Errata to 'Continuous Random Variables'

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Proposition V.1 of [GV11] is false. This was found by Mike W. Mislove and Tyler C. Barker: see [Mis13]. This does not seem to be mendable.

What is true is that the domain of continuous random variables (resp., of uniform continuous random variables) over a bc-domain is a bc-domain; similarly, Scott domains are preserved.

Also, the monad laws hold… but we do not obtain a monad because the so-called extension $h^\dagger$ of h given in Proposition V.1 is not monotonic, hence not Scott-continuous. The following is Mislove and Barker's counterexample. Take $h\colon X \to \theta\mathbf{R}_1(Y)$, where $X = Y = \Omega$ (the Cantor tree), defined by $h(\omega) = (\delta_\omega, \mathrm{id})$. For every continuous random variable (ν, f) on $X = \Omega$ with ν simple, i.e., $\nu = \sum_{i=1}^n a_i \delta_{\omega_i}$, $h^\dagger(\nu, f) = (\sum_{i=1}^n a_i \delta_{\omega_i . f(\omega_i)}, \_)$ (we write $\_$ to denote a function which we do not care about). Take f to be the constant function mapping every word to the one-letter word 1 (on whatever subdomain of Ω). Then $h^\dagger(\delta_\epsilon, f) = (\delta_1, \_)$ and $h^\dagger(\delta_0, f) = (\delta_{01}, \_)$; although $(\delta_\epsilon, f) \leqq (\delta_0, f)$, $(\delta_1, \_) \not\leqq (\delta_{01}, \_)$ because the projection of 01 onto $\operatorname{supp} \delta_1 = \downarrow 1$ is ϵ, not 1.

This also affects *uniform* continuous random variables. This time, take $h\colon X \to \upsilon\mathbf{R}_1(Y)$ with $X = Y = \Omega$ defined by $h(\omega) = (\nu_0, \mathrm{id})$ (a constant map), where $\nu_0 = 1/2\delta_0 + 1/4\delta_{11} + 1/8\delta_{100} + 1/8\delta_{101}$. Here $h^\dagger(\sum_{i=1}^n a_i\delta_{\omega_i}, \mathrm{id}) = \sum_{i=1}^n a_i(1/2\delta_{\omega_i 0} + 1/4\delta_{\omega_i 11} + 1/8\delta_{\omega_i 100} + 1/8\delta_{\omega_i 101}), \_)$. In particular, $h^\dagger(\delta_\epsilon, \mathrm{id}) = (\nu_0, \_)$, while $h^\dagger(1/2\delta_0 + 1/2\delta_1, \mathrm{id}) = (1/4\delta_{00} + 1/8\delta_{011} + 1/16\delta_{0100} + 1/16\delta_{0101} + 1/4\delta_{10} + 1/8\delta_{111} + 1/16\delta_{1100} + 1/16\delta_{1101}, \_)$. Since $(\delta_0, \mathrm{id}) \leqq (1/2\delta_0 + 1/2\delta_1, \mathrm{id})$, we would expect a similar relation on their images by $h^\dagger$, but this would imply that the support $F_0 = \downarrow\{0, 11, 100, 101\}$ of ν_0 is the image by the projection p_{F_0} of the support $F_1 = \downarrow\{00, 011, 0100, 0101, 10, 111, 1100, 1101\}$ of the first component of $h^\dagger(1/2\delta_0 + 1/2\delta_1, \mathrm{id})$. However, 100 is in F_0, and is not the prefix of any word in F_1.

This impacts the rest of the paper: we cannot characterize equational theories as in Section VI, we cannot give semantics to probabilistic higher-order languages using continuous random variables as in Section VII, M. Escardó's question on semi-decidability of testing (Section VIII) remains unsolved, and the connection with indexed valuations (Section IX) is lost.

References

[GV11] Jean Goubault-Larrecq and Daniele Varacca. Continuous random variables. In *Proceedings of the 26th Annual IEEE Symposium on Logic in Computer Science (LICS'11)*, pages 97–106, Toronto, Canada, June 2011. IEEE Computer Society Press.

[Mis13] Mike W. Mislove. Anatomy of a domain of continuous random variables II. `http://www.entcs.org/mislove/anatomy2.pdf`, 2013. Accepted.