Errata to ‘Continuous Random Variables’

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Proposition V.1 of [GV11] is false. This was found by Mike W. Mislove and Tyler C. Barker: see [Mis13]. This does not seem to be mendable.

What is true is that the domain of continuous random variables (resp., of uniform continuous random variables) over a bc-domain is a bc-domain; similarly, Scott domains are preserved.

Also, the monad laws hold... but we do not obtain a monad because the so-called extension $h^\dagger$ of $h$ given in Proposition V.1 is not monotonic, hence not Scott-continuous. The following is Mislove and Barker’s counterexample. Take $h: X \to \nu_1(\Theta(\nu))$, where $X = \Theta = \Omega$ (the Cantor tree), defined by $h(\nu) = (\delta_\nu, \text{id})$. For every continuous random variable $(\nu, f)$ on $X = \Omega$ with $\nu$ simple, i.e., $\nu = \sum_{i=1}^n a_i \delta_{\omega_i}$, $h^\dagger(\nu, f) = (\sum_{i=1}^n a_i \delta_{\omega_i} f(\omega_i), \text{id})$ (we write $\text{id}$ to denote a function which we do not care about). Take $f$ to be the constant function mapping every word to the one-letter word 1 (on whatever subdomain of $\Omega$).

Then $h^\dagger(\delta_\epsilon, f) = (\delta_1, \text{id})$ and $h^\dagger(\delta_0, f) = (\delta_{01}, \text{id})$; although $(\delta_\epsilon, f) \leq (\delta_0, f)$, $(\delta_1, \text{id}) \not\leq (\delta_{01}, \text{id})$ because the projection of 01 onto $\text{supp} \delta_1 = \downarrow 1$ is $\epsilon$, not 1.

This also affects uniform continuous random variables. This time, take $h: X \to \nu_1(\Theta(\nu))$ with $X = \Theta = \Omega$ defined by $h(\nu) = (\nu_0, \text{id})$ (a constant map), where $\nu_0 = \sum_{i=1}^n a_i \delta_{\omega_i} \text{id}) = \sum_{i=1}^n a_i (1/2 \delta_{\omega_i} + 1/4 \delta_{\omega_i} + 1/8 \delta_{100} + 1/8 \delta_{101})$. Here $h^\dagger(\sum_{i=1}^n a_i \delta_{\omega_i}, \text{id}) = \sum_{i=1}^n a_i (1/2 \delta_{\omega_i} + 1/4 \delta_{\omega_i} + 1/8 \delta_{100} + 1/8 \delta_{101}) \text{id})$. In particular, $h^\dagger(\delta_0, \text{id}) = (\nu_0, \text{id})$, while $h^\dagger(1/2 \delta_0 + 1/2 \delta_1, \text{id}) = (1/4 \delta_0 + 1/8 \delta_{01} + 1/16 \delta_{100} + 1/16 \delta_{101} + 1/4 \delta_{10} + 1/4 \delta_{11} + 1/16 \delta_{111} + 1/16 \delta_{110} + 1/16 \delta_{101})$. Since $(\delta_0, \text{id}) \leq (1/2 \delta_0 + 1/2 \delta_1, \text{id})$, we would expect a similar relation on their images by $h^\dagger$, but this would imply that the support $F_0 = \downarrow \{0, 11, 100, 101\}$ of $\nu_0$ is the image by the projection $p_{F_0}$ of the support $F_1 = \downarrow \{00, 011, 0100, 0101, 10, 111, 1100, 1101\}$ of the first component of $h^\dagger(1/2 \delta_0 + 1/2 \delta_1, \text{id})$. However, 100 is in $F_0$, and is not the prefix of any word in $F_1$.

This impacts the rest of the paper: we cannot characterize equational theories as in Section VI, we cannot give semantics to probabilistic higher-order languages using continuous random variables as in Section VII, M. Escardó’s question on semi-decidability of testing (Section VIII) remains unsolved, and the connection with indexed valuations (Section IX) is lost.
References
