Can chimps go it alone?

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Abstract. Consider a smart chimpanzee named M from a tribe afflicted with a form of Alzheimer's disease. Think of M as a logspace-bounded Turing machine. M can do simple things like integer arithmetic and matrix multiplication, but M turns sullen and calls for help when asked to perform seemingly equally simple tasks, such as simulating deterministic tree and dag automata.
Is M acting difficult or is she just not smart enough?
Even before the **P** versus **NP** question, Cook [Coo71] conjectured that no amount of smarts can compensate for Alzheimer's disease[1].
We will review some of the attempts at separating **L** from **P** inspired by pebbling arguments. Emphasis will be placed on branching programs for the tree evaluation problem, recently studied anew [CMW$^+$12]. The problem consists of determining the value that percolates to the root of a (binary) tree when a value from a domain D is prescribed at each tree leaf and an explicit function $f : D \times D \to D$ is prescribed at each internal node. In a nutshell, lower bounds for restricted branching programs can be proved, but approaches to attack the general model strangely come up against the same barrier that Nečiporuk encountered in a two-page note 50 years ago and that still stands today.
Tree evaluation naturally extends to tree generation [Cha13], where the functions $f : D \times D \to D$ at internal tree nodes are replaced with functions $f : D \times D \to \{S : S \subseteq D\}$. This is interpreted as allowing to pick, as the D-value of a node labelled f with left child ℓ and right child r, any value from $f(D$-value of ℓ, D-value of $r)$. Tree generation can then be turned into a monotone boolean function. Strong lower bounds for this function have been derived from pebbling intuition [CP12,Cha13] and we will further discuss some of these.
For a suitable bibliography please consult [CMW$^+$12,CP12,Cha13].

References

[Cha13] Siu Man Chan. Just a pebble game. *Electronic Colloquium on Computational Complexity (ECCC)*, 20:42, 2013.

[CMW$^+$12] S. Cook, P. McKenzie, D. Wehr, M. Braverman and R. Santhanam. Pebbles and branching programs for tree evaluation. *TOCT*, 3(2):4, 2012.

[Coo71] S. A. Cook. Characterizations of pushdown machines in terms of time-bounded computers. *J. ACM*, 18:4–18, 1971.

[CP12] Siu Man Chan and A. Potechin. Tight bounds for monotone switching networks via fourier analysis. *Electronic Colloquium on Computational Complexity (ECCC)*, 19:185, 2012.

[1] Steve said this in a different language, thankfully.