optimization of the movements of the vehicle tokens. In fact, experiments such as Coffey et al. (2012) have shown that even small perturbations of the vehicle movements might significantly improve the network performance without the need to abandon existing timetables. For this, the model could be translated into a machine readable Mixed Logical Dynamical system in the sense of Bemporad and Morari (1999), and then integrated into a mixed integer linear programming problem, as shown in Jùlvez et al. (2014) for a classical timed hybrid Petri net. Of course, this requires to linearize all passenger flows so as to obtain a global piece-wise affine dynamics.

4. CONCLUSIONS AND FUTURE WORK

In this article, we have introduced mFPNs for capturing passenger flows in a hybrid dynamical nets-within-nets model of a MPTS. We have assumed given the knowledge about attribution of passengers to trip profiles. Future work will include the use of *estimated* trip profiles, corresponding to statistical predictions, and keep track of the update of these estimates under the net's dynamics. Analyzing the sensitivity of both the uncontrolled and the controlled systems will then be possible in a probabilistic framework; thus allowing us to evaluate our approach in some simulation runs. Also, it may be worthwhile to systematically examine simplifications, such as network contractions or decompositions, with the purpose of improving computational efficiency.

REFERENCES

Balmer, M. (2007). *Travel demand modeling for multi-agent traffic simulations: Algorithms and systems Dissertation.* Ph.D. thesis, ETH Zurich.

Bednarczyk, M.A., Bernardinello, L., Pawlowski, W., and Pomello, L. (2005). Modelling mobility with petri hypernets. In *Proceedings of the 17th International Conference on Recent Trends in Algebraic Development Techniques.*

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica.*

Coffey, C., Nair, R., Pinelli, F., Pozdnoukhov, A., and Calabrese, F. (2012). Missed connections: Quantifying and optimizing multi-modal interconnectivity in cities. In *Proceedings of the 5th ACM SIGSPATIAL International Workshop on Computational Transportation Science.*

Daamen, W. (2004). *Modelling Passenger Flows in Public Transport Facilities.* Ph.D. thesis, Delft University of Technology.

David, R. and Alla, H. (2010). *Discrete, Continuous, and Hybrid Petri Nets.* Springer Publishing Company, Heidelberg.

Di Febbraro, A., Giglio, D., and Sacco, N. (2004). Urban traffic control structure based on hybrid petri nets. *IEEE Transactions on Intelligent Transportation Systems.*

Ding, Y. and Chien, S. (2001). Improving transit service quality and headway regularity with real-time control. *Transportation Research Record: Journal of the Transportation Research Board*, 1760.

Dotoli, M., Fanti, M., and Iacobellis, G. (2008). An urban traffic network model by first order hybrid petri nets. In *IEEE International Conference on Systems, Man and Cybernetics.*

Fu, Q., Liu, R., and Hess, S. (2012). A review on transit assignment modelling approaches to congested networks: A new perspective. *Procedia - Social and Behavioral Sciences.*

Fujiyama, T., Thoreau, R., and Tyler, N. (2014). The effects of the design factors of the train-platform interface on pedestrian flow rates. In *Pedestrian and Evacuation Dynamics 2012.* Springer International Publishing.

Júlvez, J. and Boel, R. (2010). A continuous petri net approach for model predictive control of traffic systems. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on.*

Jùlvez, J., Di Cairano, S., Bemporad, A., and Mahulea, C. (2014). Event-driven model predictive control of timed hybrid petri nets. *International Journal of Robust and Nonlinear Control.*

Köhler, M., Moldt, D., and Rölke, H. (2003). Modelling mobility and mobile agents using nets within nets. In *Proceedings of the 24th International Conference on Applications and Theory of Petri Nets.*

Nait-Sidi-Moh, A., Manier, H., Manier, M.A., and El-Moudni, A. (2002). Control of a public transport network by the max-plus algebra: case of a system constrained by maximal connection times. In *2nd WSEAS Int. Conf. on Simulation, Modeling and Optimization (ICOSMO 2002).*

Silva, M. and Recalde, L. (2002). Petri nets and integrality relaxations: A view of continuous petri net models. *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews.*

Silva, M. and Recalde, L. (2005). Continuization of timed petri nets: From performance evaluation to observation and control. *Lecture Notes in Computer Science.*

Transportation Research Board (ed.) (2013). *Transit Capacity and Quality of Service Manual.* National Academy of Sciences., 3 edition.