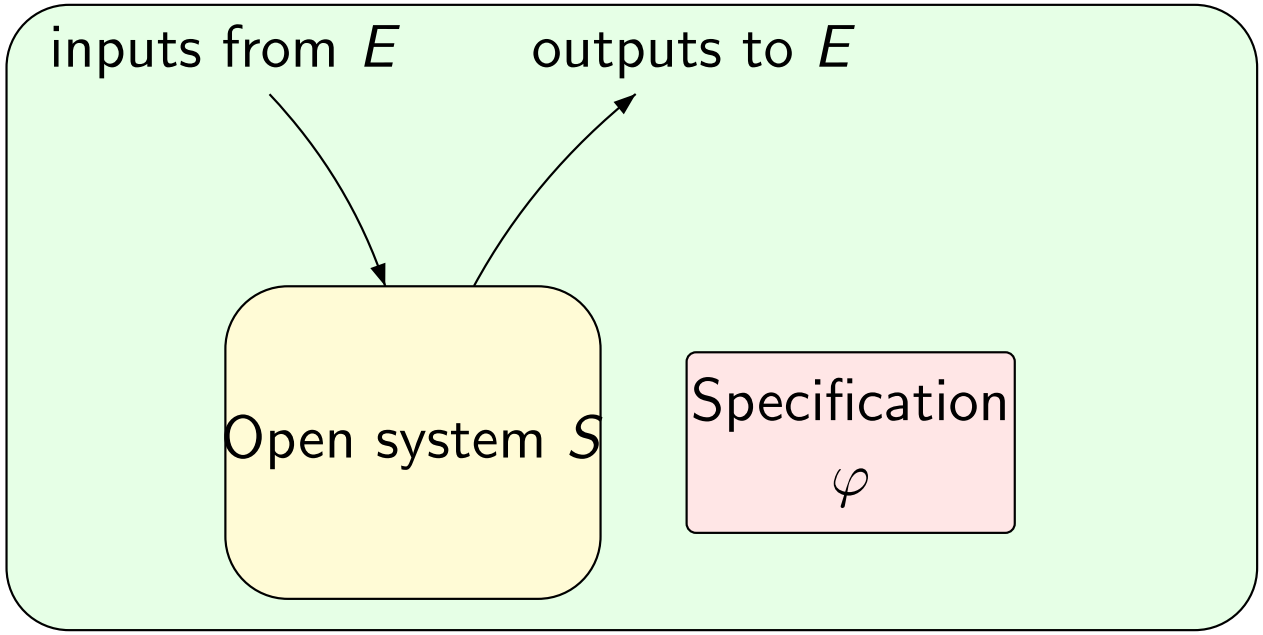
Distributed Synthesis for Well Connected Architectures

Paul Gastin, Nathalie Sznajder and Marc Zeitoun

March 13th 2006
ACI Cortos Persee Versydis

Synthesis of a reactive system



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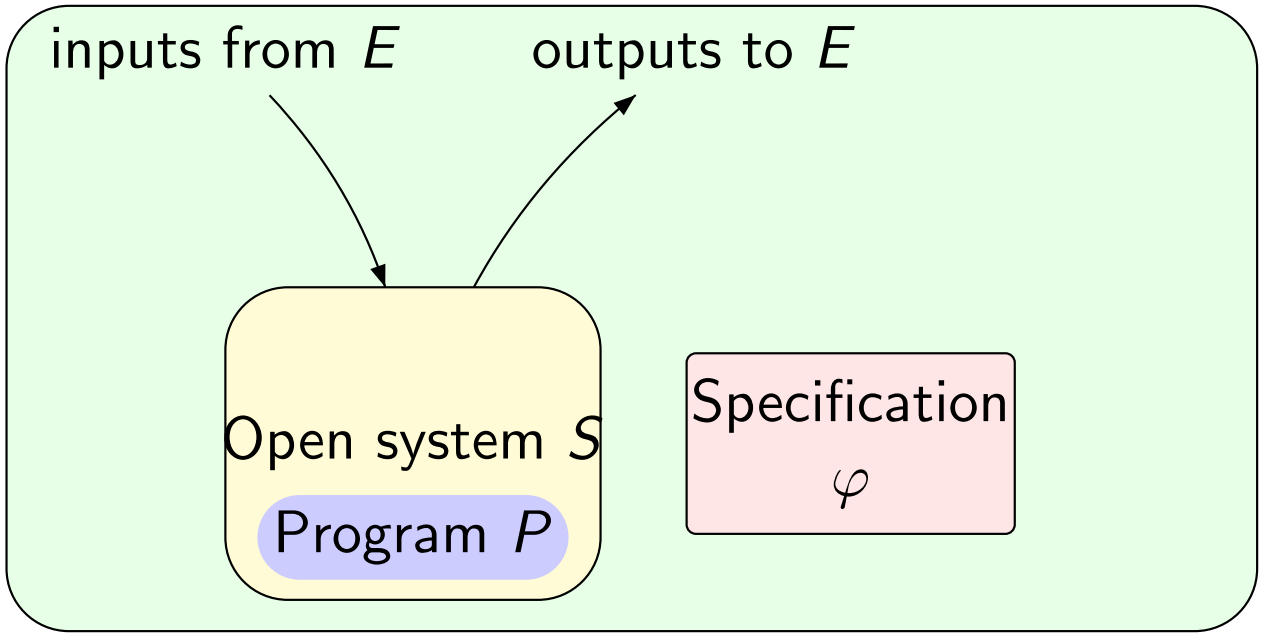
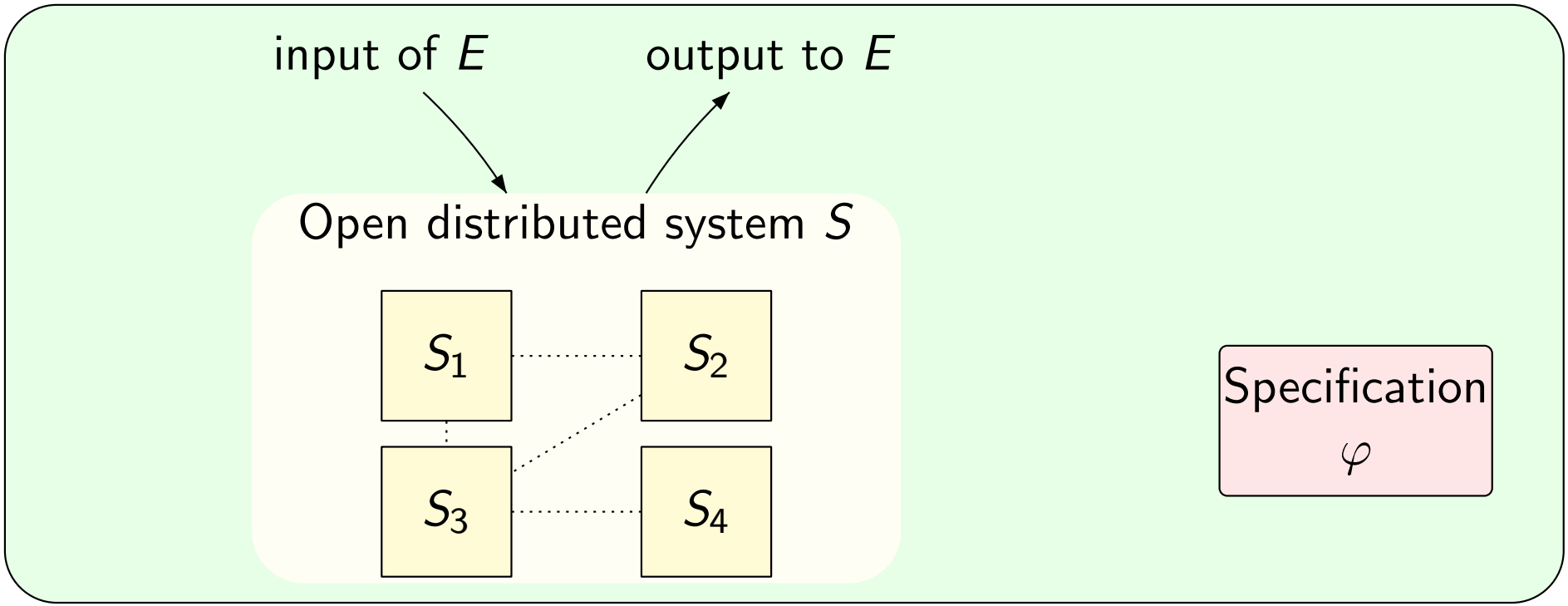


Two problems

- Decide whether there exists a program st. $P||E \models \varphi, \qquad \forall E$.
- Synthesis: If so, compute such a program.

For reasonable systems and specifications, the problems are decidable.

Distributed synthesis



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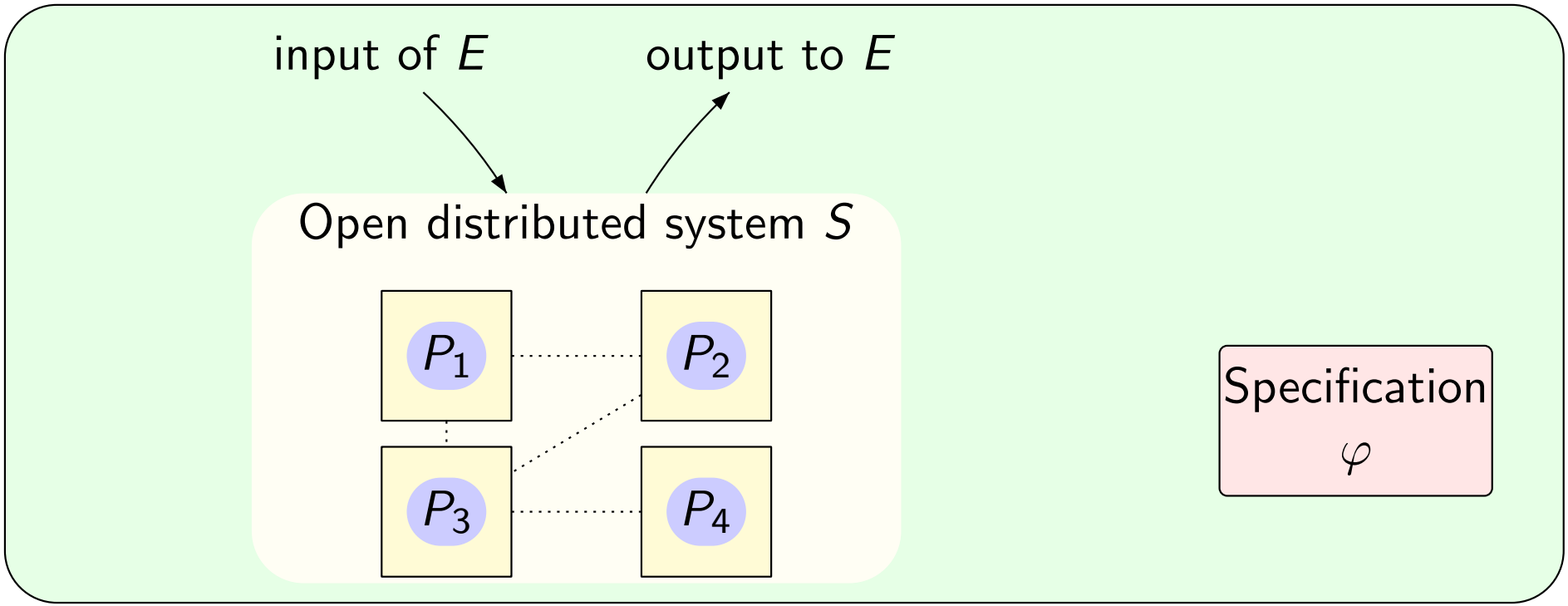

Two problems

- Decide the existence of a distributed program such that their joint behavior $P_1||P_2||P_3||P_4||E$ satisfies φ, for all E.
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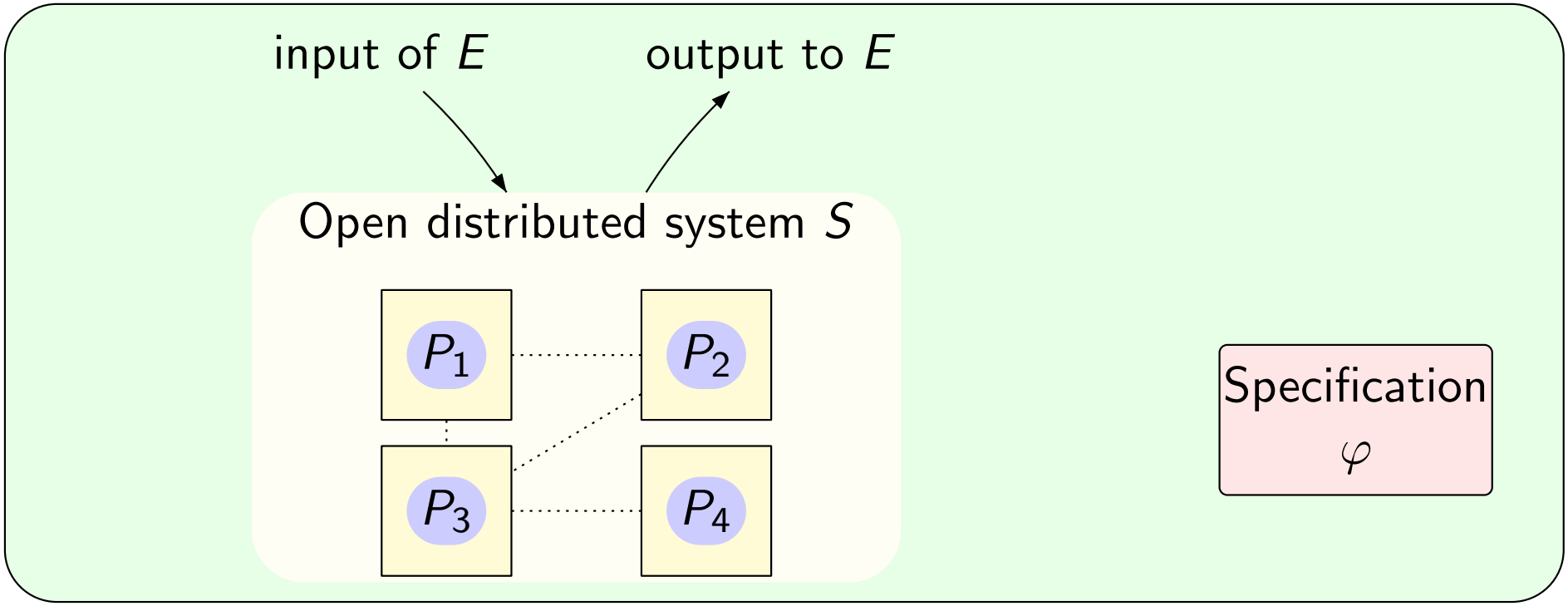
Distributed synthesis



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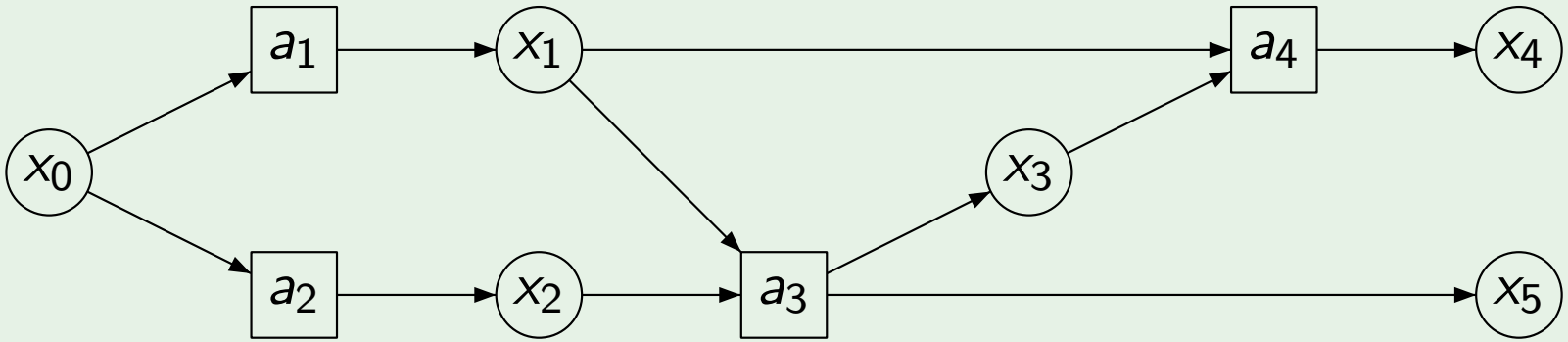
- Synthesis : If it exists, compute such a distributed program.

Peterson-Reif 1979, Pnueli-Rosner 1990

In general, the problem is undecidable.

The model

Example



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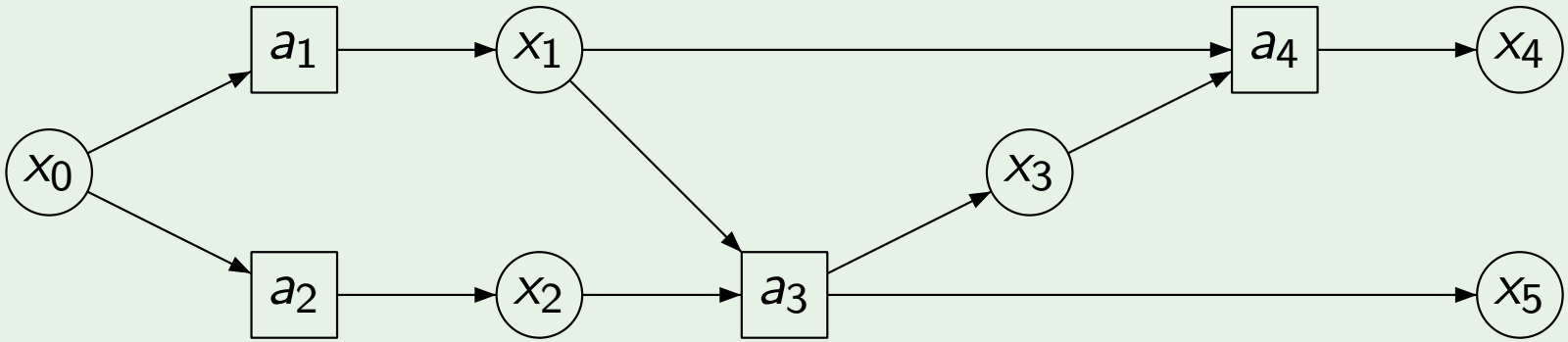

- Synchronous behavior

The model

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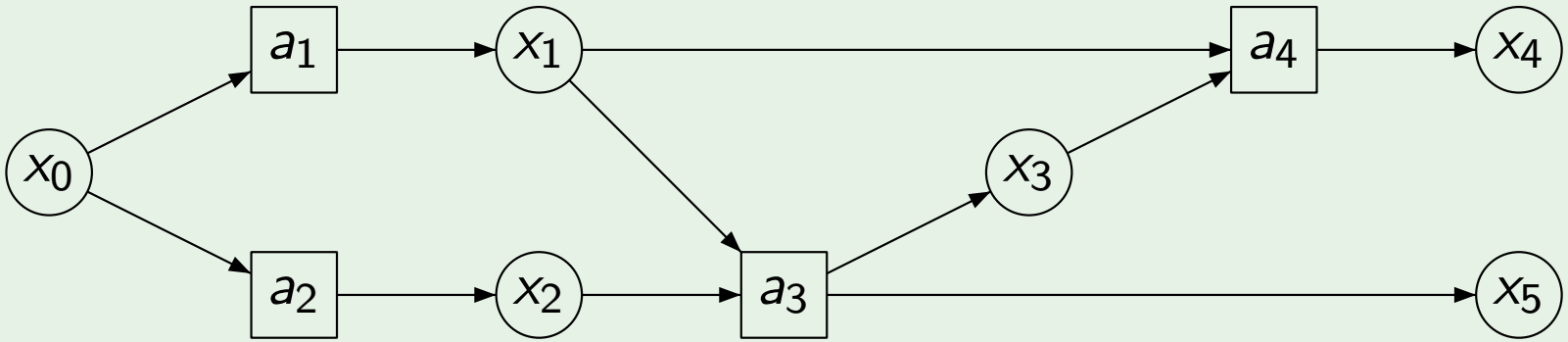

- Synchronous behavior
- Strategies with local memory

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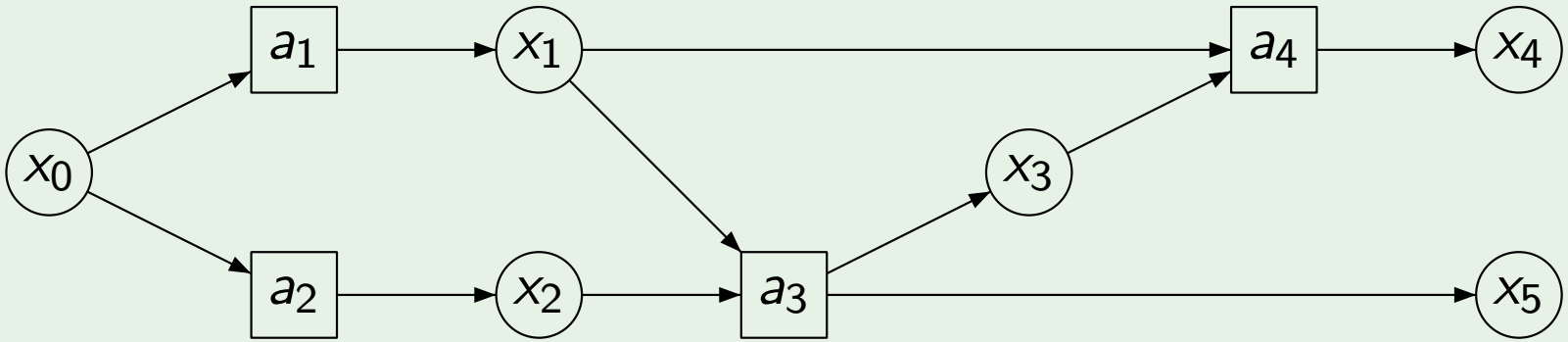

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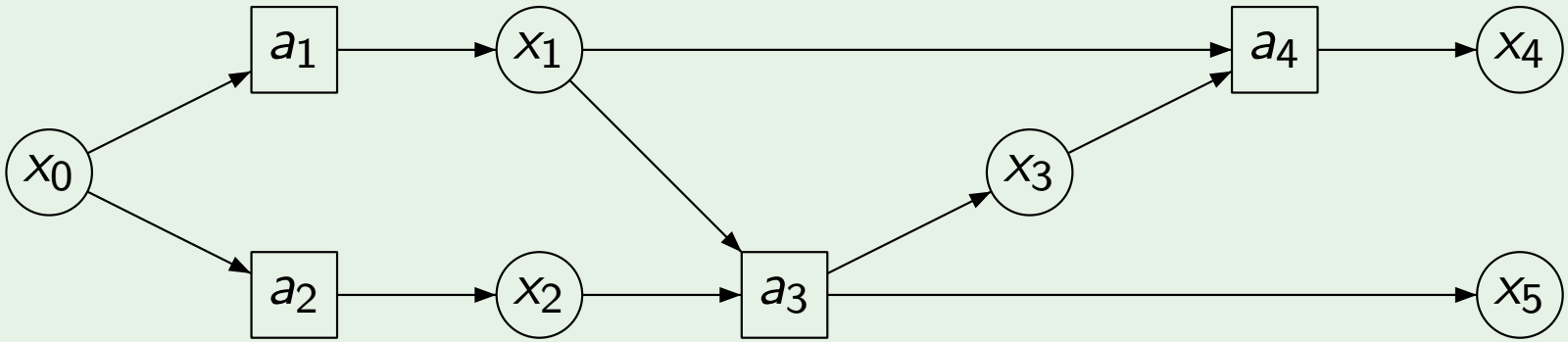
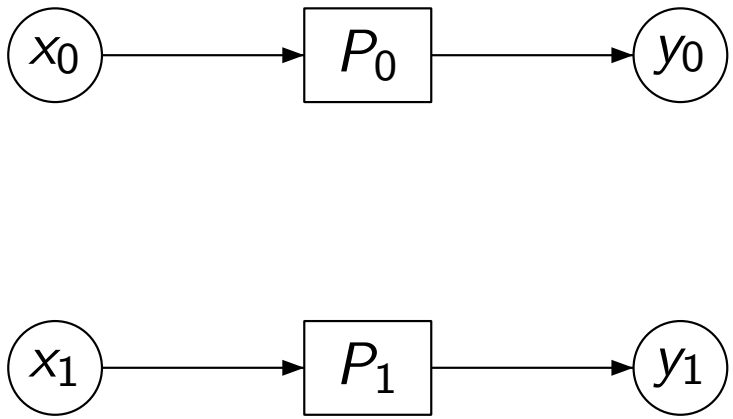


- Synchronous behavior
- Strategies with local memory
- 0-delay semantics
- Input-output specifications

Undecidable architecture (Pnueli–Rosner '90)



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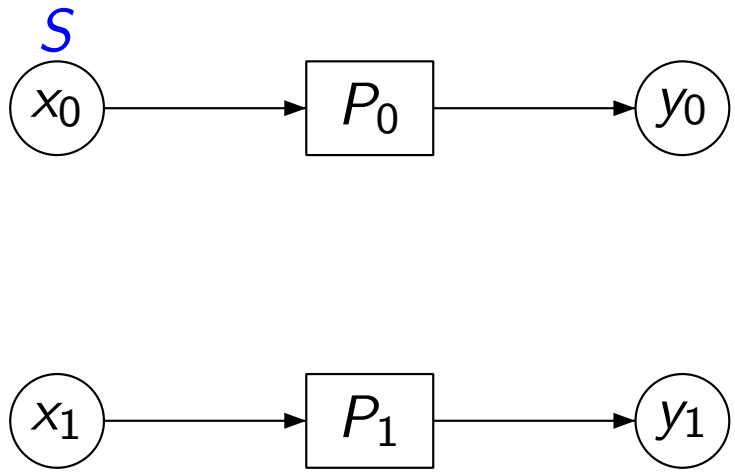

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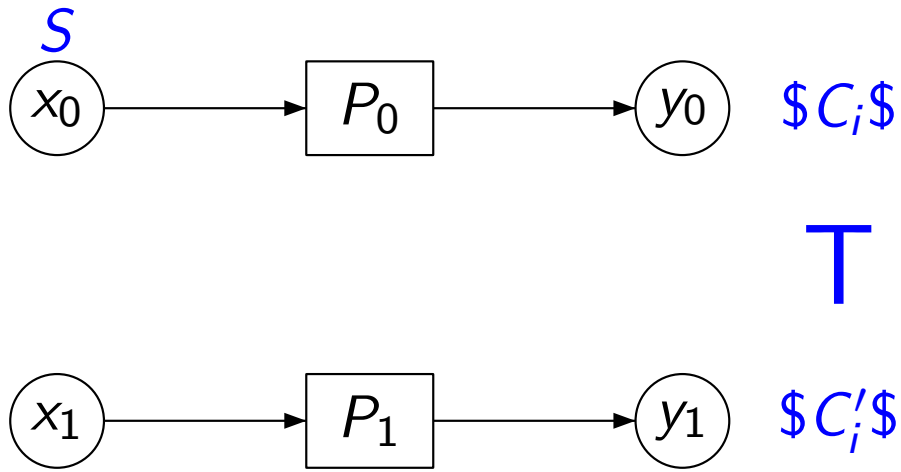

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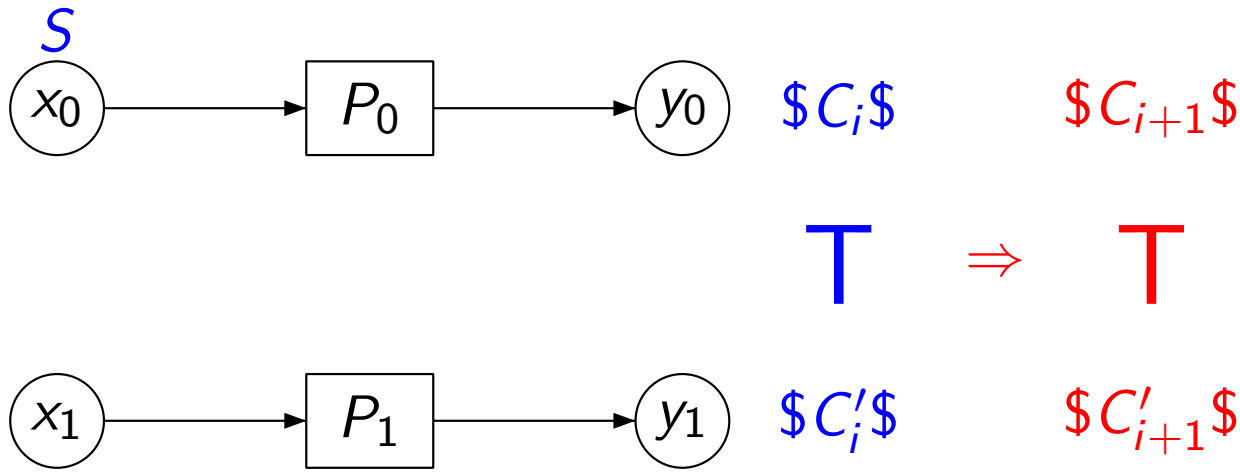
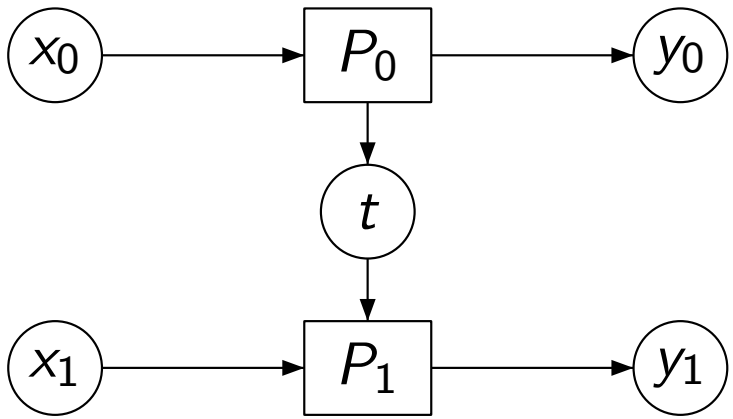
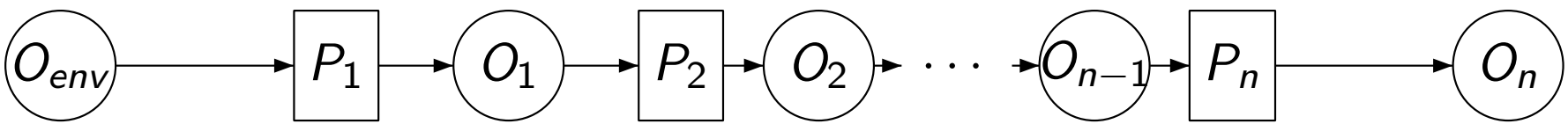


Decidable architecture



Pipe-line decidable for global specifications (Kupferman–Vardi '01)



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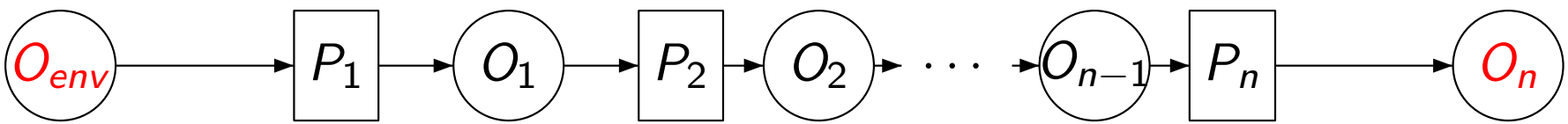

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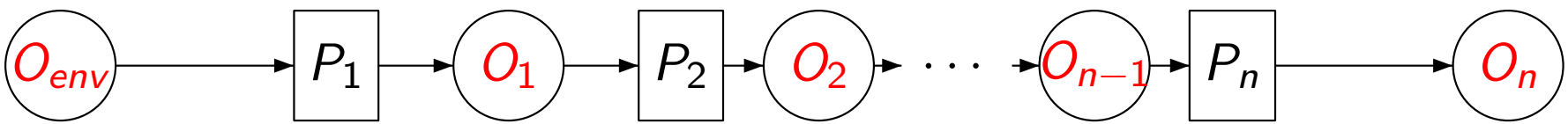
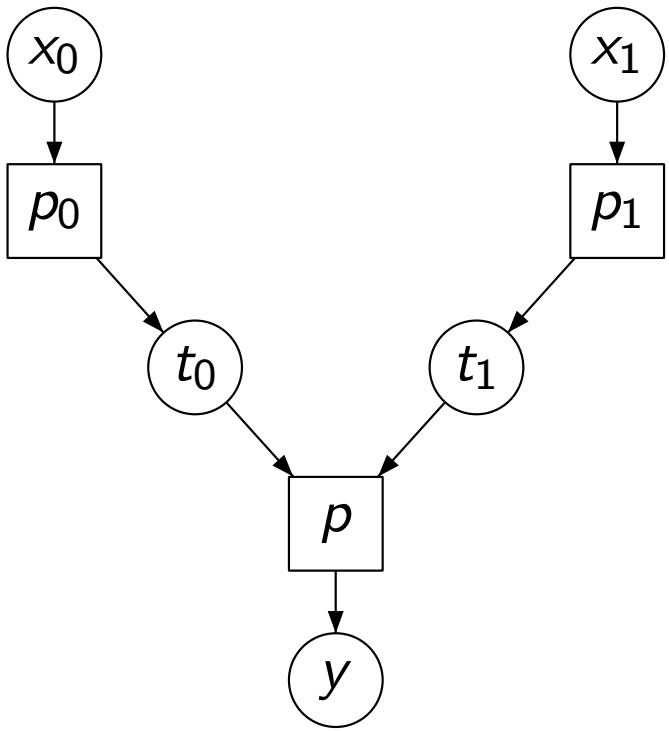


Information fork criterion (Finkbeiner–Schewe '05)



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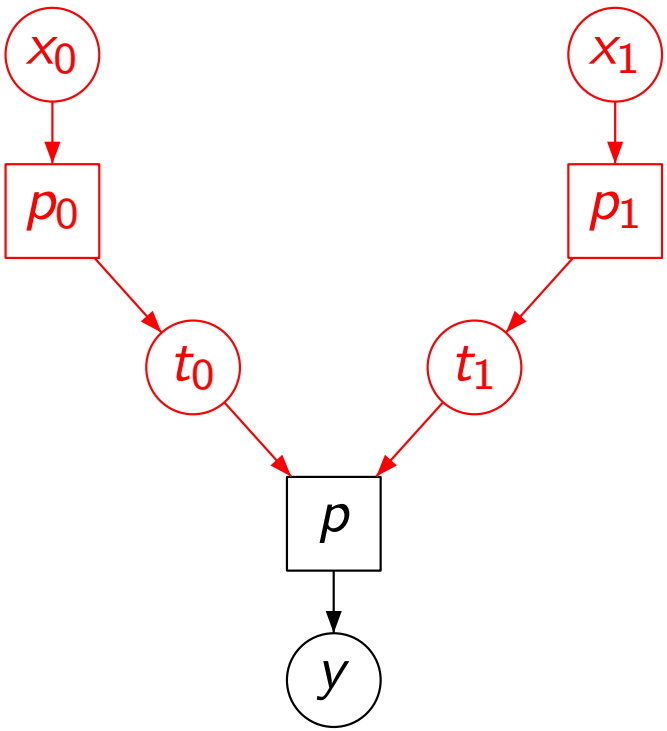
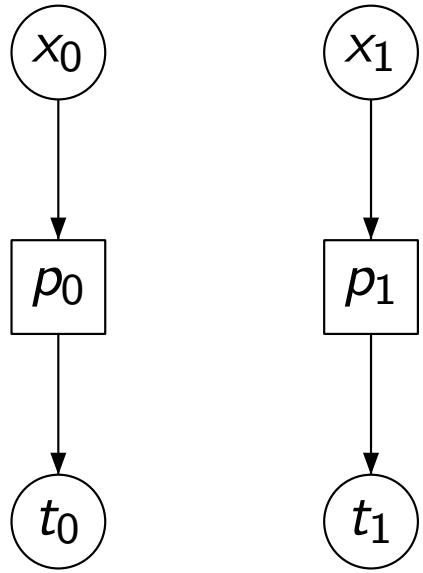




Outline

Outline

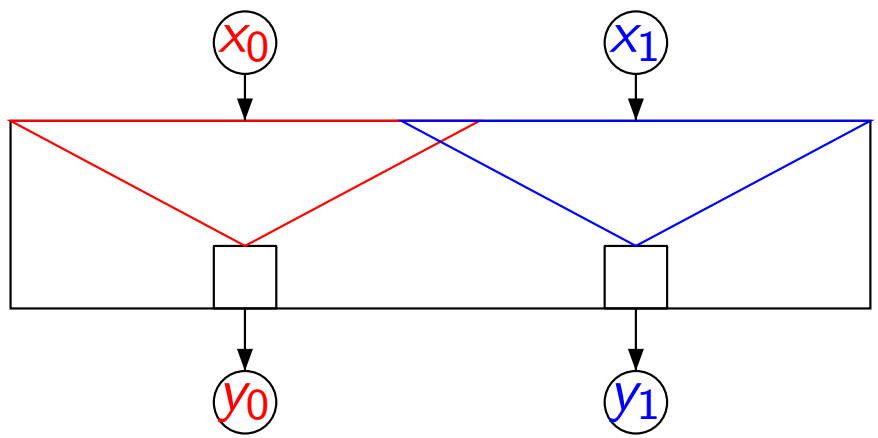
Uncomparable information yields undecidability

Definition

For an output variable v, View(v) is the set of input variables u such that v is accessible from u.

Definition

An architecture has uncomparable information if there exist x,y output variables such that $\text{View}(x) \setminus \text{View}(y) \neq \emptyset$ and $\text{View}(y) \setminus \text{View}(x) \neq \emptyset$. Otherwise it is said to have preordered information.



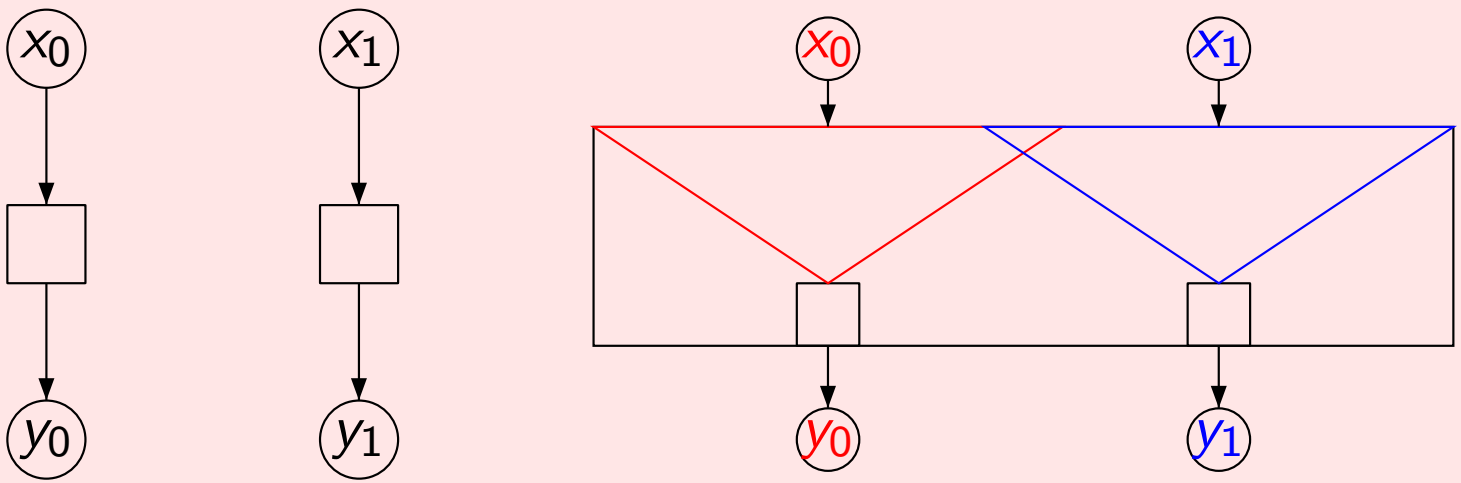
Uncomparable information yields undecidability

Theorem

Architectures with uncomparable information are undecidable for LTL or CTL input-output specifications.

Proof

LTL specifications :



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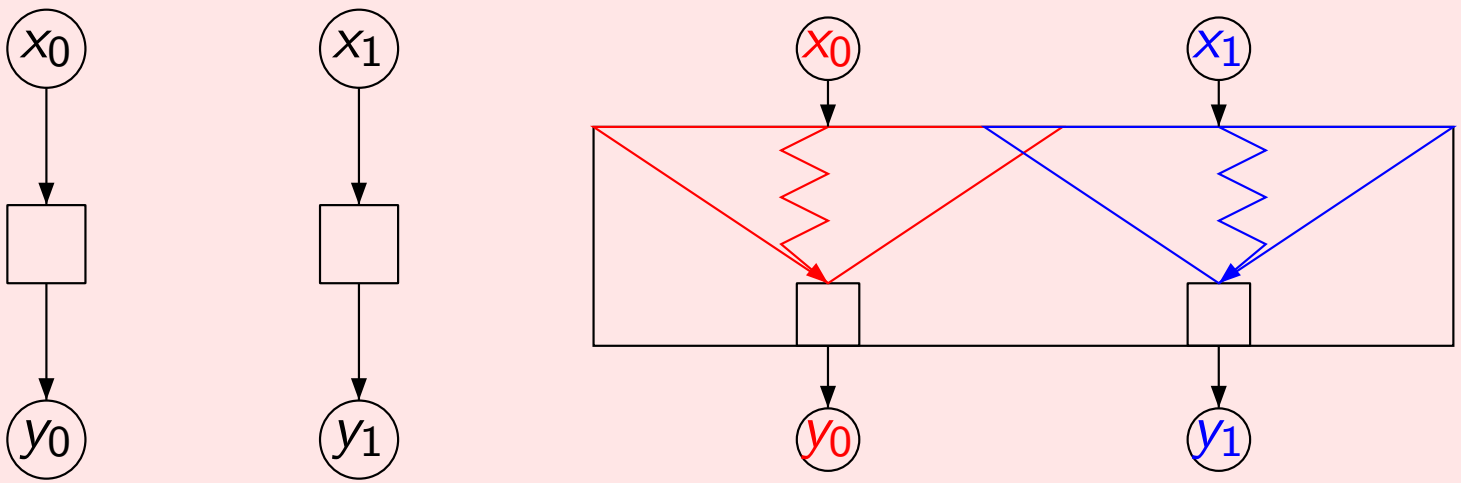

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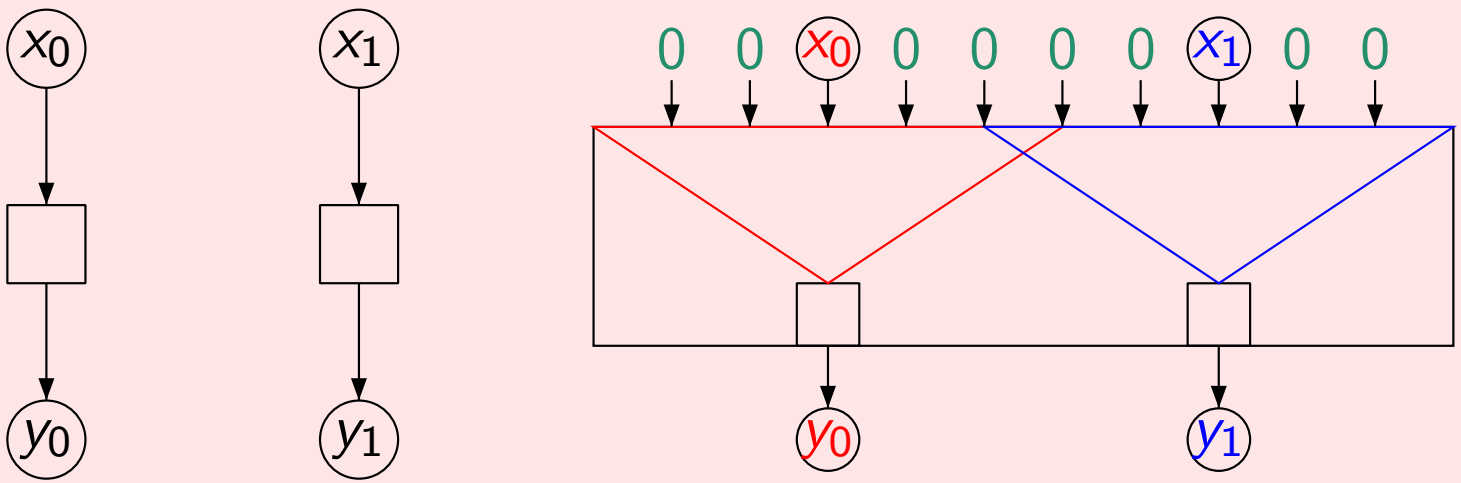

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Uniformly well connected architectures

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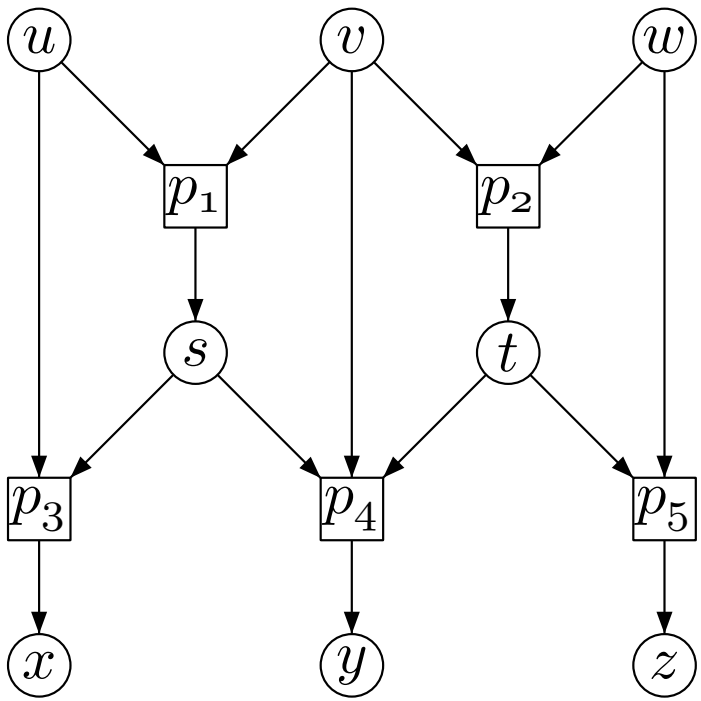
An architecture is uniformly well connected if there is a uniform way to route variables in View(v) to v for each output variable v.



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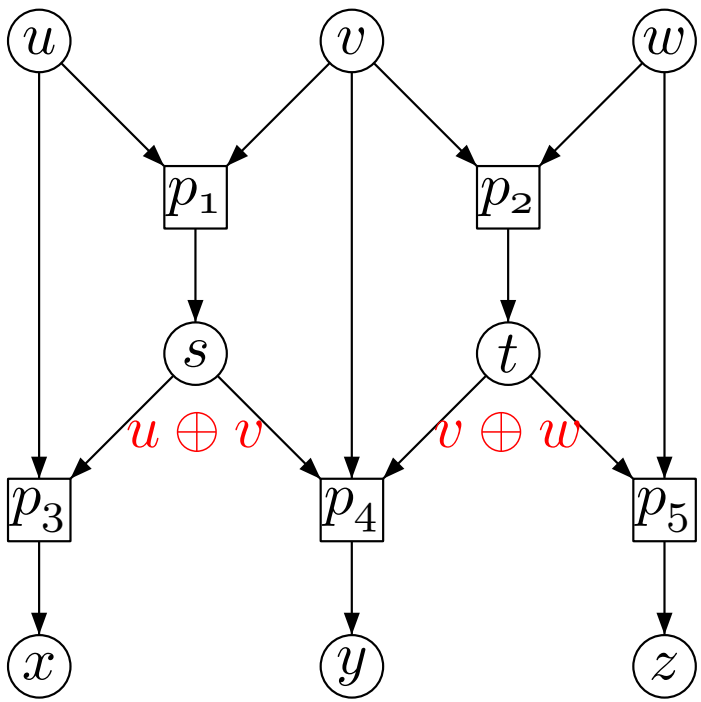
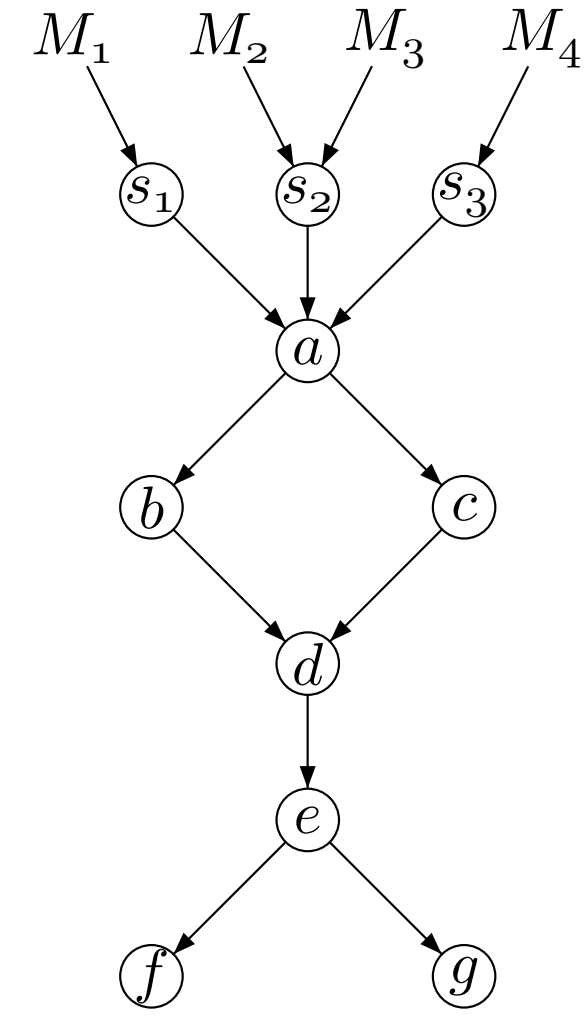
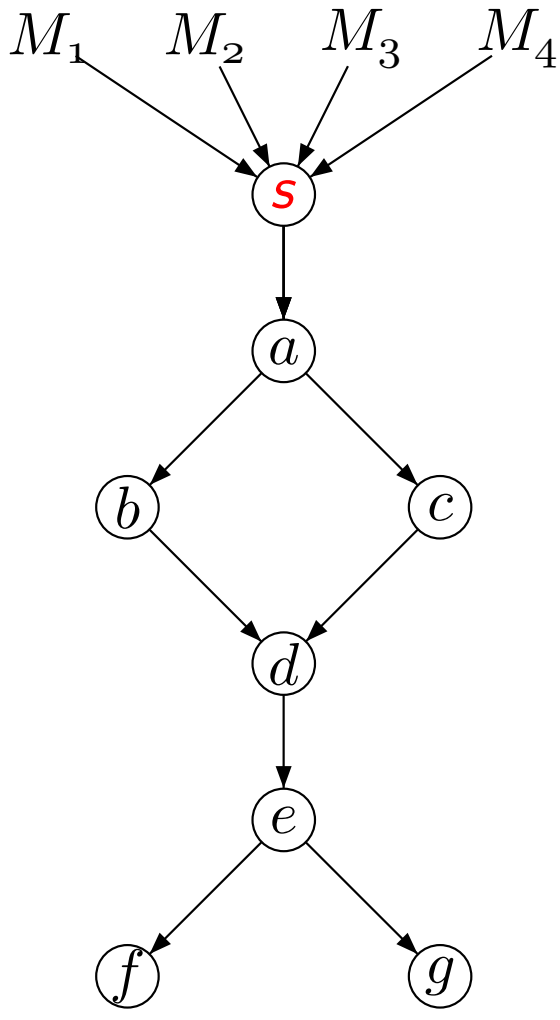


Network Information Flow



Network Information Flow : Multicast



demands
M_1, M_2, M_3, M_4

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Relations between the two problems

Theorem

The multicast reduces to checking uniform well connectedness.

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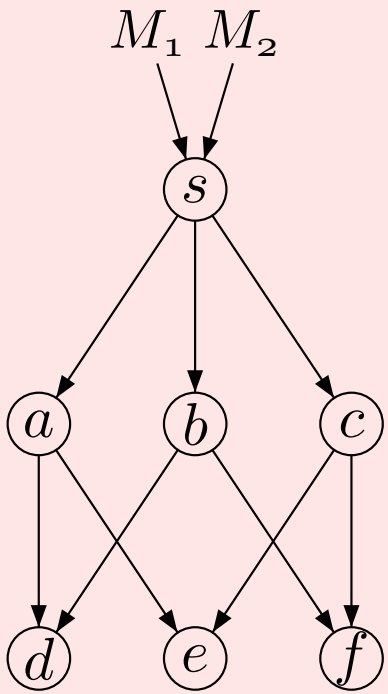

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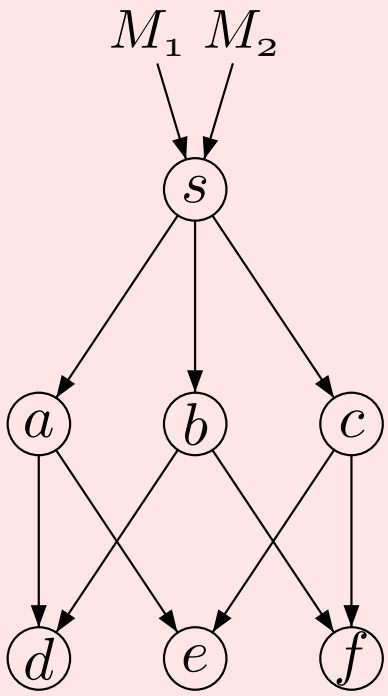

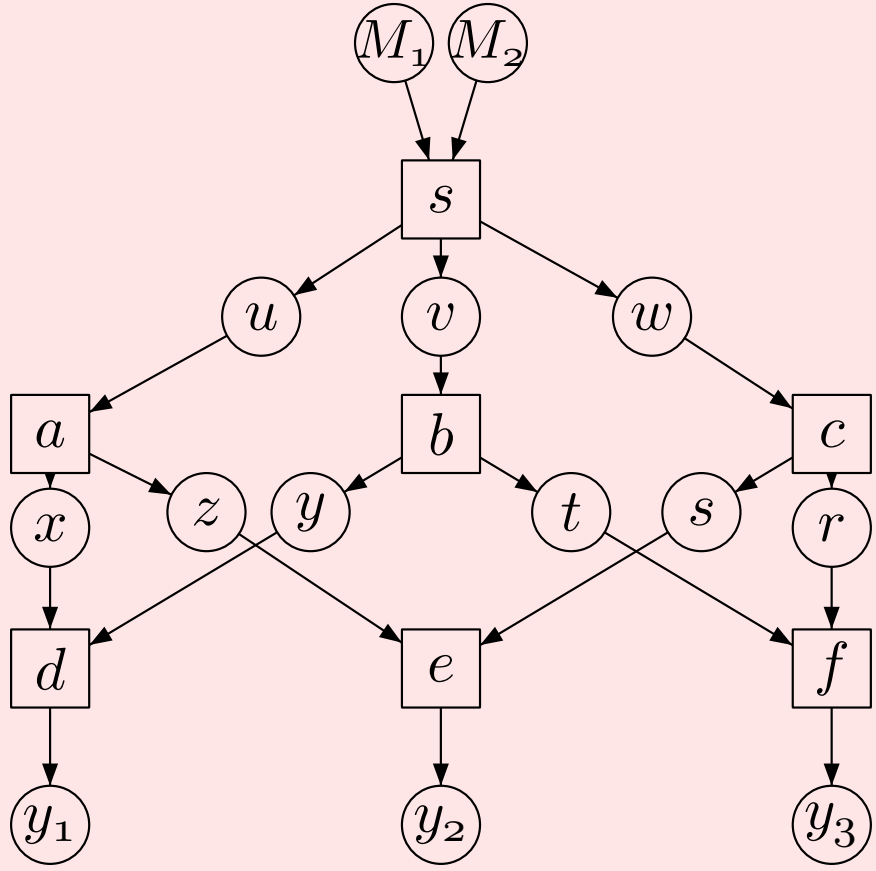

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Checking uniform well connectedness reduces to the network information flow.

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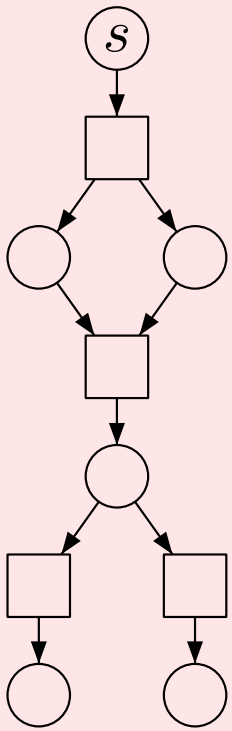

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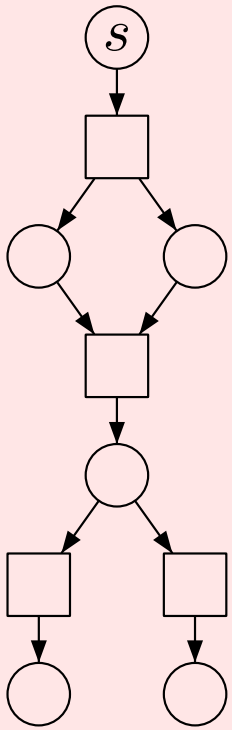

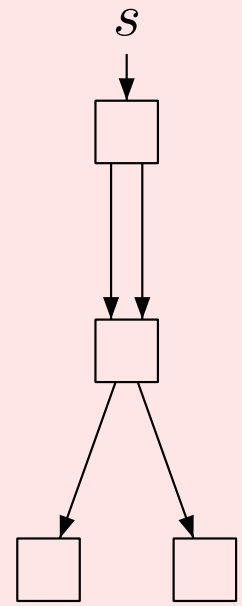

demands s demands s

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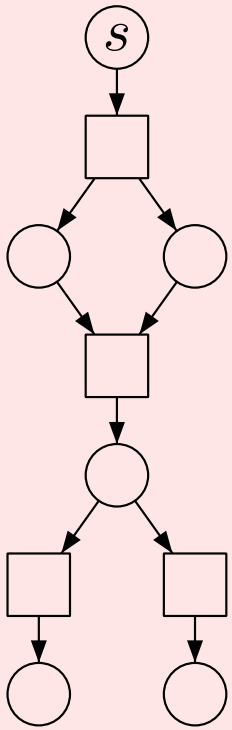

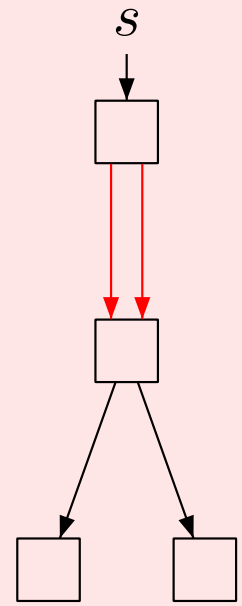

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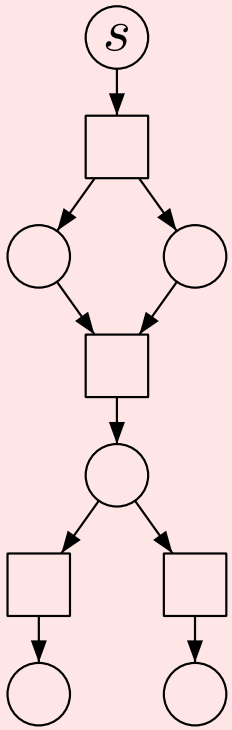

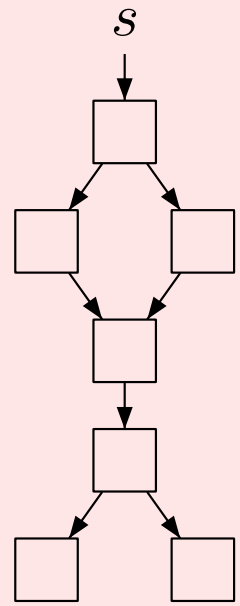

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Complexity

Rasala Lehman-Lehman 2004

Multicast whith alphabet size $q = p^k$ (where p is prime) is NP-hard.

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It is trivially NP. Reduction from multicast gives NP-hardness.

Decidability criterion for uniformly well connected architectures

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Architectures with preordered information are decidable for CTL specifications.*

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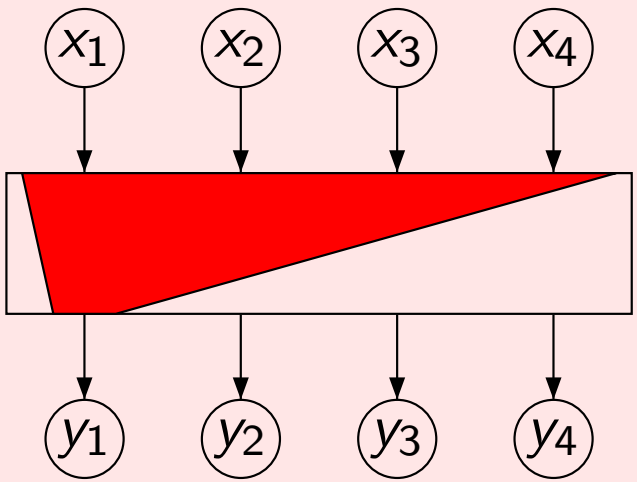

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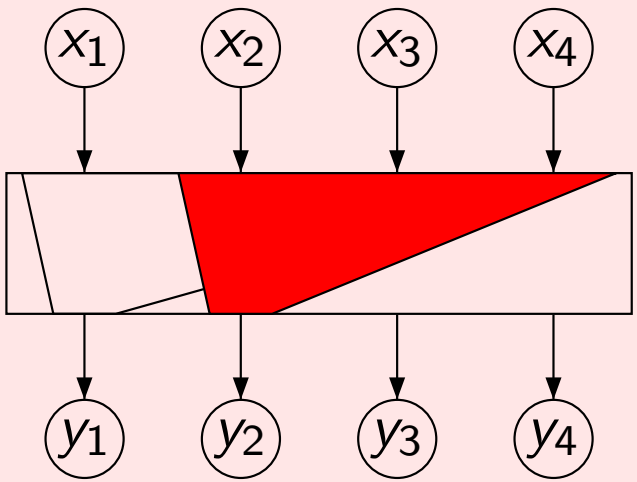

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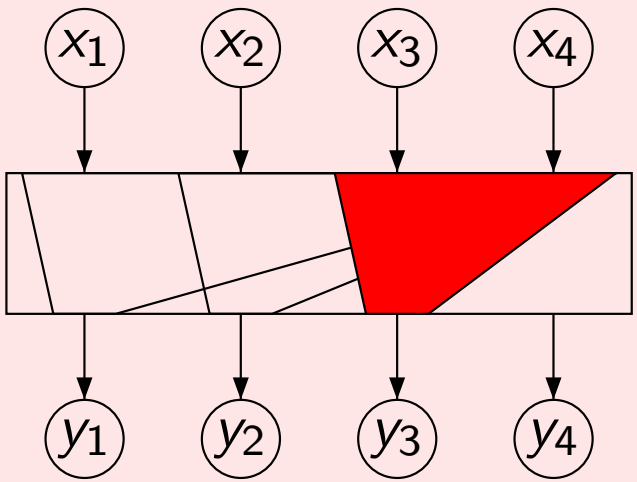

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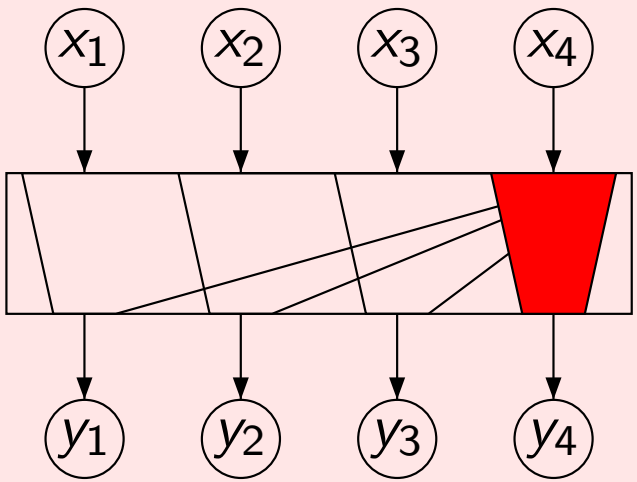

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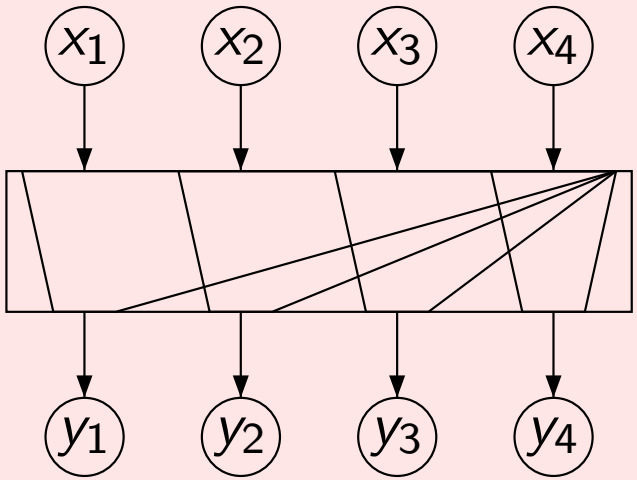
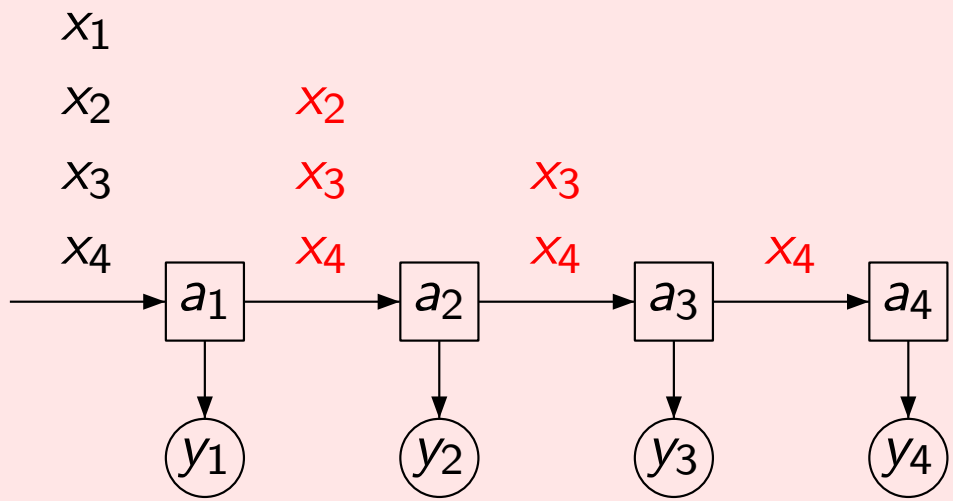




Theorem: Kupferman-Vardi (LICS'01)

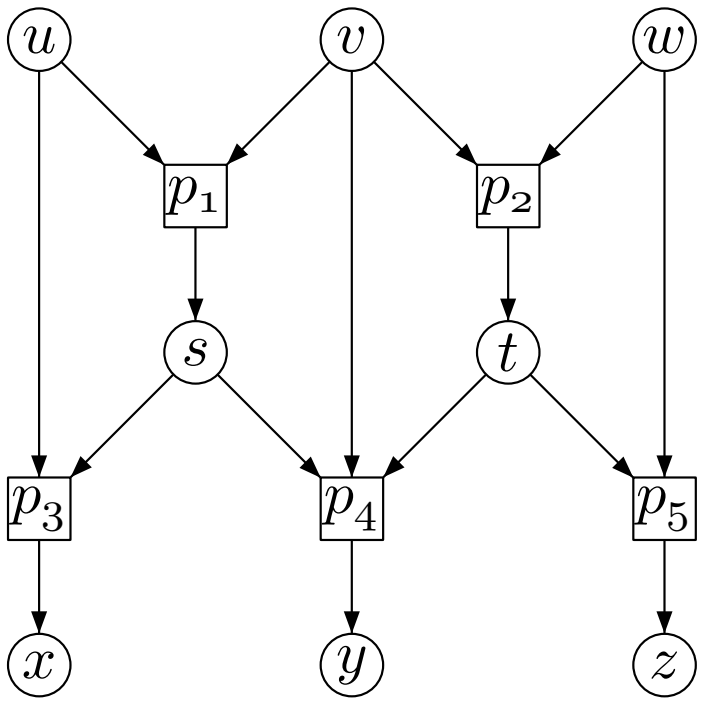
The synthesis problem with local strategies is decidable for pipeline architectures and CTL* specifications (or tree-automata specifications) on all variables.

Outline

Well connected architectures

Definition

An architecture is well connected if, for each output variable y, the subarchitecture formed by $E^{*-1}(y)$ is uniformly well connected.



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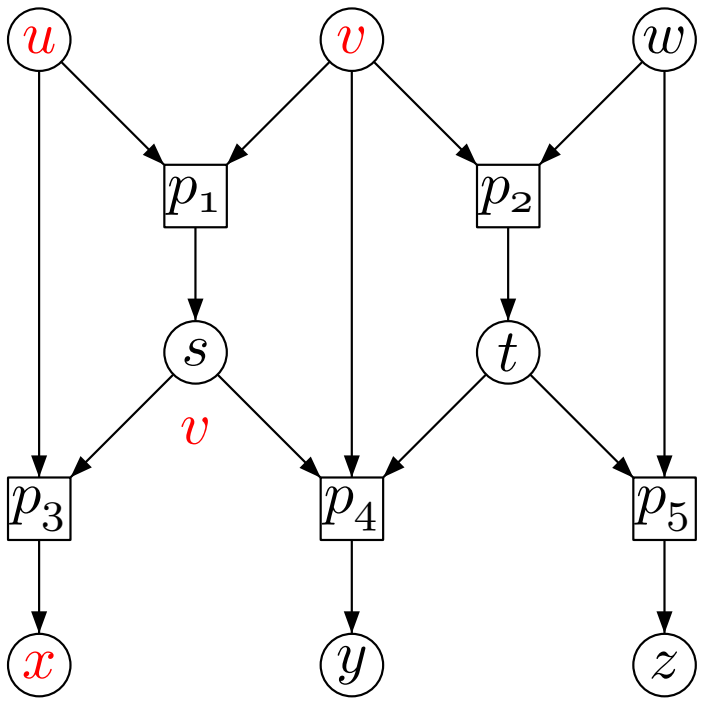

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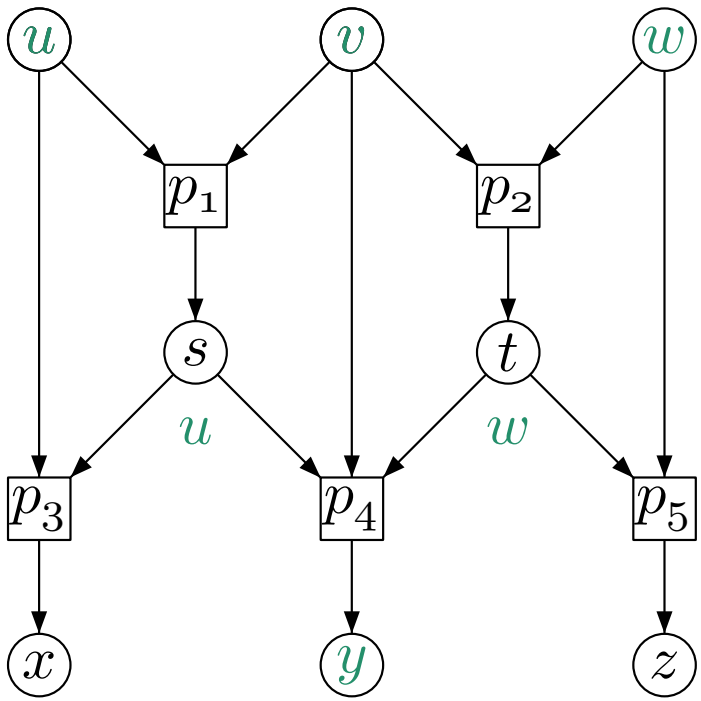

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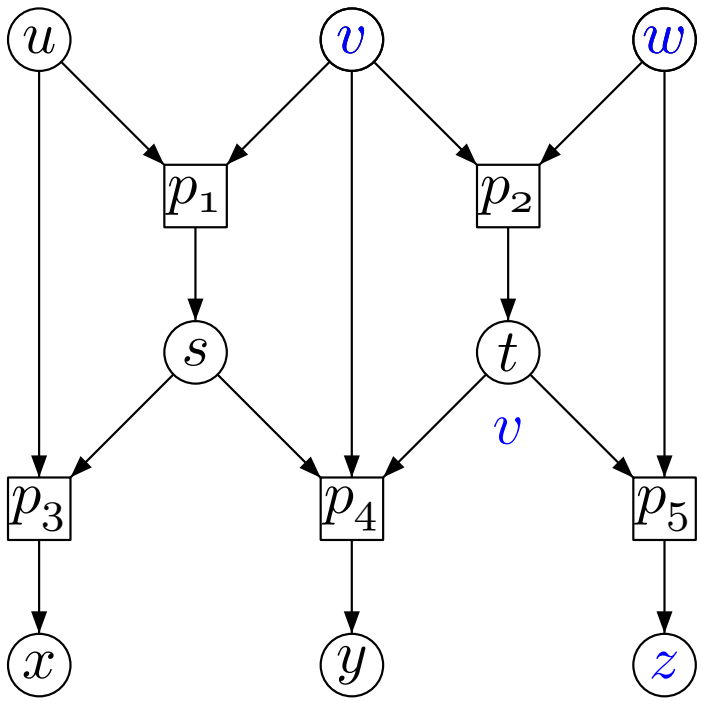

Well connected architectures

Theorem

One can decide whether an architecture is well-connected in polynomial time.

Well connected architectures

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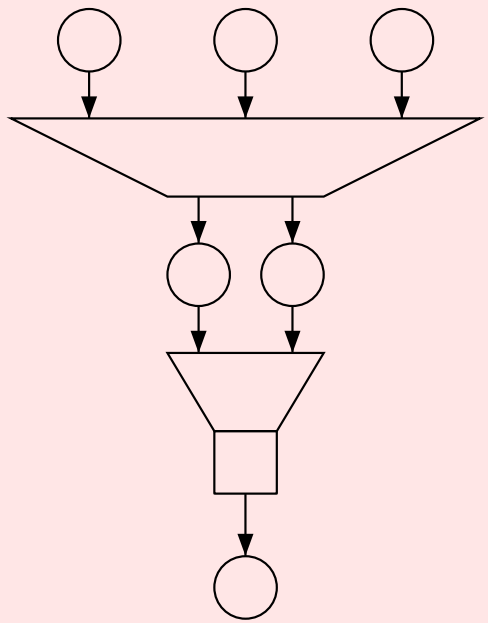

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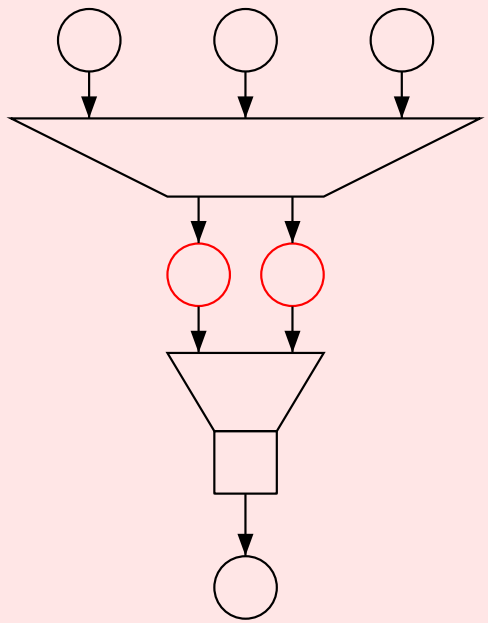

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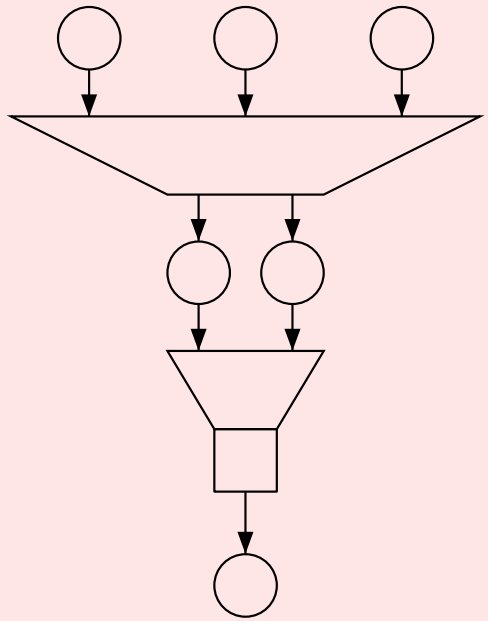

Rasala Lehman–Lehman 2004

One can solve the network information flow in the special case where there is a unique sink in polynomial time.

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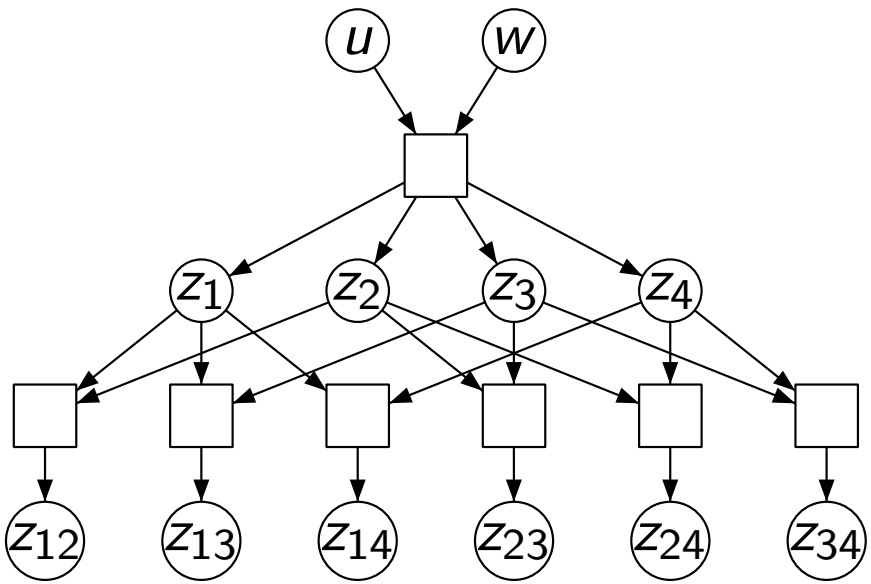

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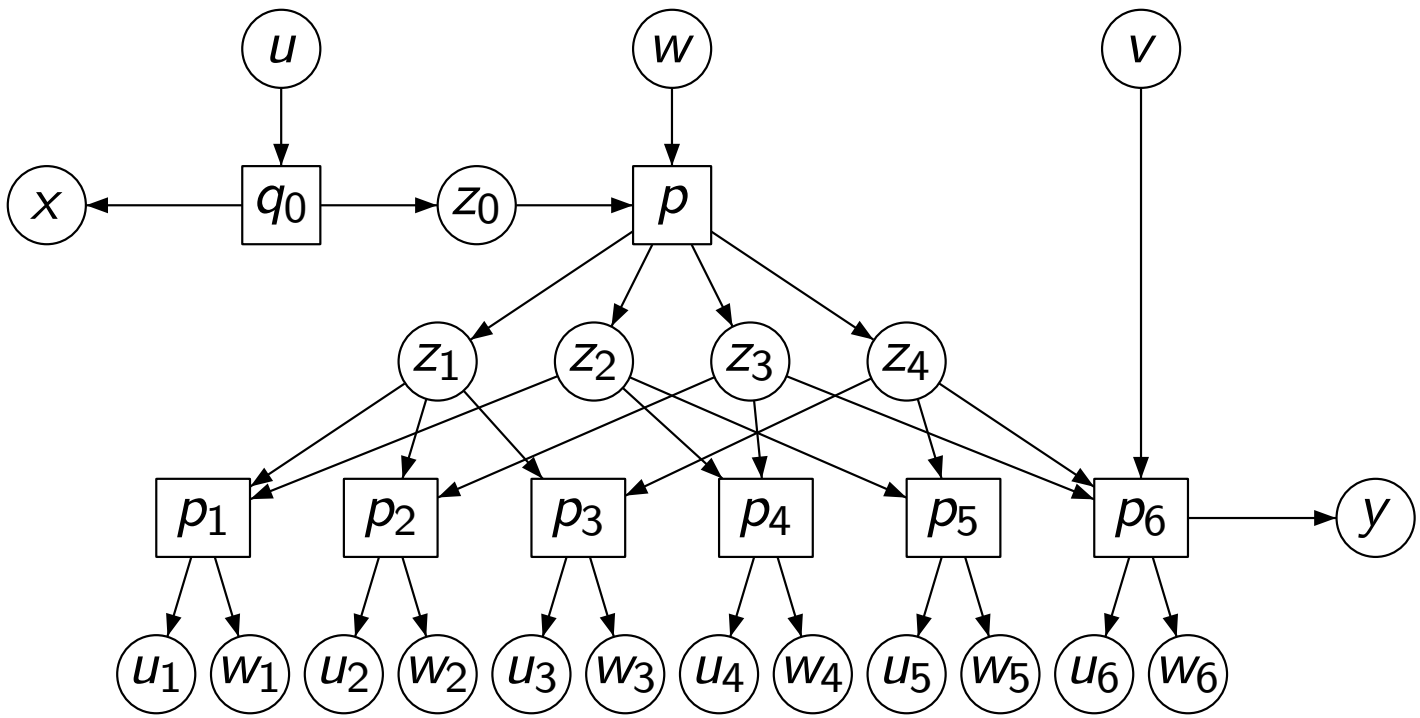

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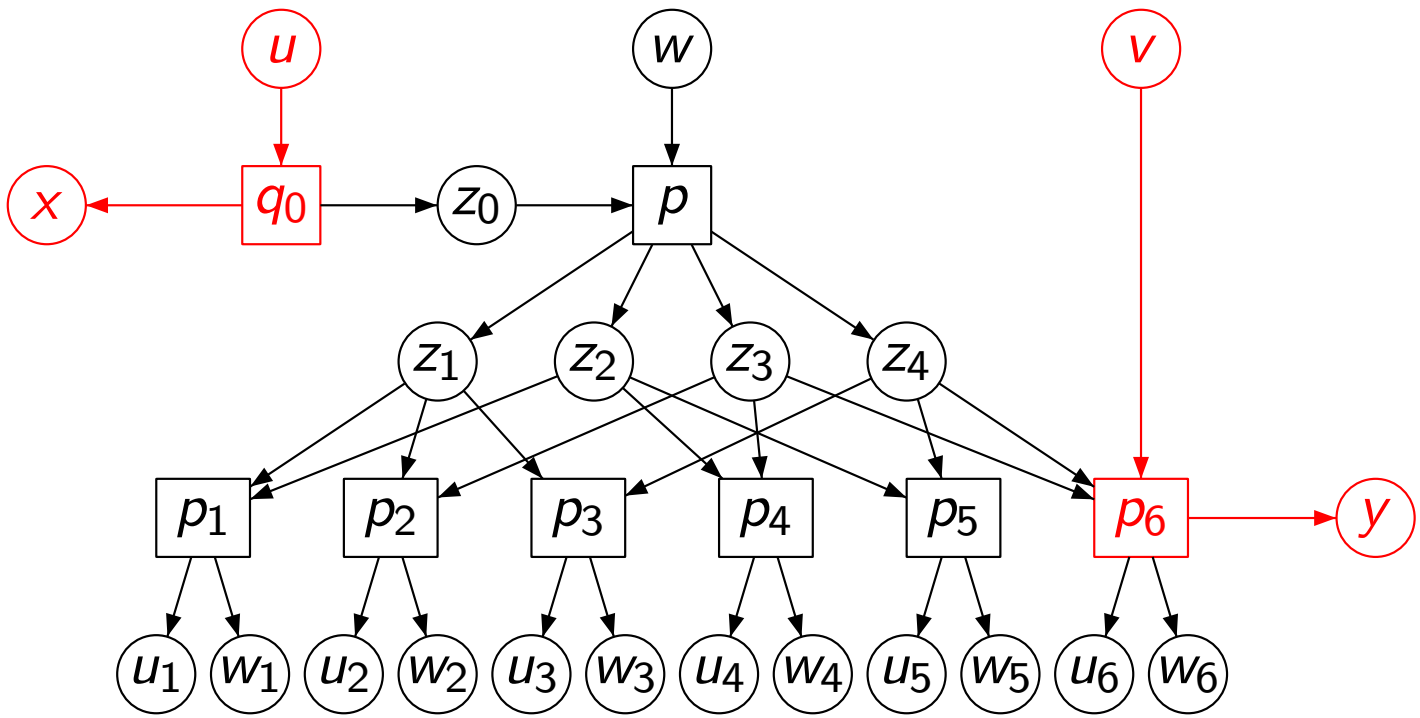

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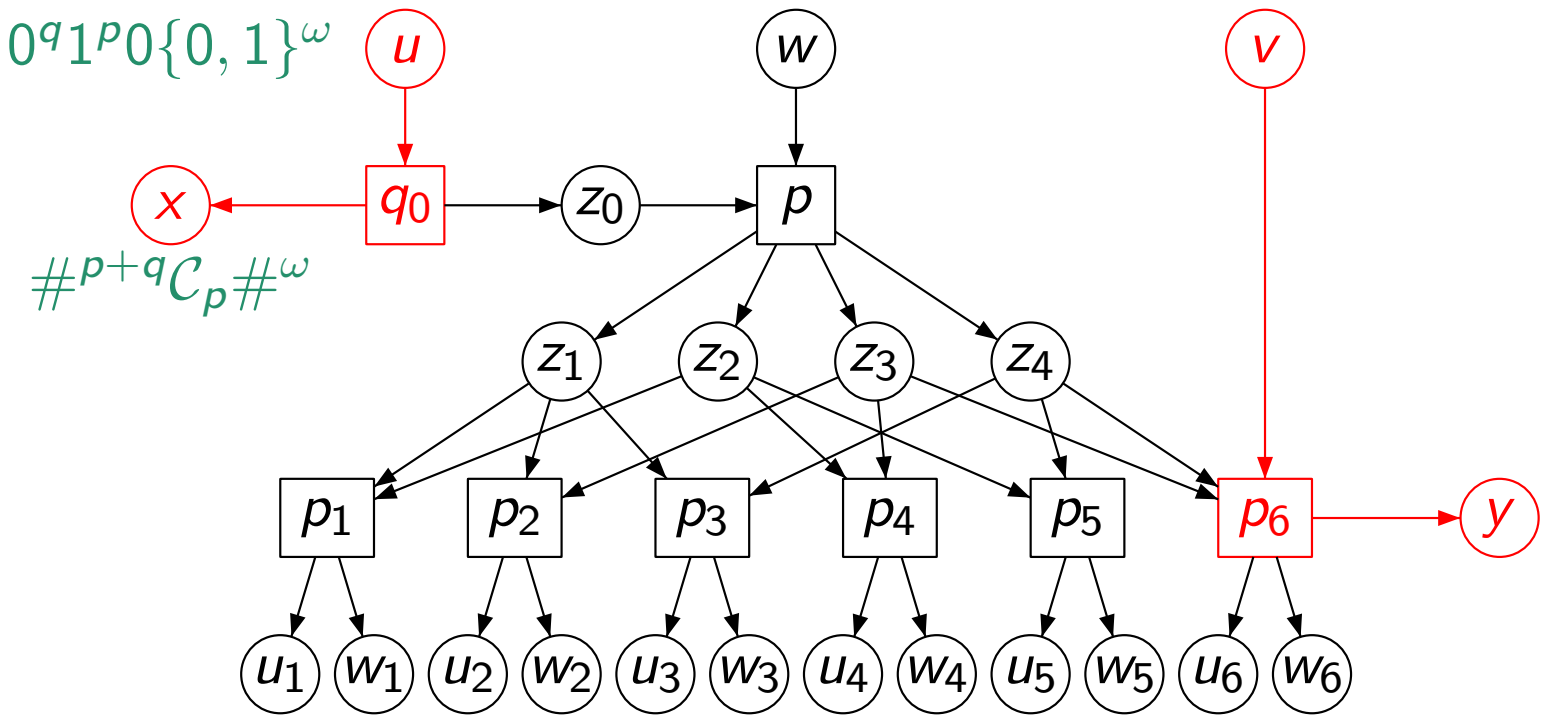
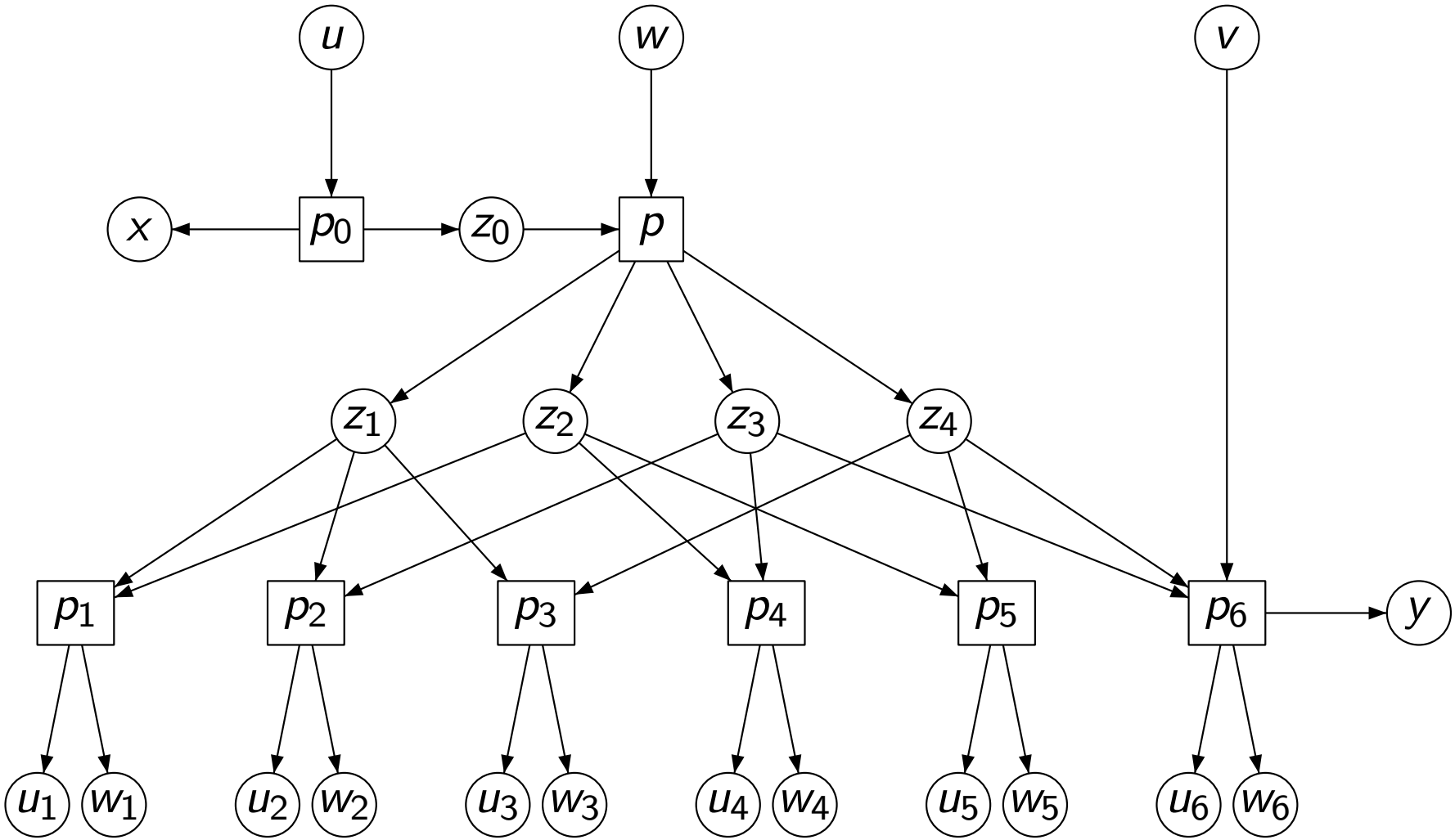


Well connected preordered architectures are undecidable : The specification



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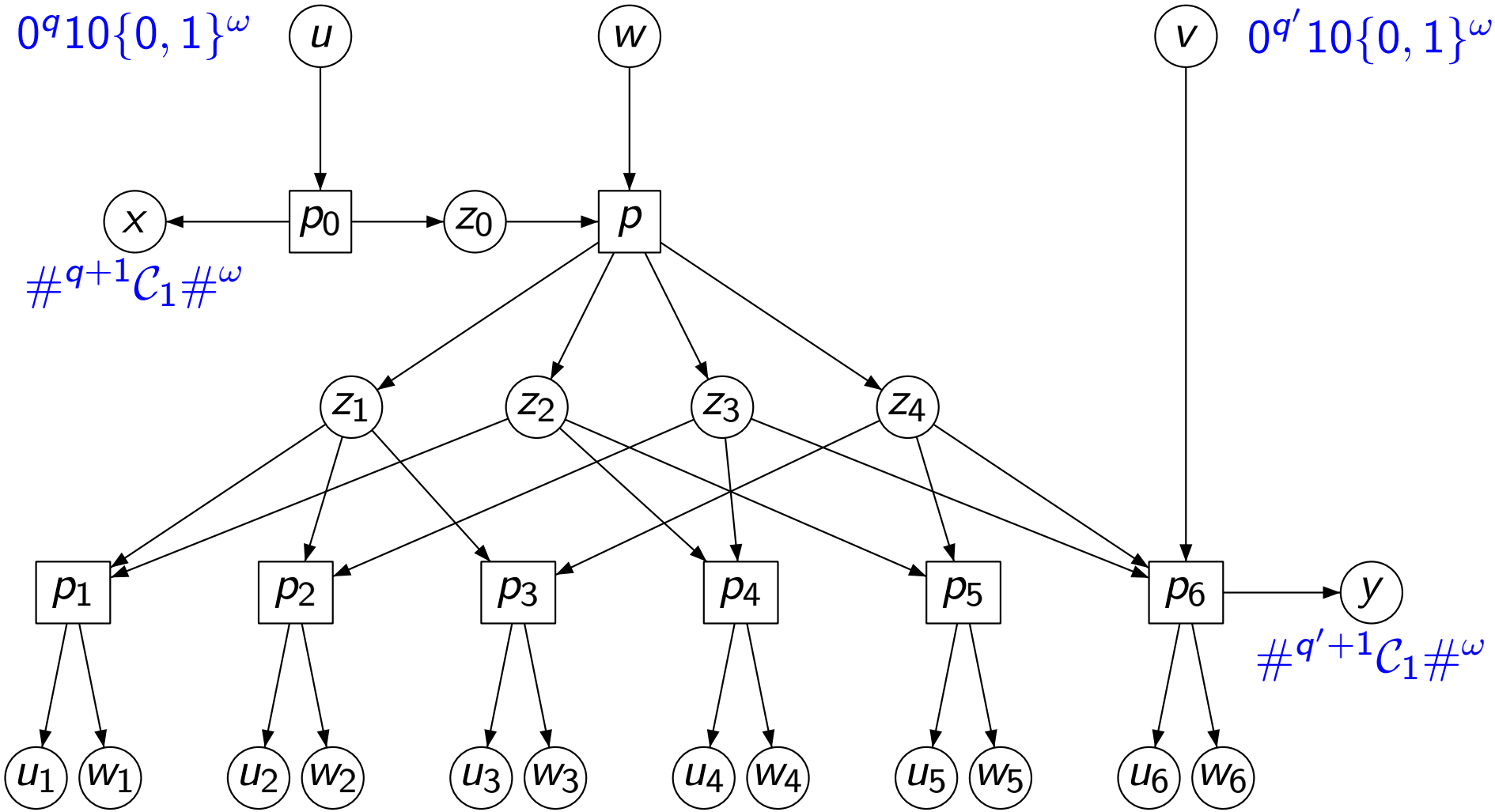

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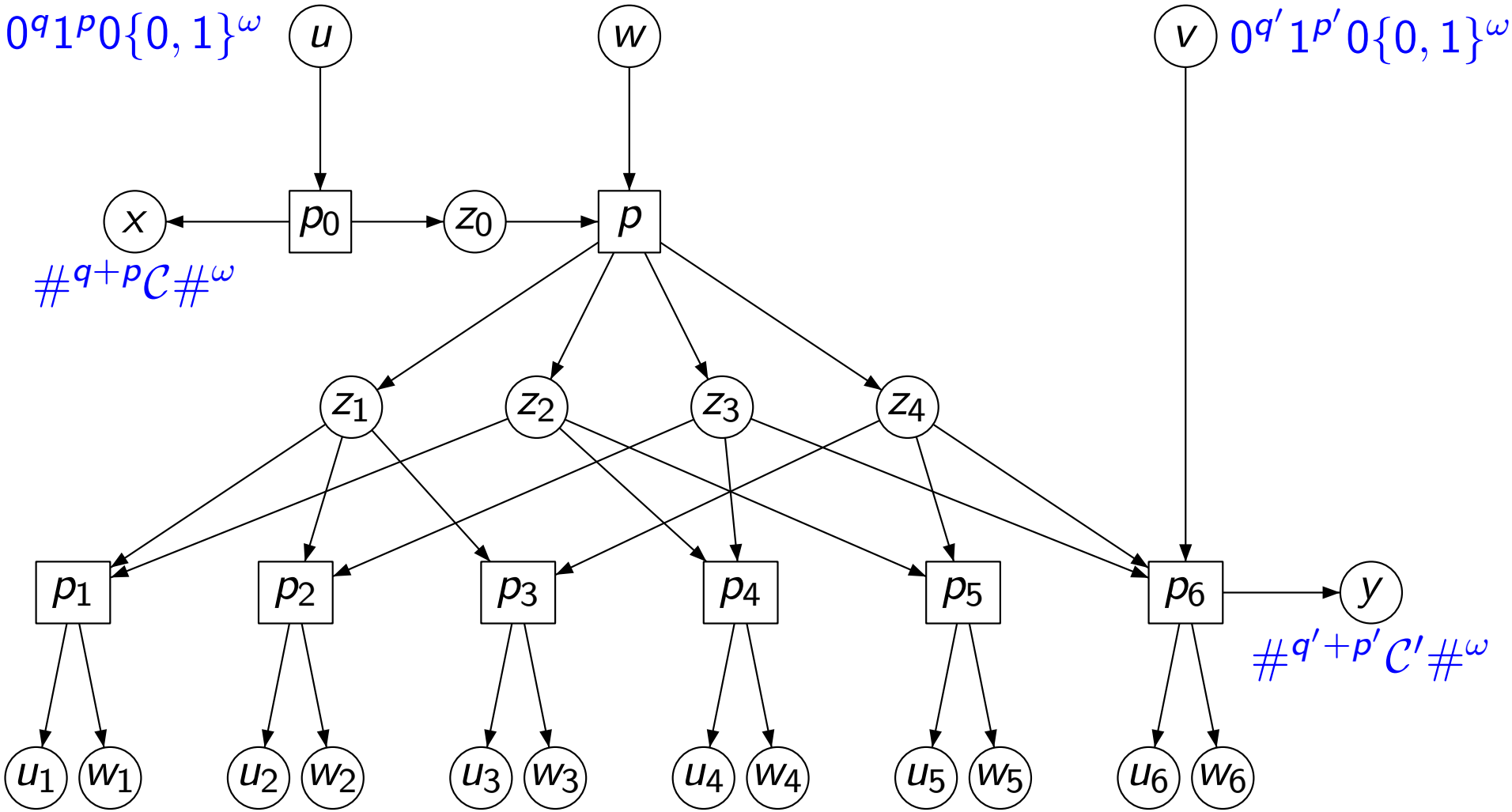

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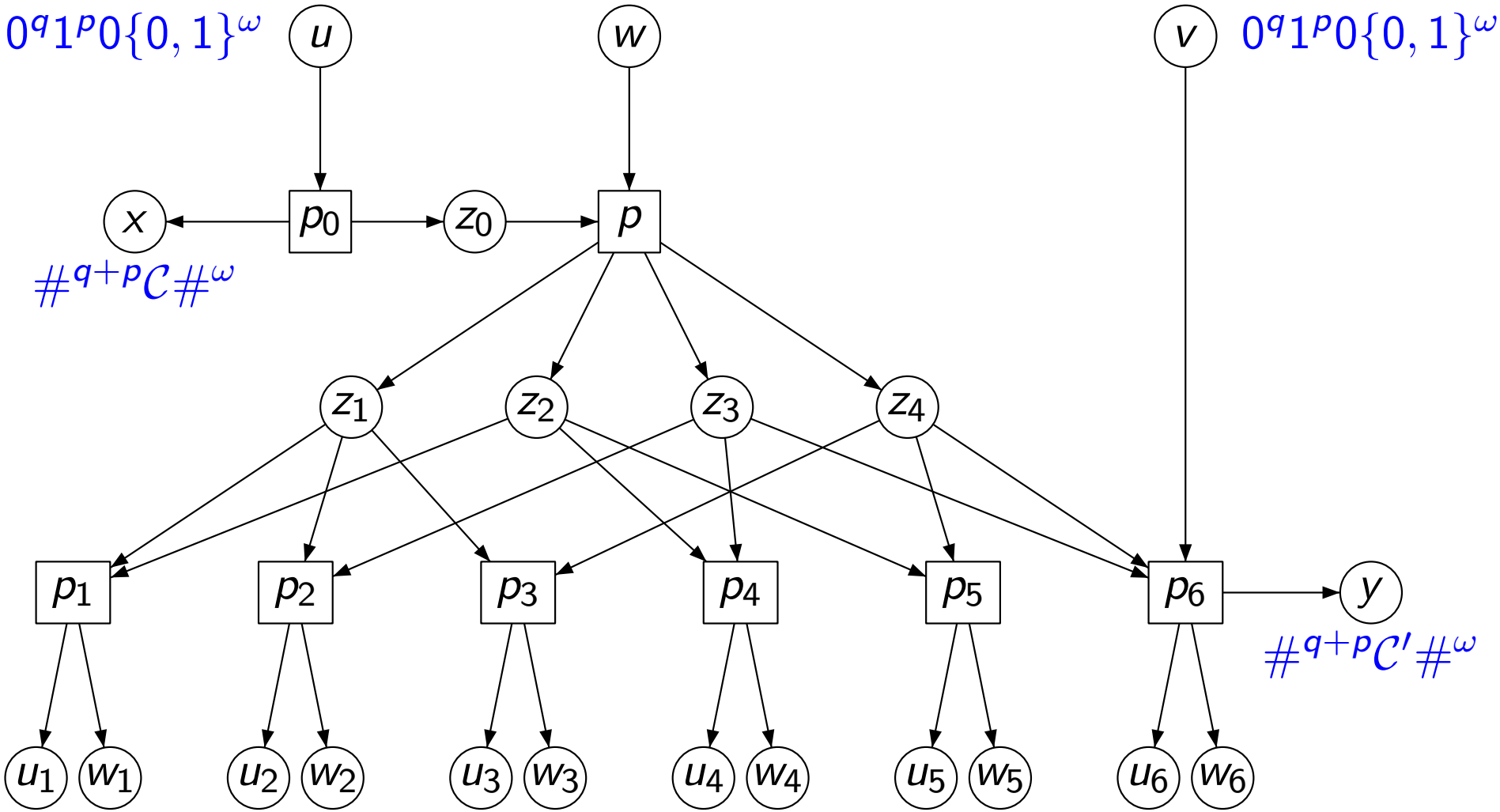

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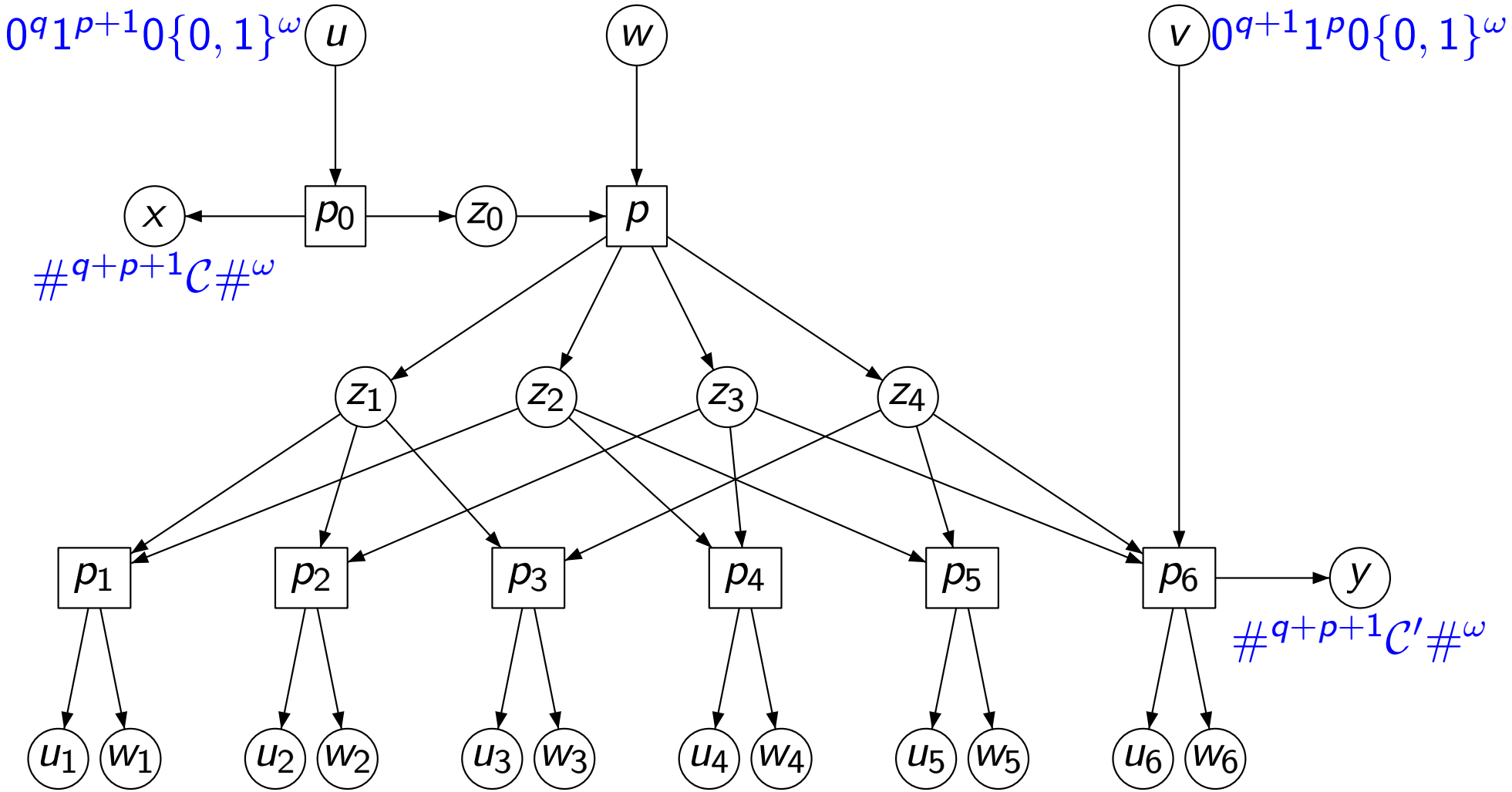

$$u = v + 1 \implies \mathcal{C}' \vdash \mathcal{C}$$

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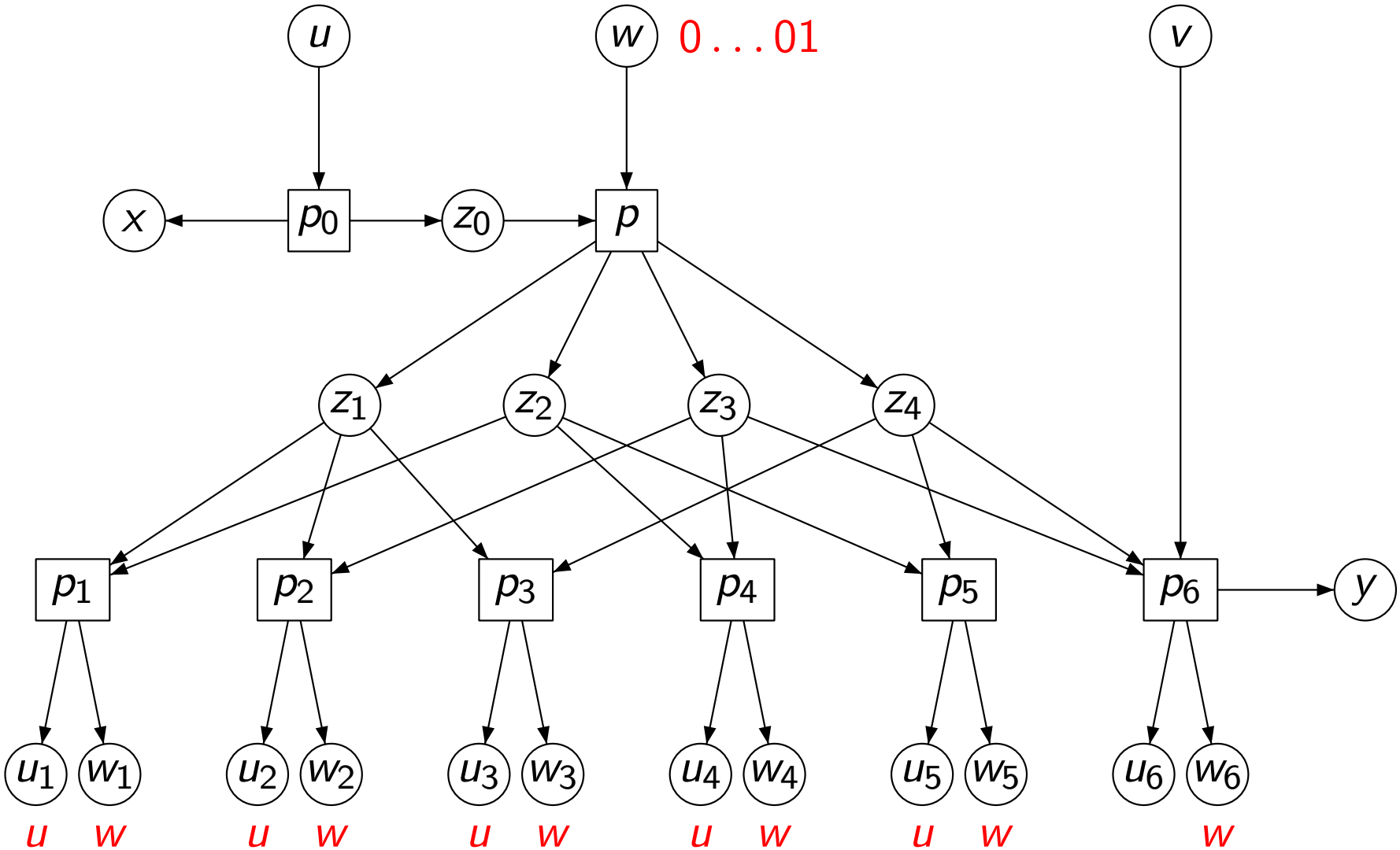
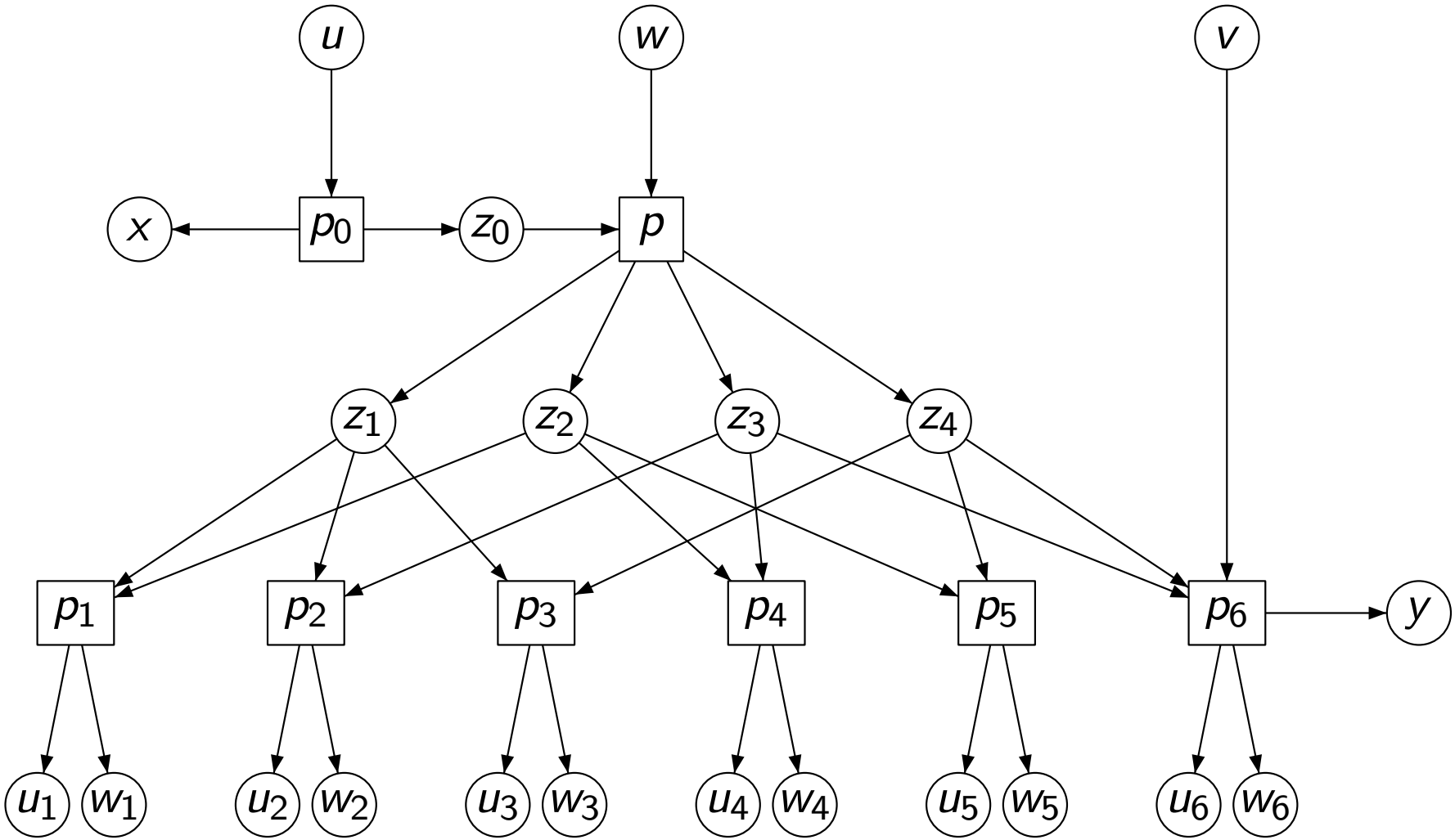


Well connected preordered architectures are undecidable : A distributed implementation



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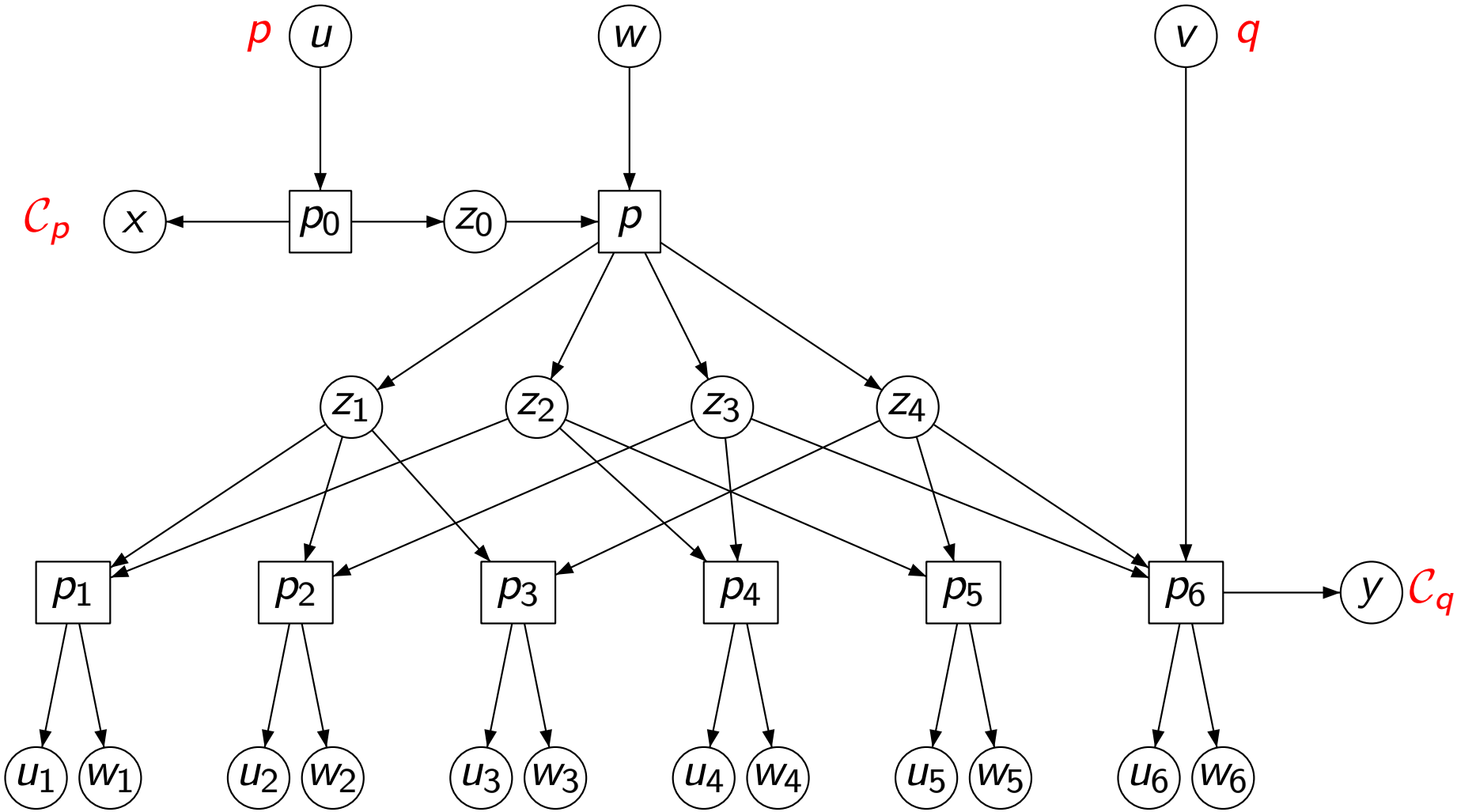

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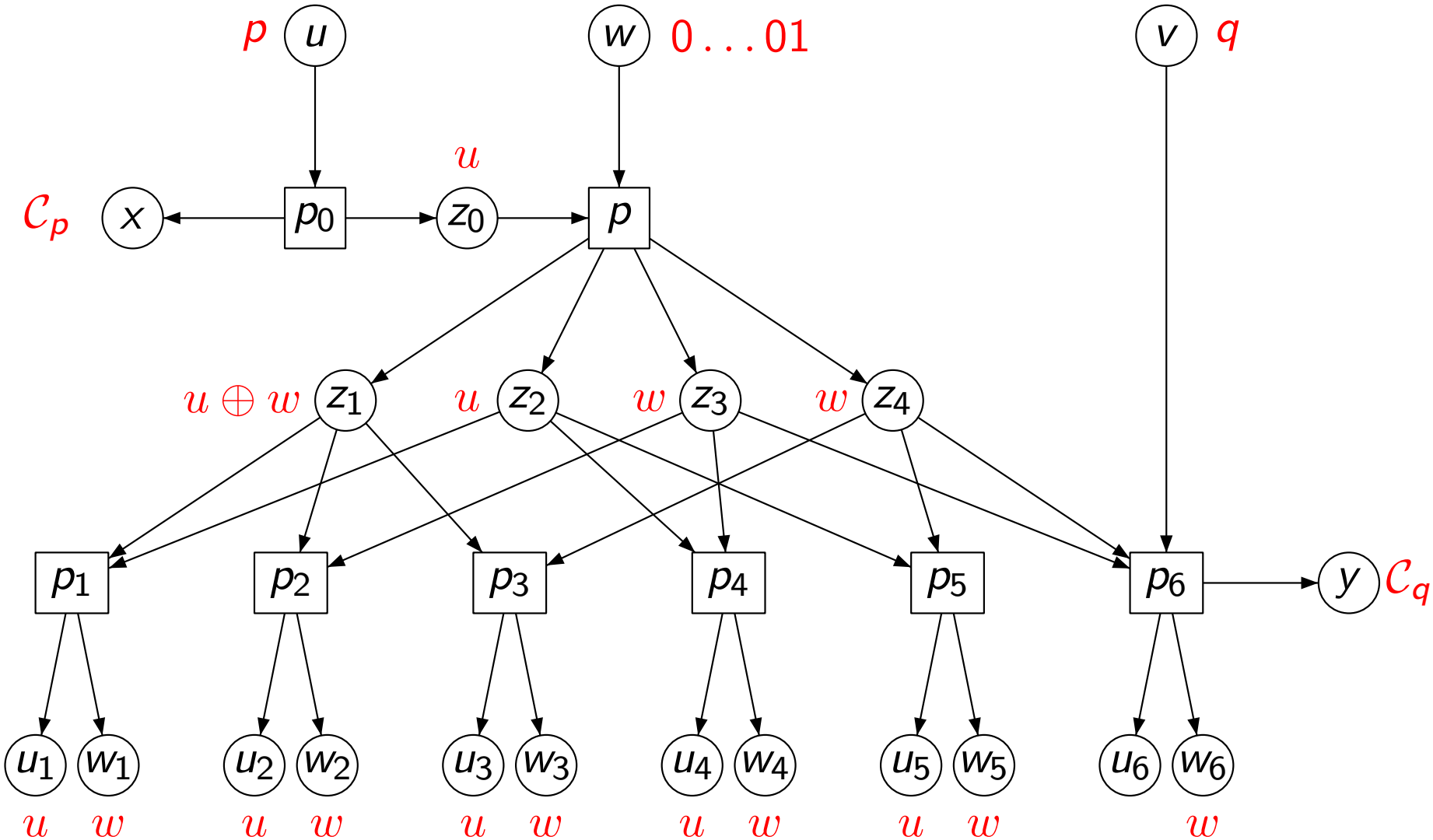

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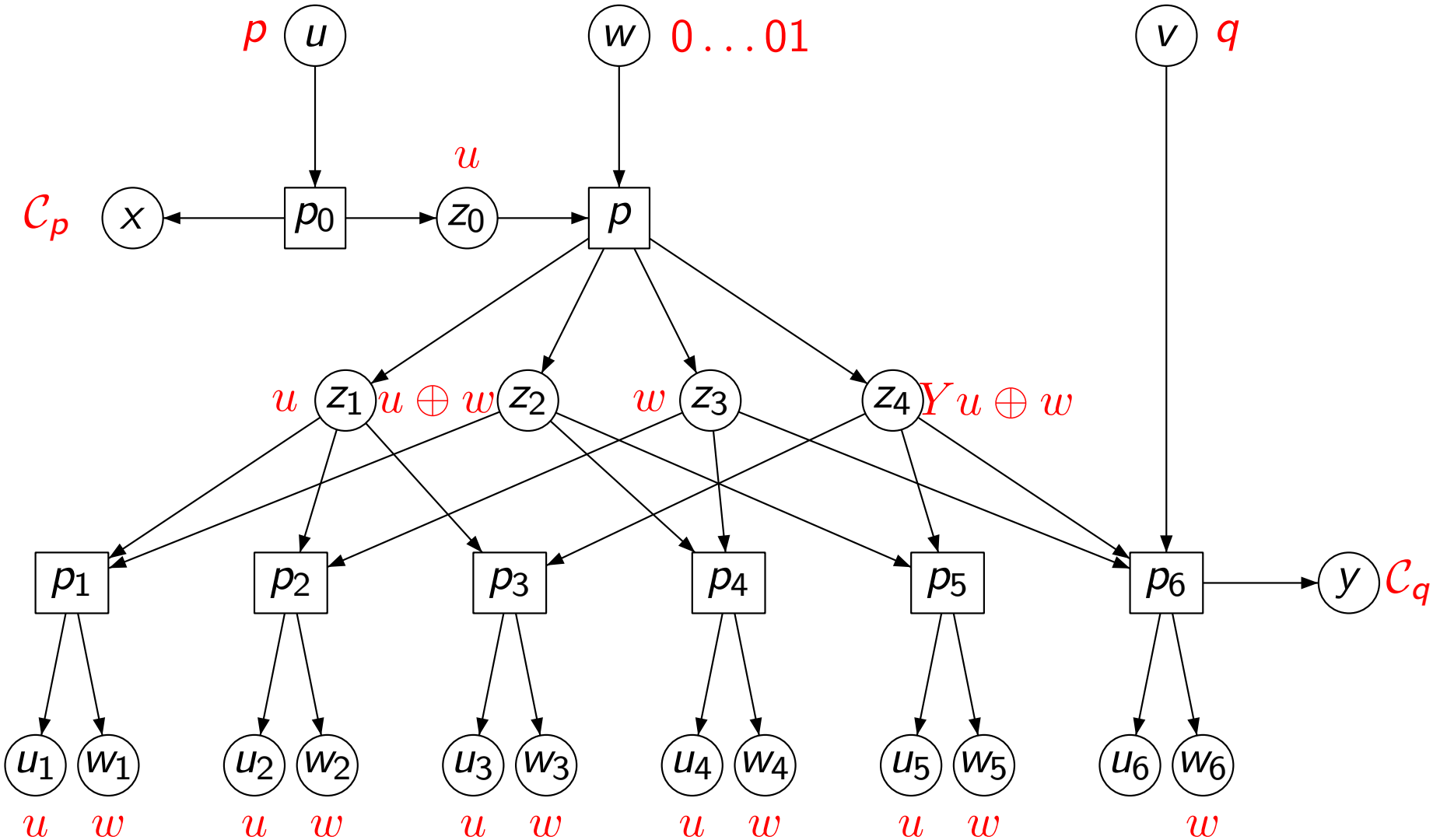

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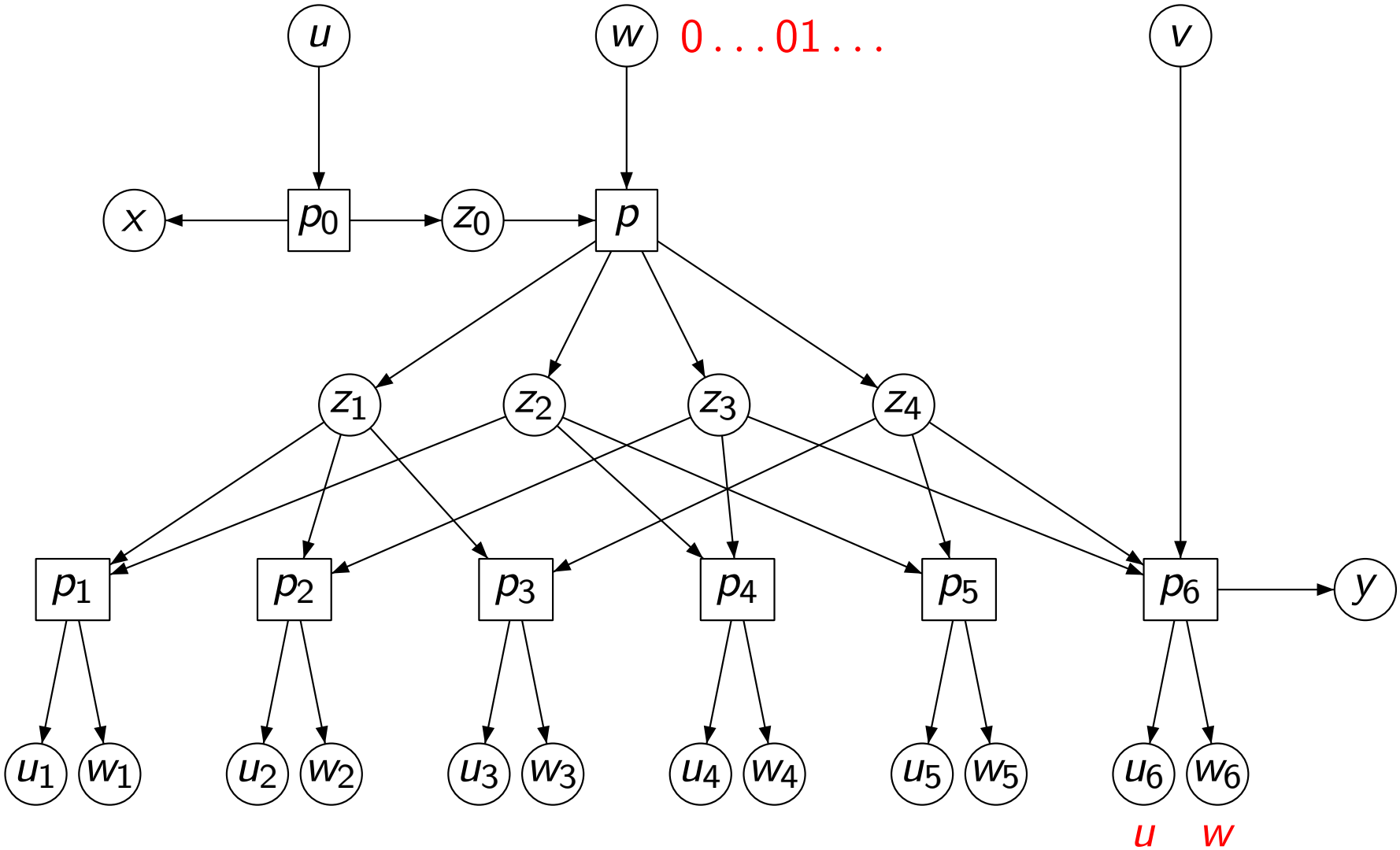
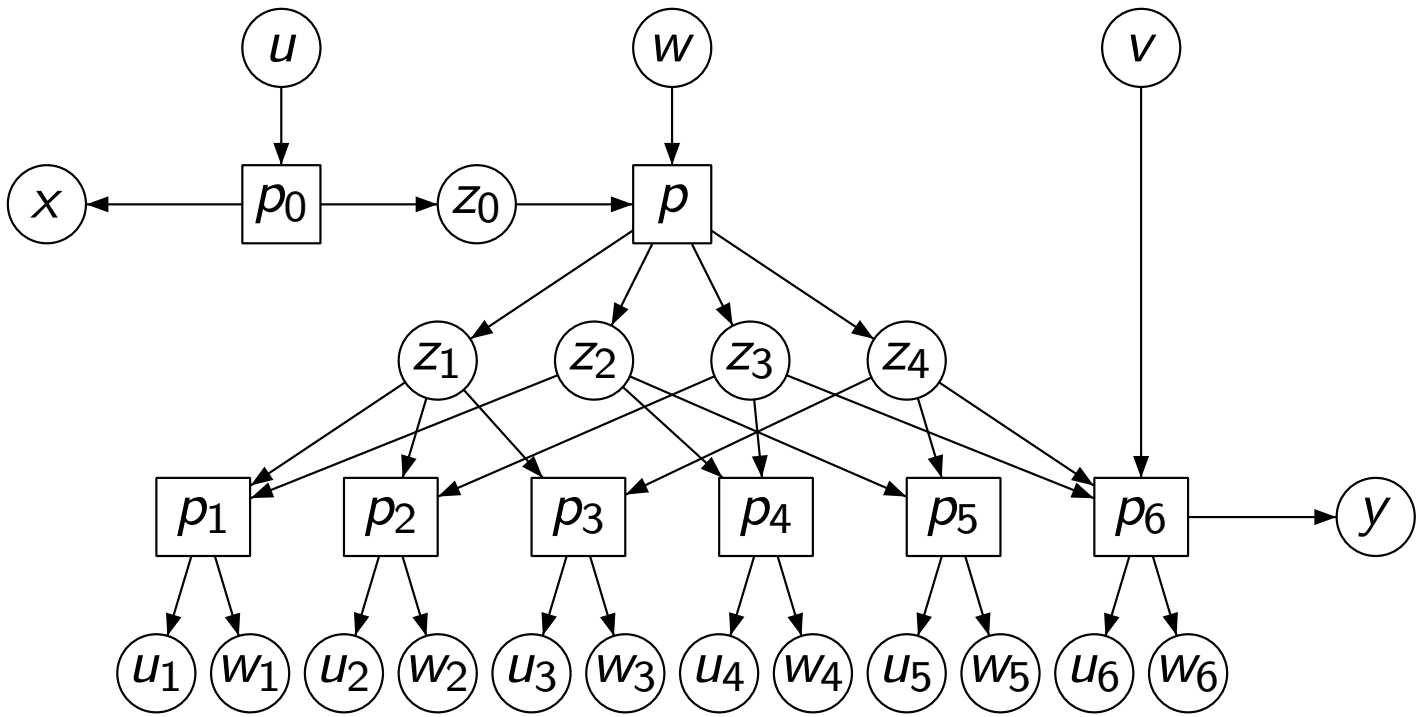


Well connected preordered architectures are undecidable: Masking one bit of u to y

Lemma

Let $u^b = 0^q b 1^p 0 u'$ *and* $w = 0^q 1 w'$. *Then*
$(\hat{f}_{z_3}, \hat{f}_{z_4})(u^0[n], w[n]) = (\hat{f}_{z_3}, \hat{f}_{z_4})(u^1[n], w[n])$



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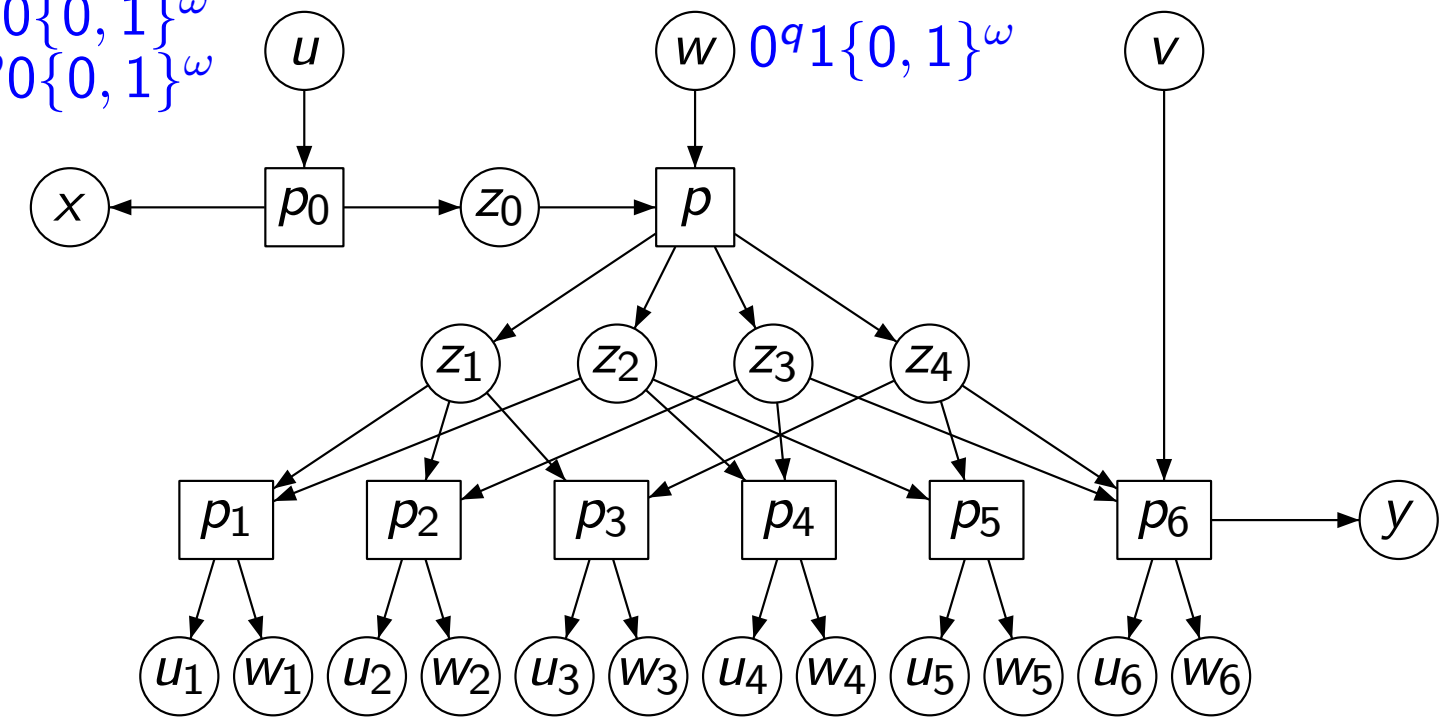

Well connected preordered architecures are undecidable: Enforcing output of the correct configuration

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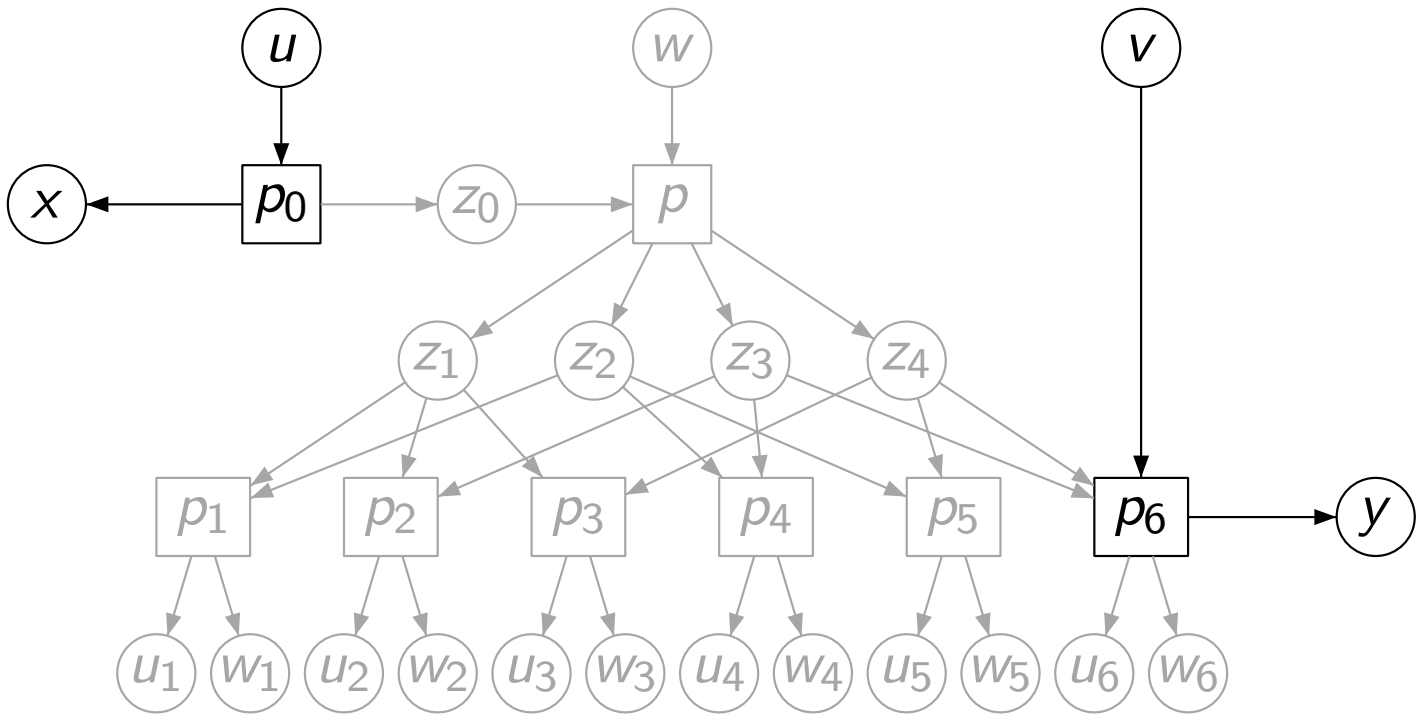
For all $q \geq 0$, $u \in 0^q 1^p 0\{0,1\}^\omega \implies x = \#^{q+p} \mathcal{C}_p \#^\omega$



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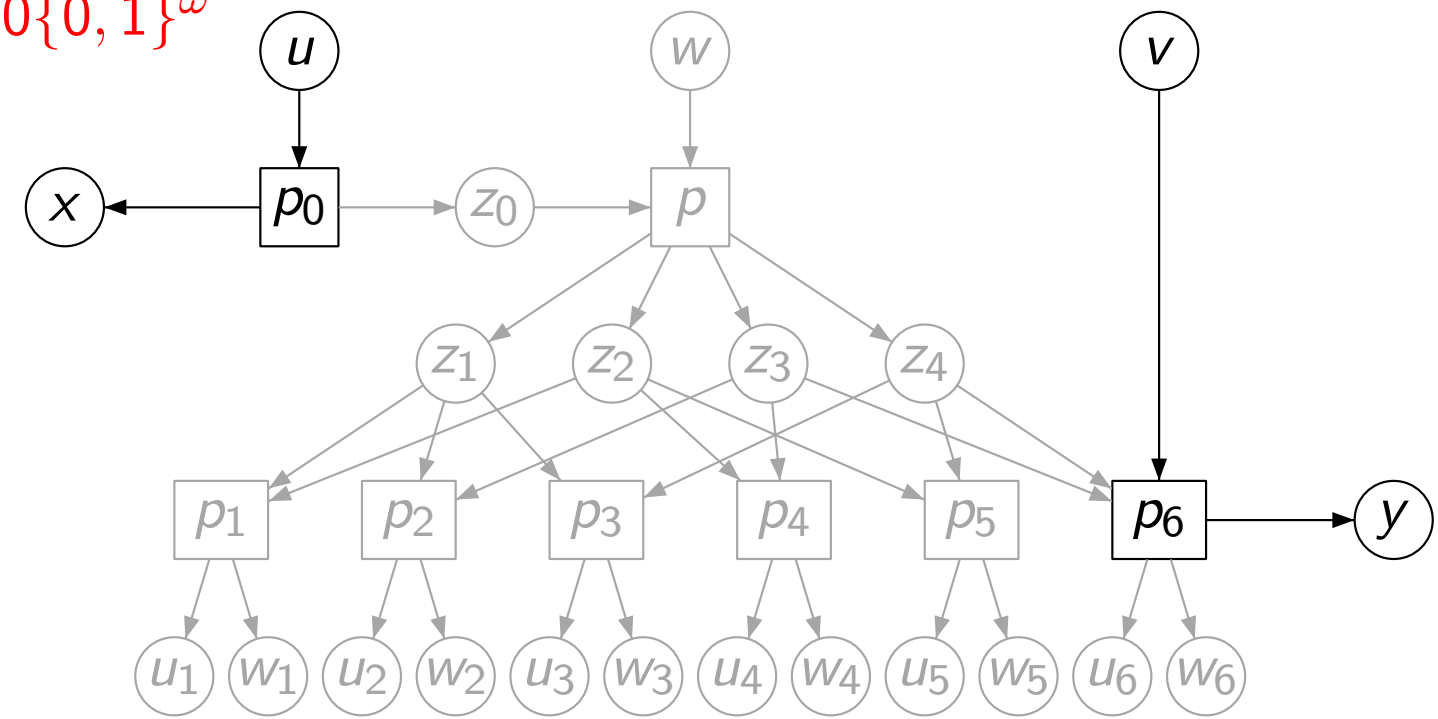

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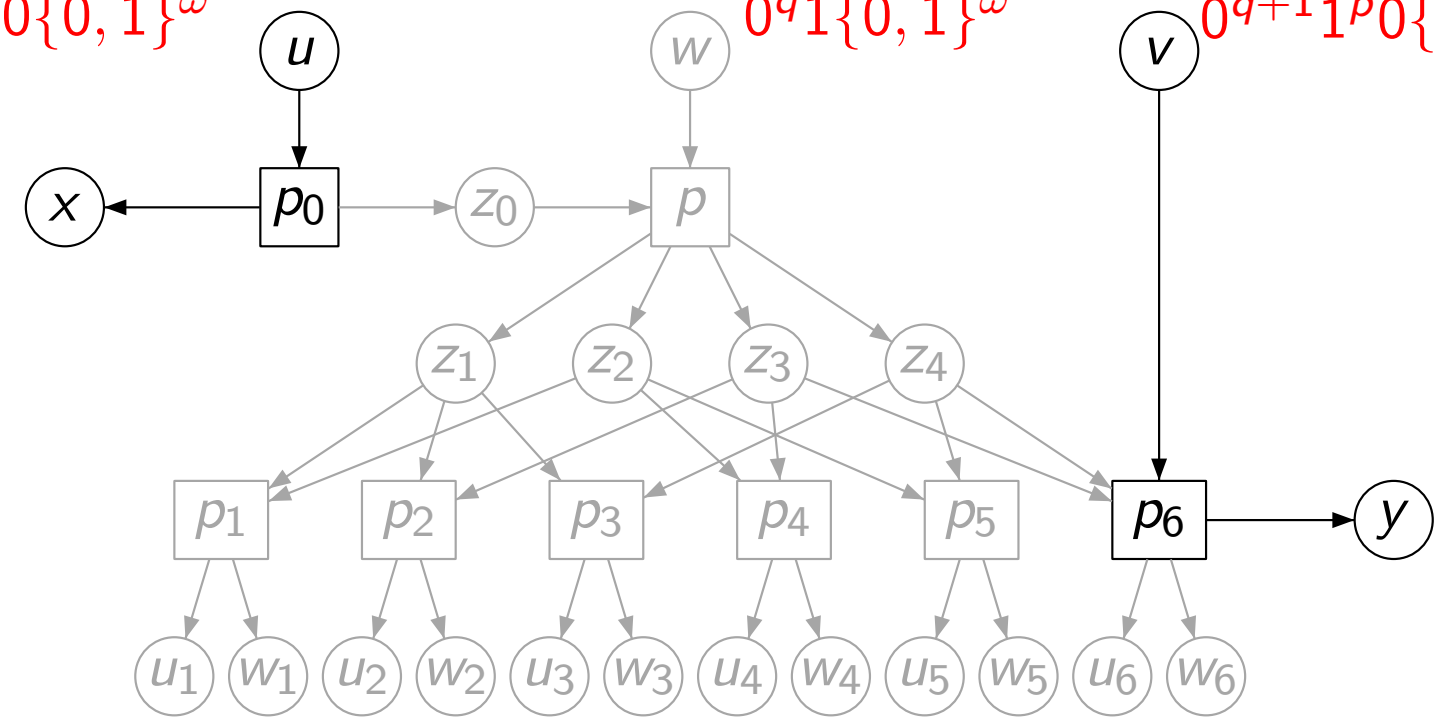

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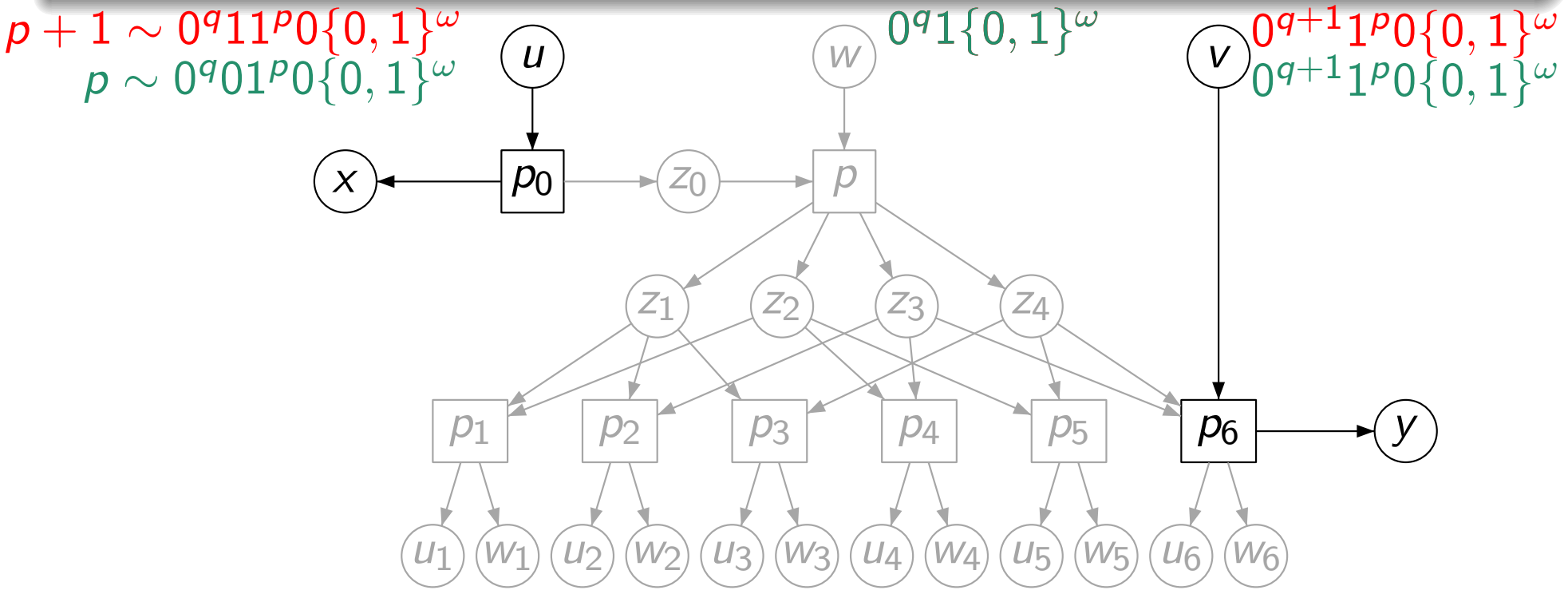

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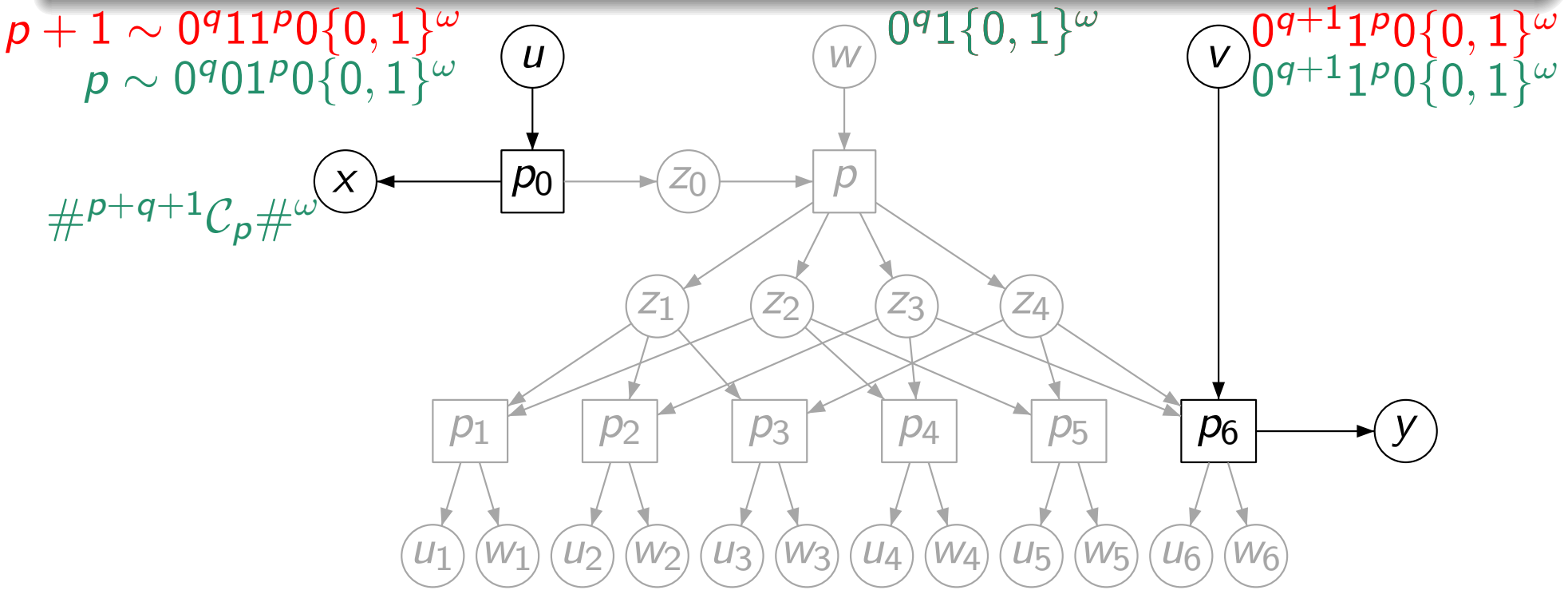

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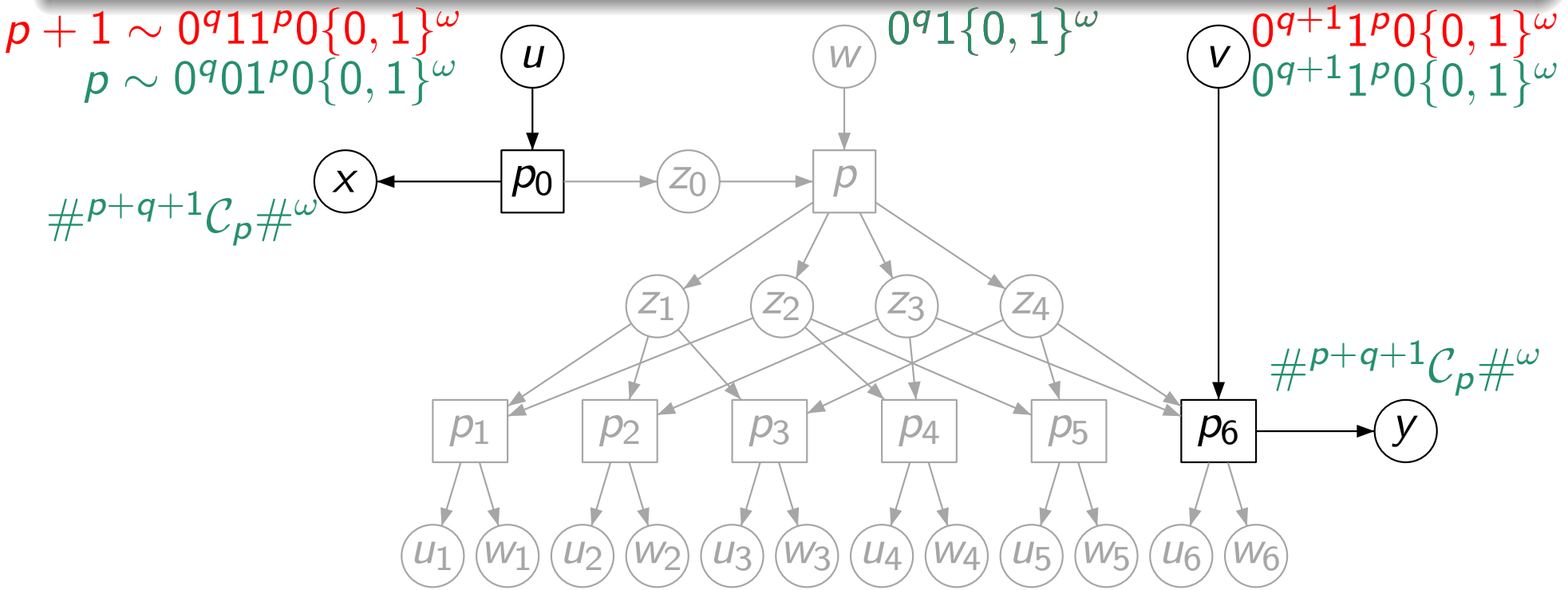

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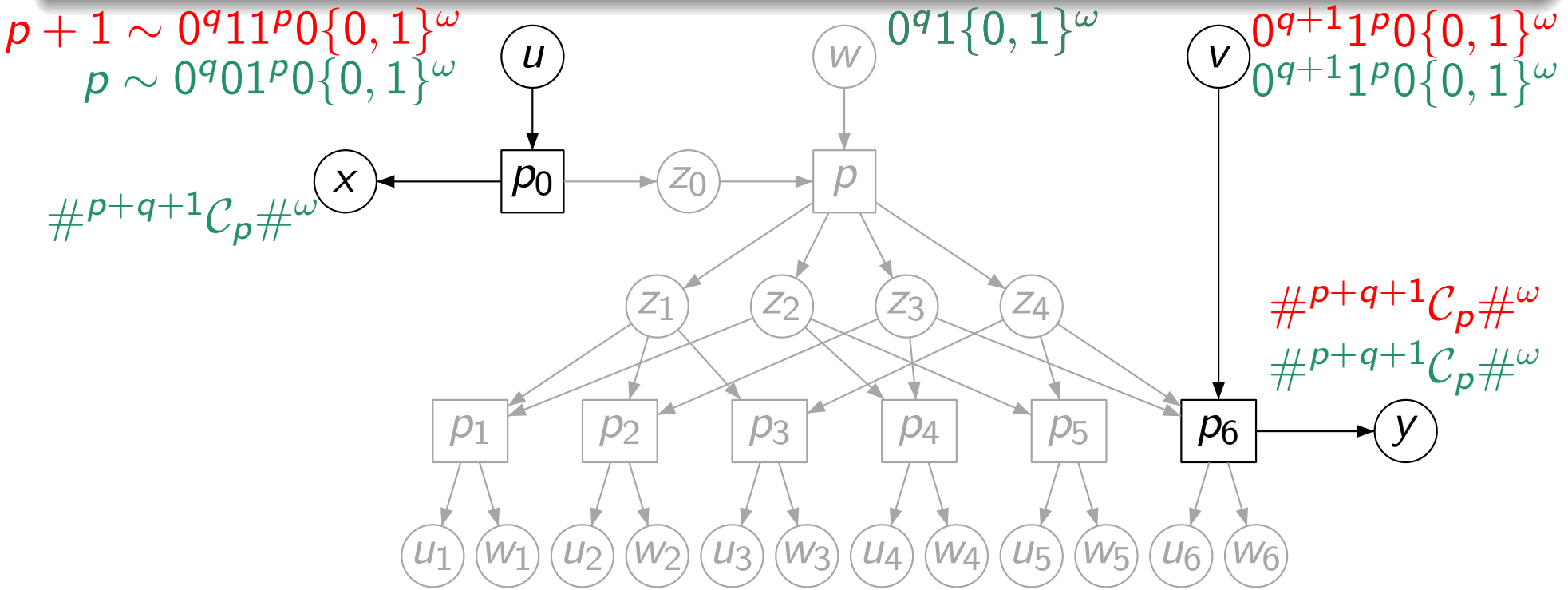

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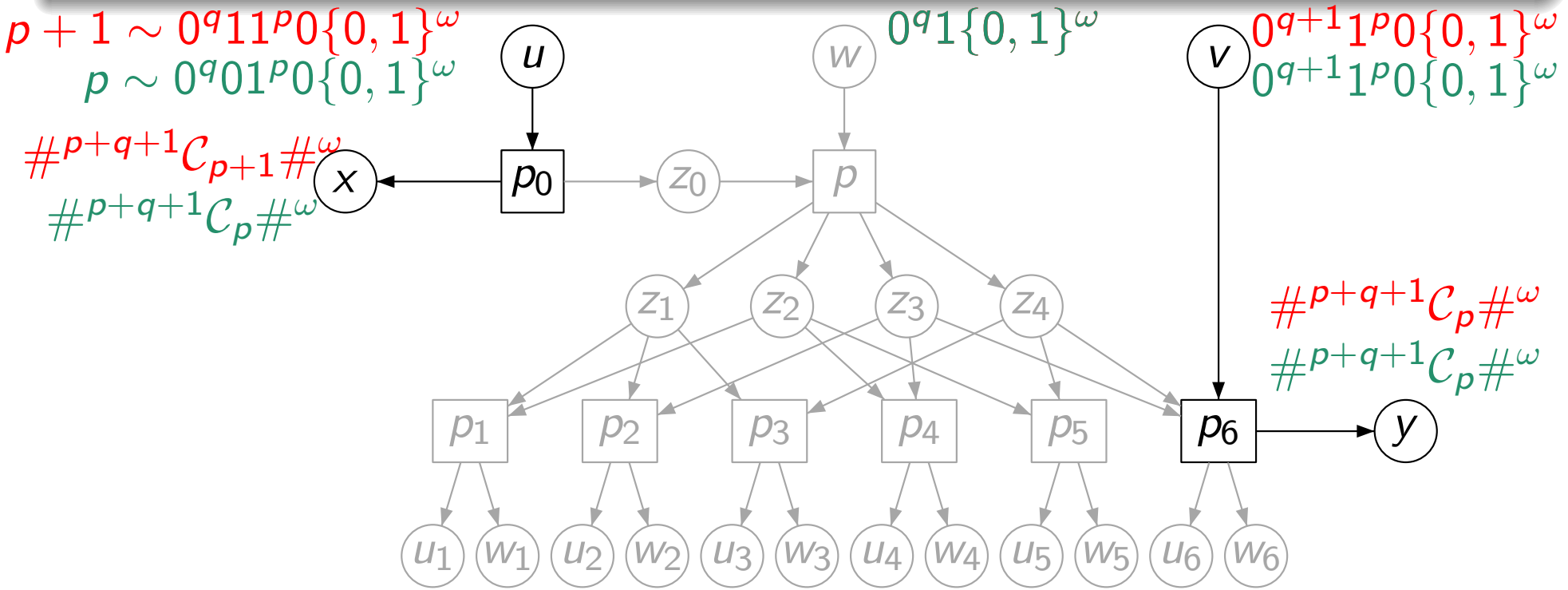


April Rasala Lehman and Eric Lehman.
Complexity classification of network information flow problems.
In *SODA*, pages 142–150, 2004.