Errata (2) to ‘Exponentiable Streams and Prestreams’

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The mistake was found by Stefano Nicotra, who told me about it my email on January 29th, 2019.

Right before Theorem 8, I say:

We deal with Haucourt streams right away. Lemma 20 does not apply in this setting, since the Sierpiński stream is not a Haucourt stream. Indeed, because [0, 1] is connected, there is no directed path from 0 to 1, or from 1 to 0, in $S$. As a result, it may well be that there are exponentiable Haucourt streams with non-core-compact carriers.

This difficulty is, in fact, nonexistent: the Sierpiński space $S$ is connected and even path-connected, so that argument does not apply. Recounting S. Nicotra’s argument, the map $q: [0, 1] \to S$ that maps 0 to 0 and all other points to 1 is continuous, and defines a path from 0 to 1 in $S$. The map $q^\perp: [0, 1] \to S$ that maps 1 to 0 and all other points to 1 defines a path from 1 to 0.

It is easy to see that the two maps $q$, $q^\perp$ define prestream morphisms from $[0, 1]$ to $S$. Hence the Sierpiński stream $S$ is a Haucourt stream. Therefore Lemma 20 applies: the carrier of any exponentiable object in $HStr$ is core-compact. Combining this with the published proof of Theorem 8, we obtain the following strengthening of Theorem 8:

**Theorem 8 (improved—for free).** The exponential objects in $HStr$ are exactly the core-compact Haucourt streams, i.e., the Haucourt streams $\mathcal{X} = (X, (\subseteq_U)_{U \in O(X)})$ whose carrier $X$ is core-compact.

For every core-compact Haucourt stream $\mathcal{X} = (X, (\subseteq_U)_{U \in O(X)})$, and every Haucourt stream $\mathcal{Y} = (Y, (\subseteq_V)_{V \in O(Y)})$, the exponential object $\mathcal{Y}^\mathcal{X}$ in $Str$ is (up to isomorphism) the haucourtification of $([X \to Y]^\mathcal{X}, (\leq_W)_{W \in O([X \to Y]^\mathcal{X})})$.

This solves open problem 1 of the conclusion in the positive, as well.