Errata (2) to 'Exponentiable Streams and Prestreams'

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The mistake was found by Stefano Nicotra, who told me about it my email on January 29th, 2019.

Right before Theorem 8, I say:

> We deal with Haucourt streams right away. Lemma 20 does not apply in this setting, since the Sierpiński stream is not a Haucourt stream. Indeed, because $[0, 1]$ is connected, there is no directed path from 0 to 1, or from 1 to 0, in $\mathbb{S}$. As a result, it may well be that there are exponentiable Haucourt streams with non-core-compact carriers.

This difficulty is, in fact, *nonexistent*: the Sierpiński space $\mathbb{S}$ *is* connected and even path-connected, so that argument does not apply. Recounting S. Nicotra's argument, the map $q\colon [0, 1] \to \mathbb{S}$ that maps 0 to 0 and all other points to 1 is continuous, and defines a path from 0 to 1 in $\mathbb{S}$. The map $q^\perp\colon [0, 1] \to \mathbb{S}$ that maps 1 to 0 and all other points to 1 defines a path from 1 to 0.

It is easy to see that the two maps q, $q^\perp$ define prestream morphisms from $\overrightarrow{[0, 1]}$ to $\mathbb{S}$. Hence the Sierpiński stream $\mathbb{S}$ is a Haucourt stream. Therefore Lemma 20 applies: the carrier of any exponentiable object in **HStr** is core-compact. Combining this with the published proof of Theorem 8, we obtain the following strengthening of Theorem 8:

Theorem 8 (improved—for free). *The exponential objects in* **HStr** *are exactly the core-compact Haucourt streams, i.e., the Haucourt streams* $\mathcal{X} = (X, (\sqsubseteq_U)_{U \in \mathcal{O}(X)})$ *whose carrier* X *is core-compact.*

For every core-compact Haucourt stream $\mathcal{X} = (X, (\sqsubseteq_U)_{U \in \mathcal{O}(X)})$*, and every Haucourt stream* $\mathcal{Y} = (Y, (\preceq_V)_{V \in \mathcal{O}(Y)})$*, the exponential object* $\mathcal{Y}^{\mathcal{X}}$ *in* **Str** *is (up to isomorphism) the haucourtification of* $([\mathcal{X} \to \mathcal{Y}]^o, (\leq^s_W)_{W \in \mathcal{O}([\mathcal{X} \to \mathcal{Y}]^o)})$.

This solves open problem 1 of the conclusion in the positive, as well.