Errata to ‘Exponentiable Streams and Prestreams’

Jean Goubault-Larrecq

March 1, 2017

1 The Mistake

Lemma 17, according to which dipaths from x to y in U coincide with directed paths from x to y in U, is *wrong*.

Every dipath is a directed path, but the converse fails, even in a stream. Consider indeed the stream $\mathcal{X} = (X, (\sqsubseteq_U)_{U \in \mathcal{O}(X)})$ defined as $\overrightarrow{[0,1]}$, and the map $\gamma \colon [0,1] \to \mathcal{X}$ defined by $\gamma(t) = t$. This is a directed path, being just the identity stream morphism from $\overrightarrow{[0,1]}$ to itself, but this is not a dipath. The property "γ is a dipath" would mean that for every open subset V of $[0,1]$, for all $s, t \in V$ with $s \leq t$, $\gamma(s) \sqsubseteq_V \gamma(t)$ (i.e., $[s,t] \subseteq V$), but that is wrong of $s = 0$, $t = 1$, $V = [0, 1/3) \cup (2/3, 1]$ for example. Lemma 17 is wrong.

Ironically, that was stated explicitly in the preceding paragraph.

2 Changes

The definition of a Haucourt circulation and Haucourt stream (Definition 5) must be changed, as follows:

Definition 5 (corrected). *A* Haucourt circulation *on X is a precirculation $(\sqsubseteq_U)_{U \in \mathcal{O}(X)}$ such that, for every open subset U, for all $x, y \in U$, $x \sqsubseteq_U y$ if and only if there is a prestream morphism $\gamma \colon \overrightarrow{[0,1]} \to (X, (\sqsubseteq_U)_{U \in \mathcal{O}(X)})$ such that $\gamma(0) = x$, $\gamma(1) = y$, and the image of γ lies entirely inside U. (Such a map γ is called a* directed path from x to y in U.*)* *A* Haucourt stream *is a prestream whose precirculation is a Haucourt circulation.*

The only change with the definition in the published paper is that γ should be a morphism from $\overrightarrow{[0,1]}$ to X (a *directed path*) not from $([0,1], \leq)$ to X (a *dipath*).

The corrected definition accords with Haucourt's, as discussed in the Appendix, in the sense that Haucourt's category **Str*** is exactly the category of Haucourt streams, with the corrected definition.

Lemma 12, which states that every Haucourt stream is a stream, still holds with the new definition. (That is consistent with Haucourt's results just mentioned.) The proof is the same, merely changing "there is a dipath γ from x to y in U" into "there is a directed path γ from x to y in U". Indeed, the end of the proof states "the whole interval $[t_{k-1}, t_k]$ is included in $(a_{ij}, b_{ij}) \subseteq \gamma^{-1}(U \cap U_i)$, so $\gamma(t_{k-1}) \sqsubseteq_{U \cap U_i} \gamma(t_k)$", and this holds when γ is a directed path.

Example 7, which states that $\overrightarrow{\mathbb{R}}$ is a Haucourt stream, also holds with the new definition.

One should of course ignore Lemma 17. One should also ignore the paragraph preceding Lemma 17 ("We can do better using Haucourt's notion of *directed path from x to y* in $\mathcal{X}$ [...] However, it does not make a difference whether we define Haucourt streams with dipaths or with directed paths"), since the corrected definition makes it superfluous.

The developments related to the haucourtification of a prestream (Definition 7, Lemma 18, and subsequent) are left unchanged, since I have used the new definition based on directed paths already there.

Acknowledgments

The mistake was found by Jérémy Dubut on February 24th, 2017. I am indebted to him.