Verification of nondeterministic channel systems with probabilistic message losses

Joint work with C. Baier and N. Bertrand

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Outline of the Talk

Lossy Channel Systems

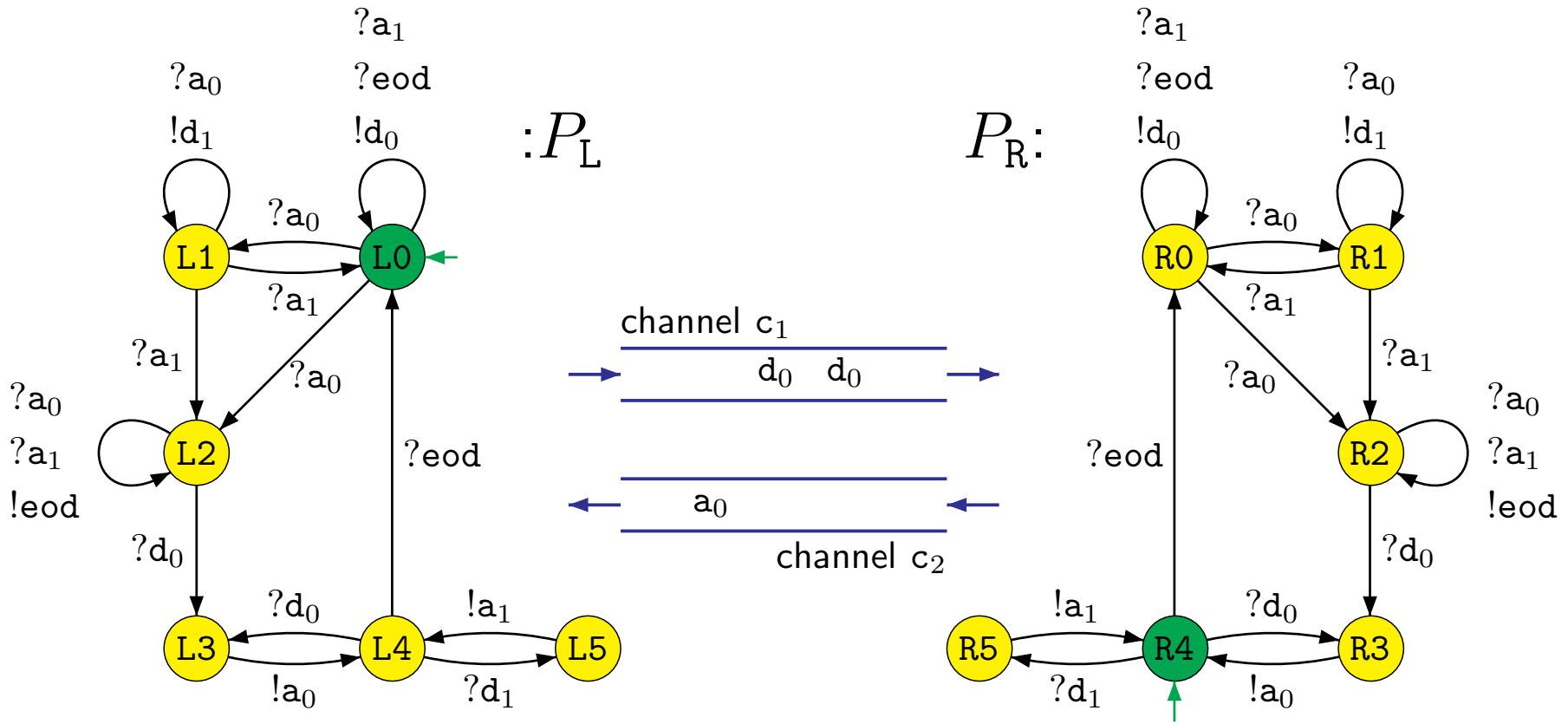
The Problem of Verifying Liveness Properties

Probabilistic Lossy Channel Systems

Nondeterministic and Probabilistic Lossy Channel Systems

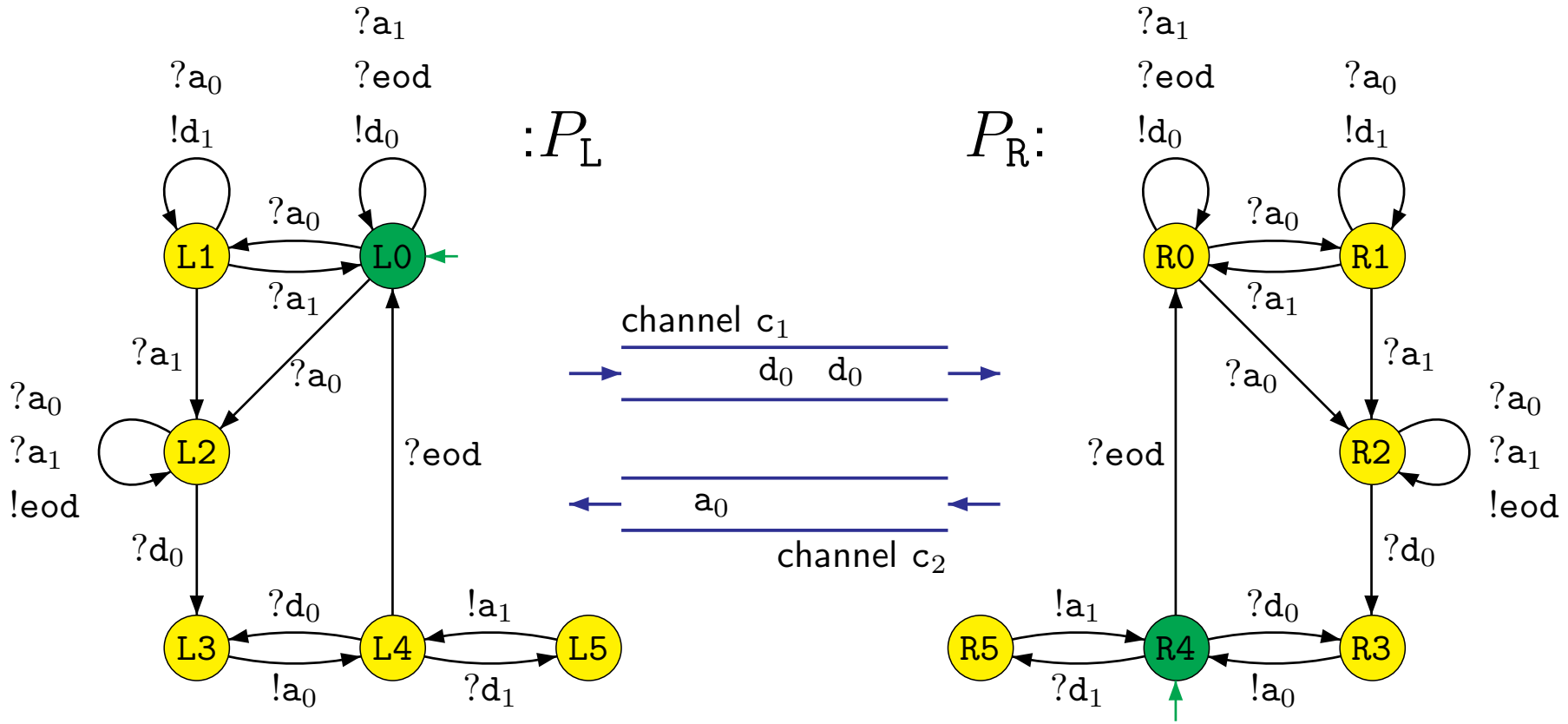
A long trial-and-error search for the right question to answer

Lossy Channel Systems



A model for asynchronous protocols that can handle unreliable communication

Lossy Channel Systems



Transition-system semantics: $\langle \mathtt{L0}, \mathtt{R4}, \mathtt{d}_0\,\mathtt{d}_0, \mathtt{a}_0 \rangle \rightarrow \langle \mathtt{L0}, \mathtt{R3}, \mathtt{d}_0, \mathtt{a}_0 \rangle$

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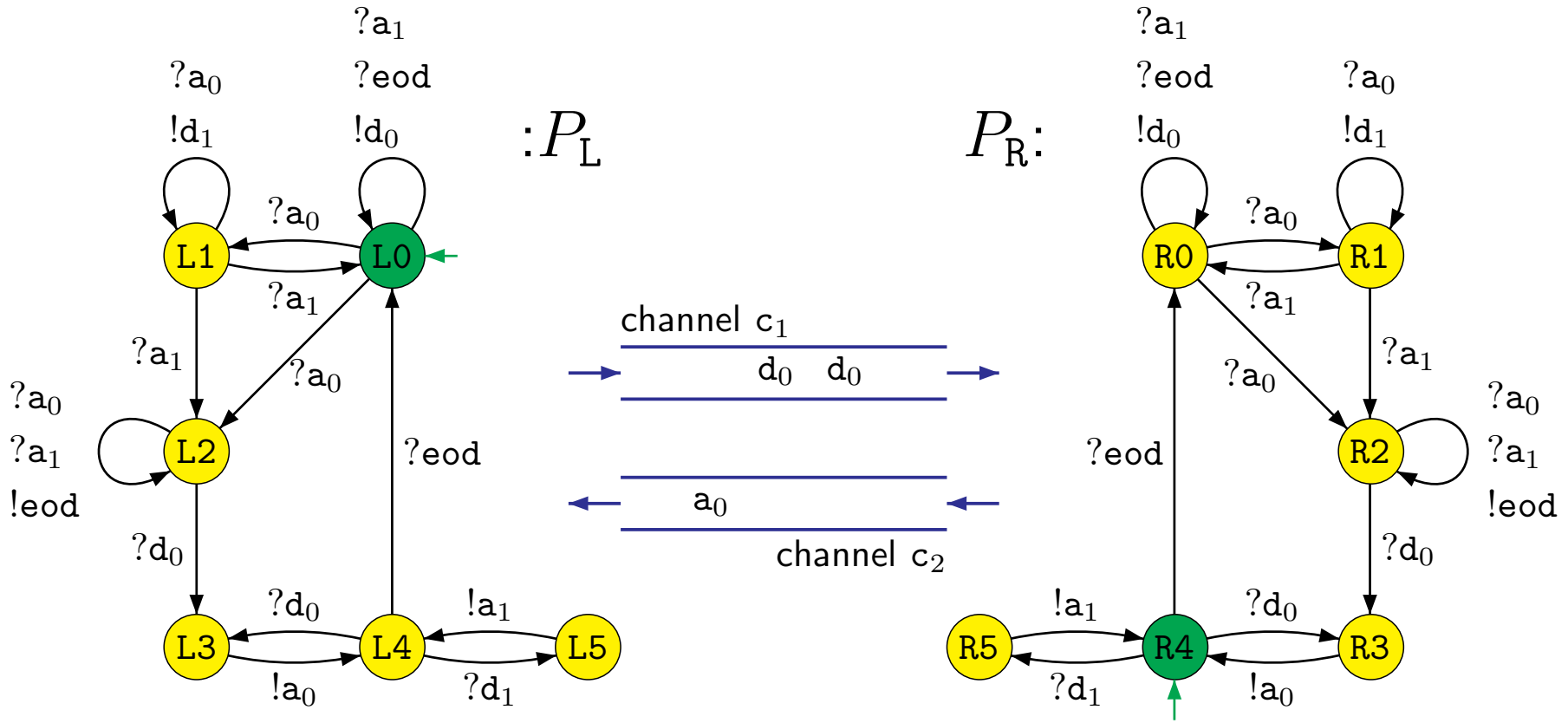

Message losses: $\langle \mathtt{L0}, \mathtt{R4}, \mathtt{d}_0\,\mathtt{d}_0, \mathtt{a}_0\rangle \rightarrow \langle \mathtt{L0}, \mathtt{R3}, \mathtt{d}_0, \varepsilon\rangle \qquad (\sqsubseteq \langle \mathtt{L0}, \mathtt{R3}, \mathtt{d}_0, \mathtt{a}_0\rangle)$

Verifying Lossy Channel Systems

• Reachability, safety properties, termination, many regular equivalences, are decidable (Abdulla & Jonsson, Finkel & Purush Iyer & Cécé, Kučera & S.)

• Regular model checking works in practice (Abdulla & Annichini & Bouajjani & Jonnson) even though worst-case complexity is nonprimitive-recursive

• Ingredients for decidability: symbolic computation of reachability set + termination based on wqo theory (here Higman's lemma)

Fine for verifying safety properties!

What About Liveness Properties?

Lossy channel systems are not so good for verifying liveness properties:

Problem 1: Liveness properties are undecidable for LCS's

Problem 2: Liveness properties are always false on LCS's

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Problem 1+: Adding fairness makes liveness even more undecidable

Fix 1+: Go for a probabilistic model

(nasty behaviours can be swept under the *measure zero* rug)

Probabilistic Lossy Channel Systems

Basic idea is to assume that *message losses follow probabilistic rules*, e.g. there is a known "failure rate" (Purush Iyer)

More realist than just non-deterministic losses (protocols are designed with the idea that losses are not that likely)

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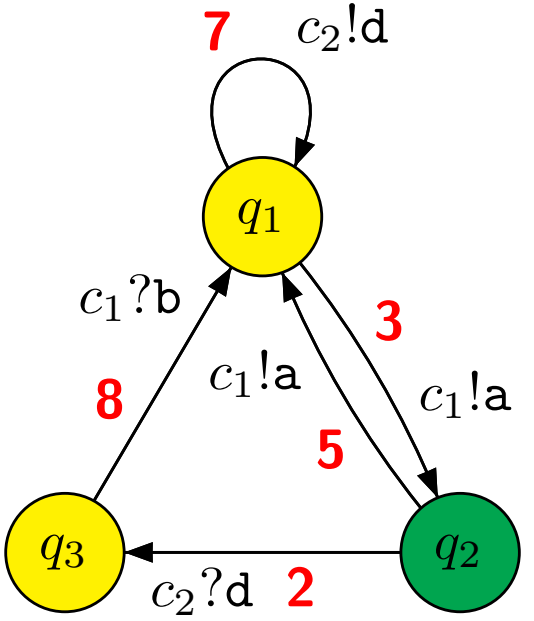
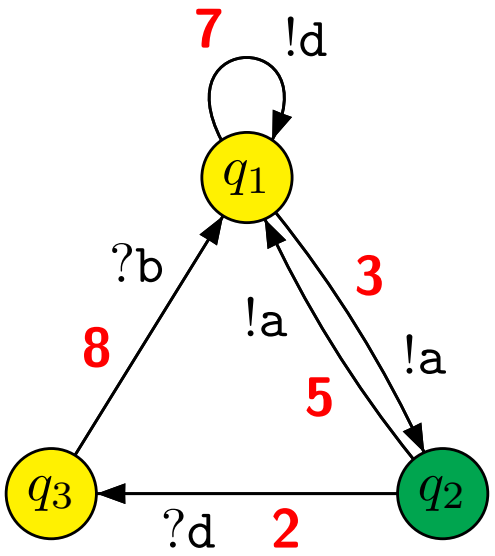
Definition: A Probabilistic LCS (a "PLCS") is
a LCS equipped with
– positive weights on rules,
– a constant probability $p_{\text{loss}} \in (0, 1)$

$$p_{\text{loss}} = .01 + \quad \frac{7}{8} \quad \frac{3}{5} \quad \frac{2}{}$$



Markovian Semantics

Semantics in form of a countable Markov chain



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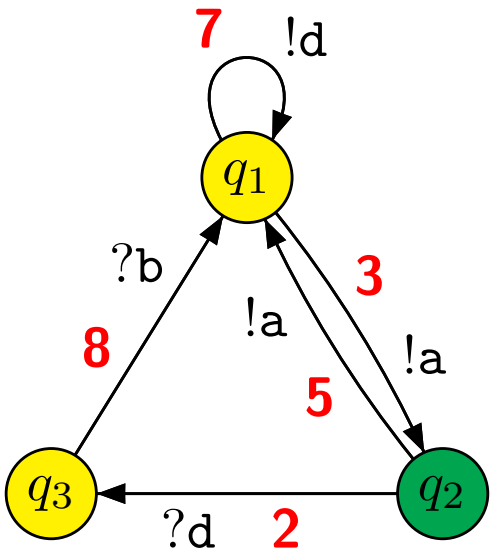

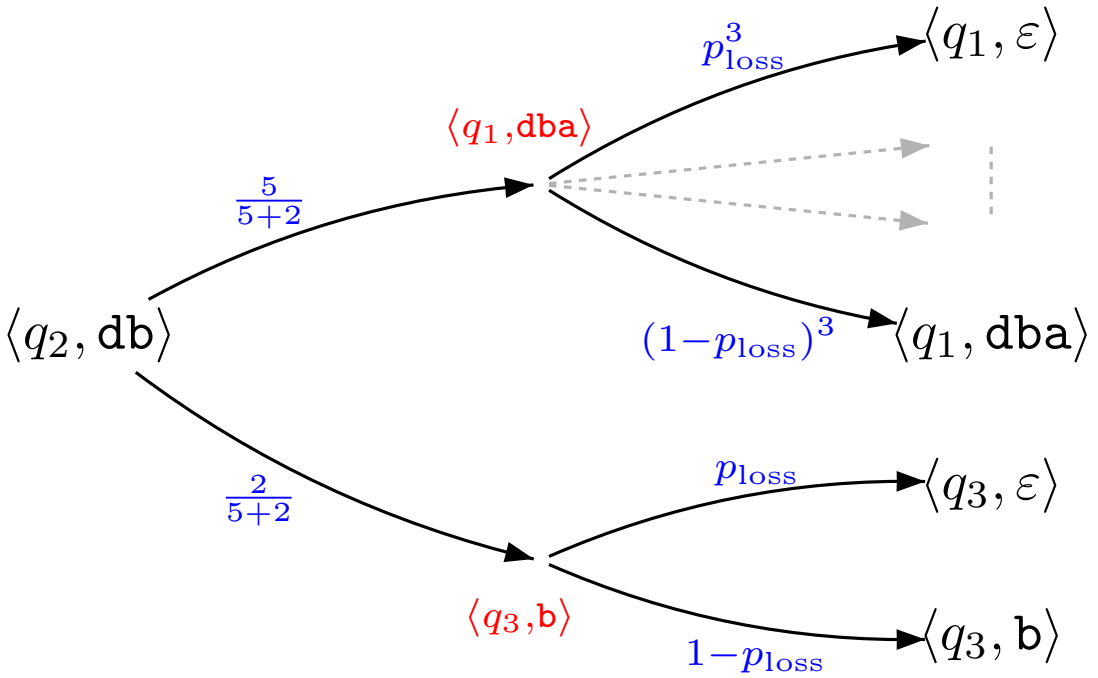

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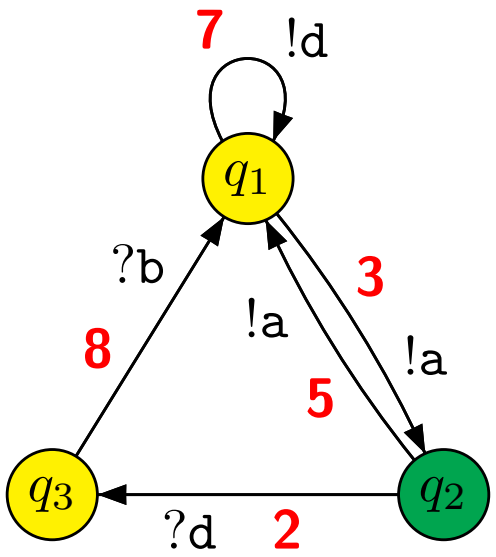

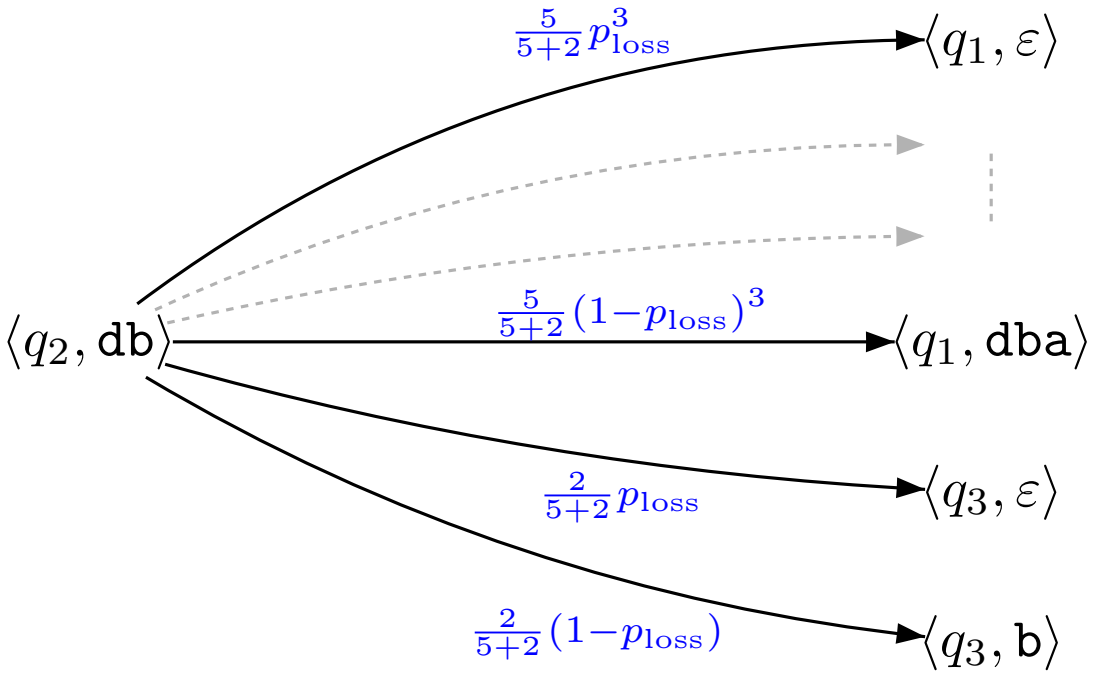

Verification of PLCS's

Thm. Qualitative verification "*does φ hold almost surely?*" of linear-time properties is decidable (Purush Iyer, Baier, Abdulla, Jonsson, Rabinovich, Bertrand, S.)

Thm. Approximate quantitative verification is decidable (Rabinovich).

Ingredients for decidability:

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2. Existence of a finite attractor, i.e., a finite set of configurations that will be visited almost surely from any starting configuration.

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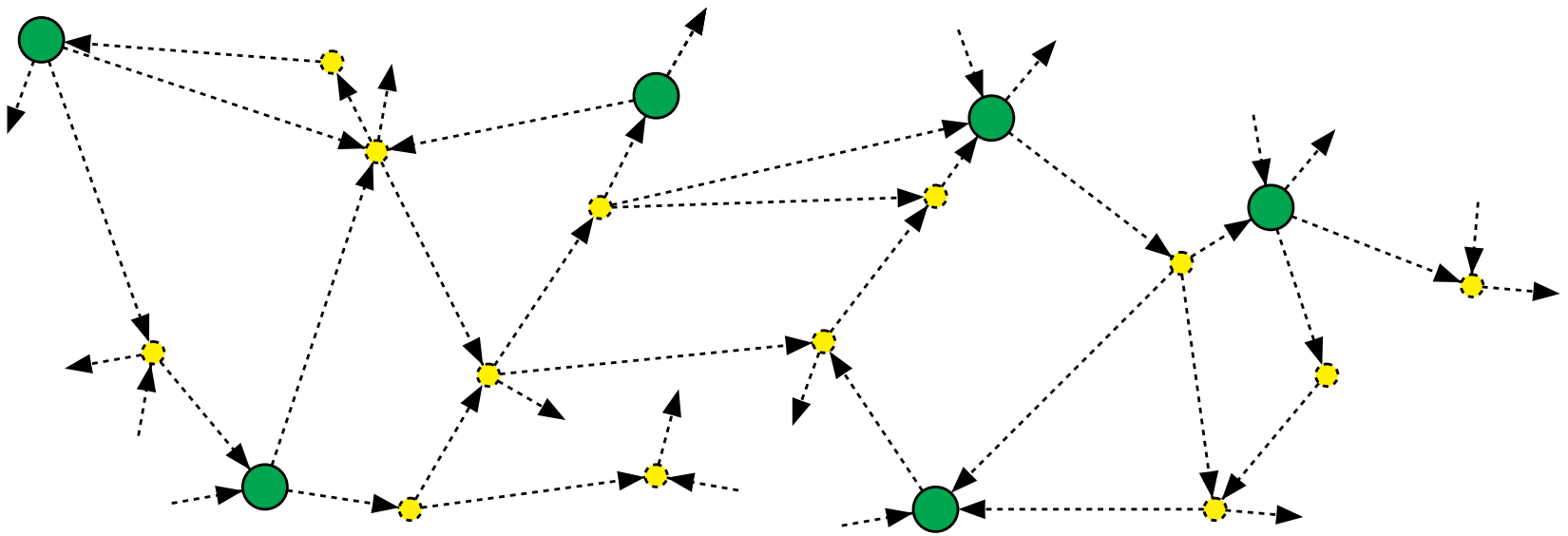

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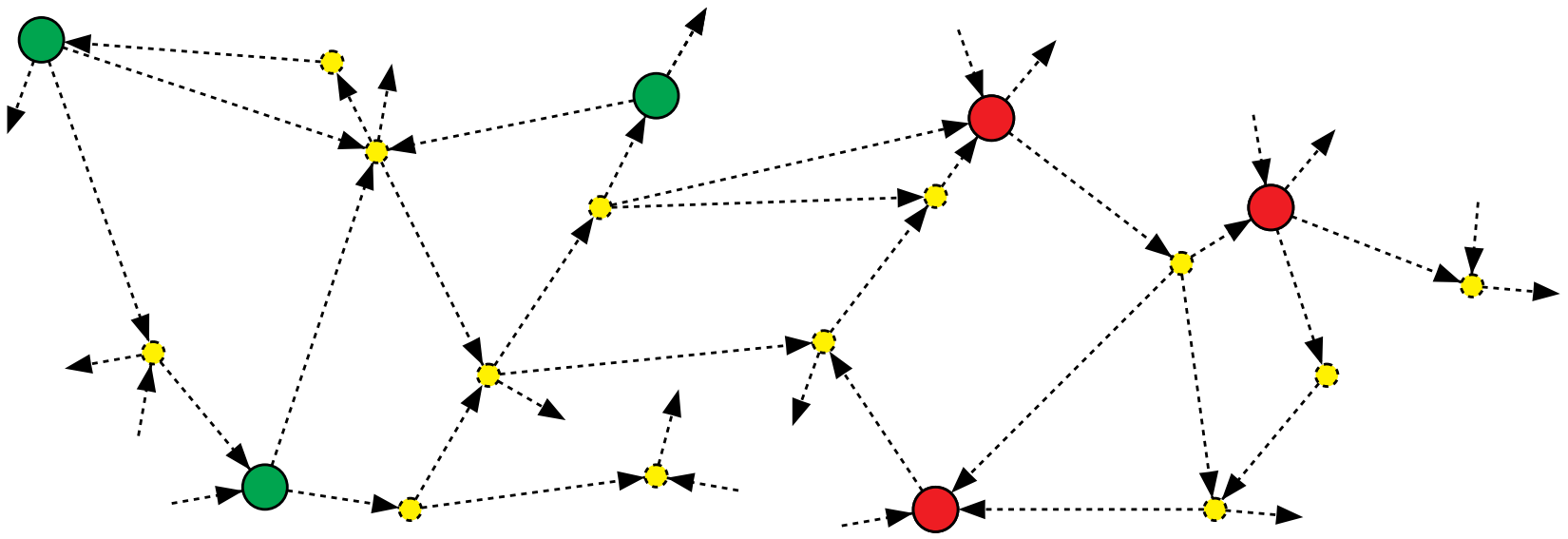

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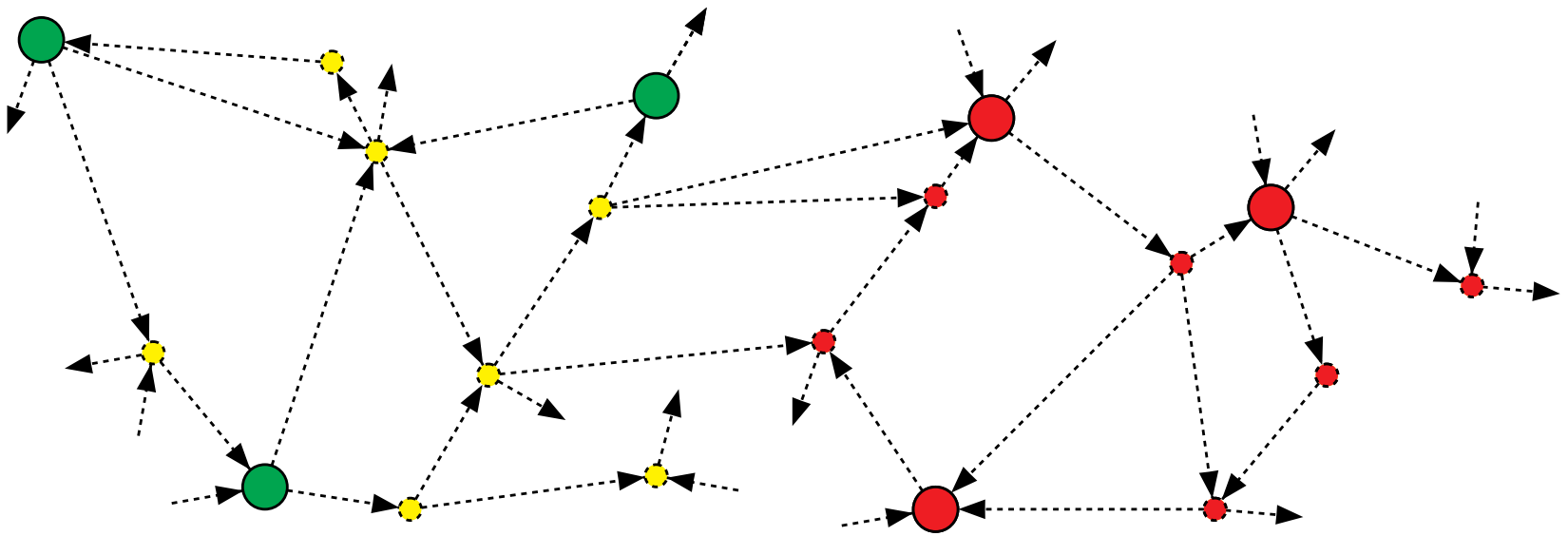

Markov-Chain Model Still not Adequate

The problem with PLCS's is that you have to view rules as probabilistic instead of nondeterministic.

Classically, nondeterminism in rules is used for:

– arbitrary interleaving of asynchronous components

– abstraction of real-life programs

– open systems

– early designs

– unpredictable message losses

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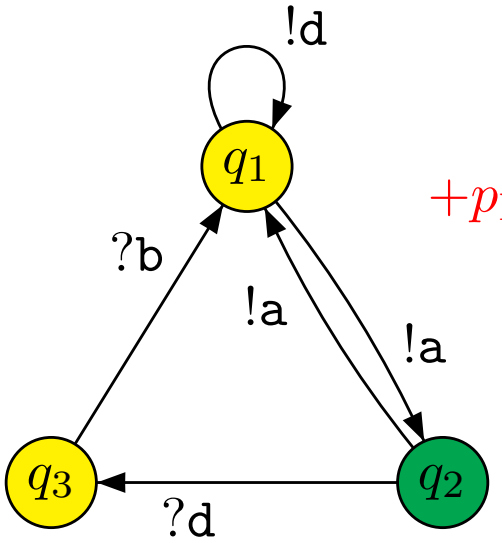
You want a *Markov decision process* model, where rules are nondeterministic and losses are probabilistic!! [Bertrand & S. 2003].

NPLCS and Markov Decision Processes



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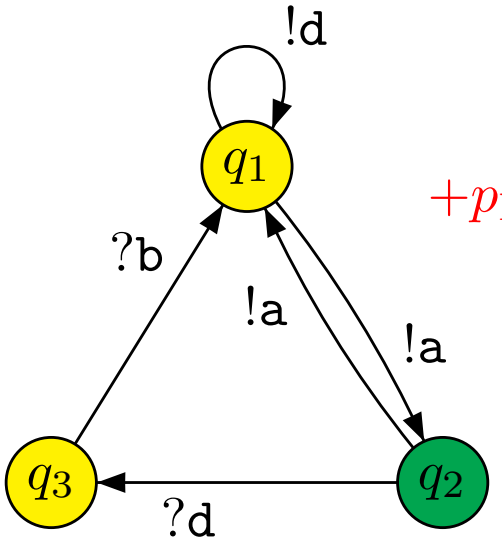

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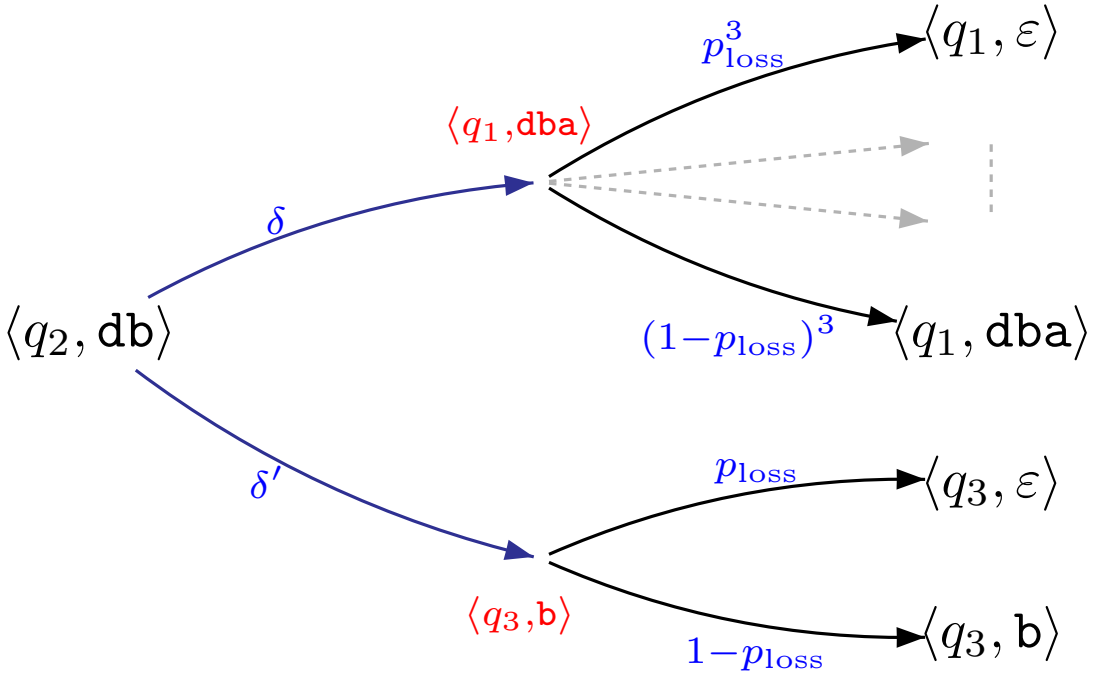

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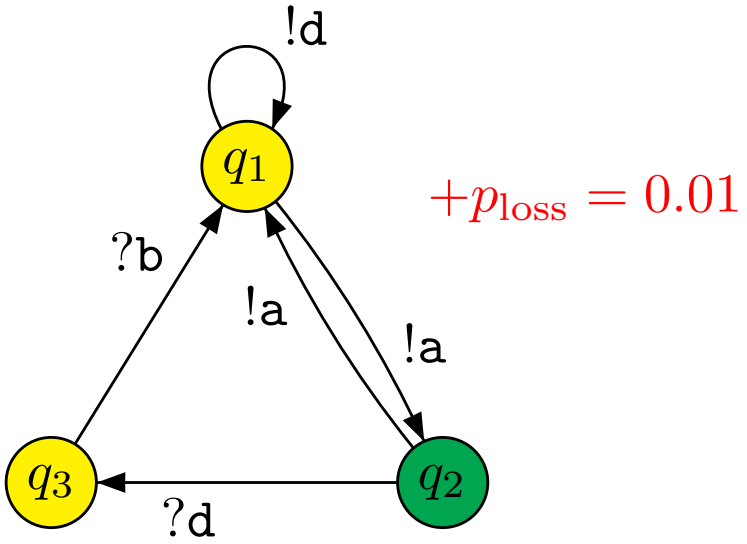

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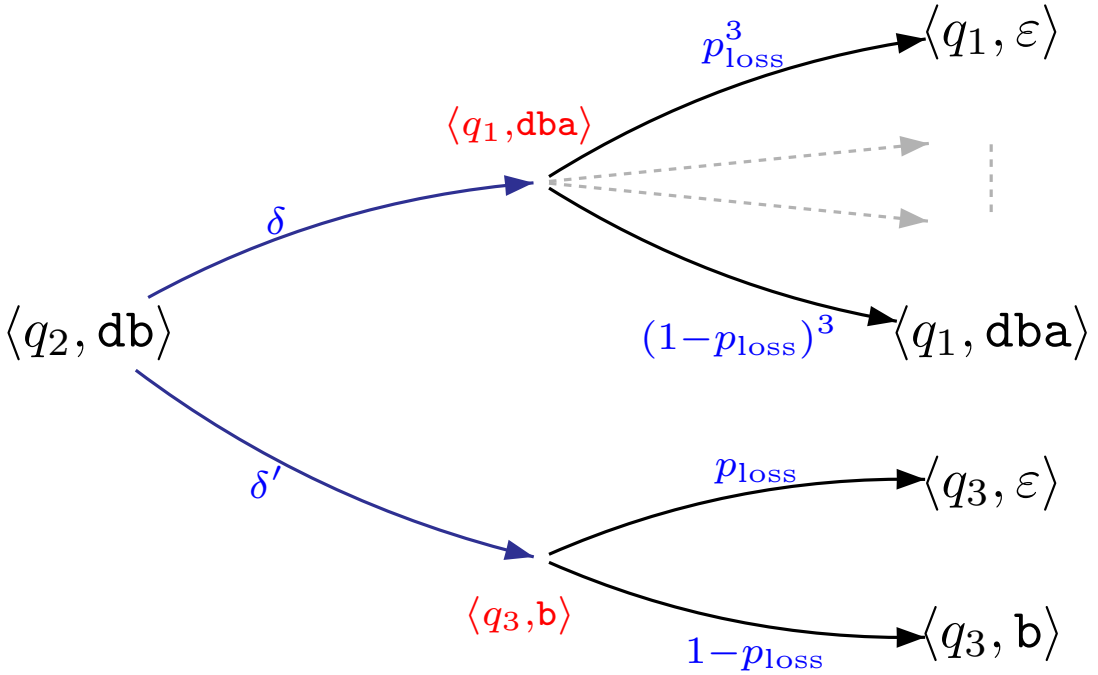

Def. A *scheduler* $\mathcal{U} : \text{Conf}^+ \to \Delta$ picks an enabled rule $\delta = \mathcal{U}(h)$ based on the complete history of the system.

Given $\mathcal{U}$, a NPLCS $\mathcal{N}$ generates a Markov chain $\mathcal{M}(\mathcal{N},\mathcal{U})$ and a probability measure $\mathbb{P}_{\mathcal{U}}(s \models \varphi)$.

Verifying NPLCS's

Thm. Qualitative verification of the form

$$\text{does } \mathbb{P}_{\mathcal{U}}(s \models \varphi) = 1 \text{ for all finite-memory } \mathcal{U}?$$

is decidable for NPLCS's (Baier, Bertrand, S.)

Ingredients for decidability:

1. restriction to finite-memory schedulers
2. existence of a finite attractor
3. reduction to constrained reachability questions with fixpoints
4. symbolic decision method for the extended reachability questions

What Unrestricted Schedulers Can Do

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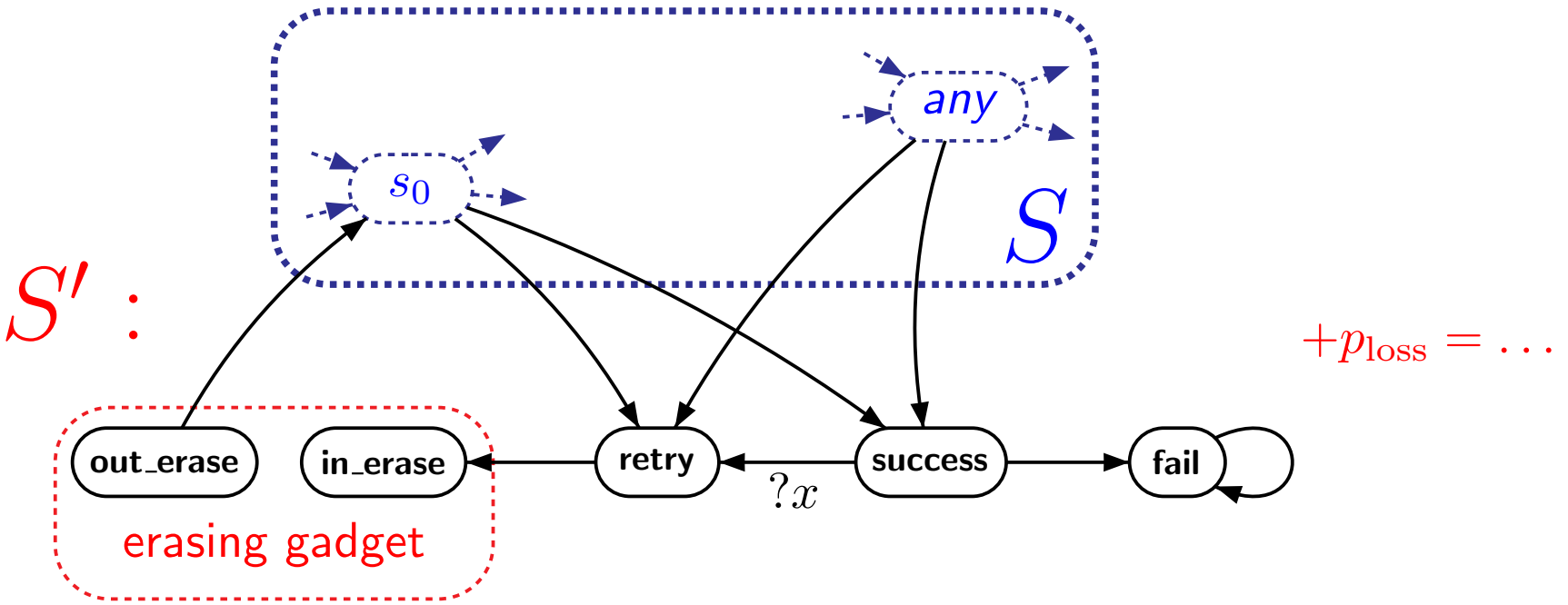

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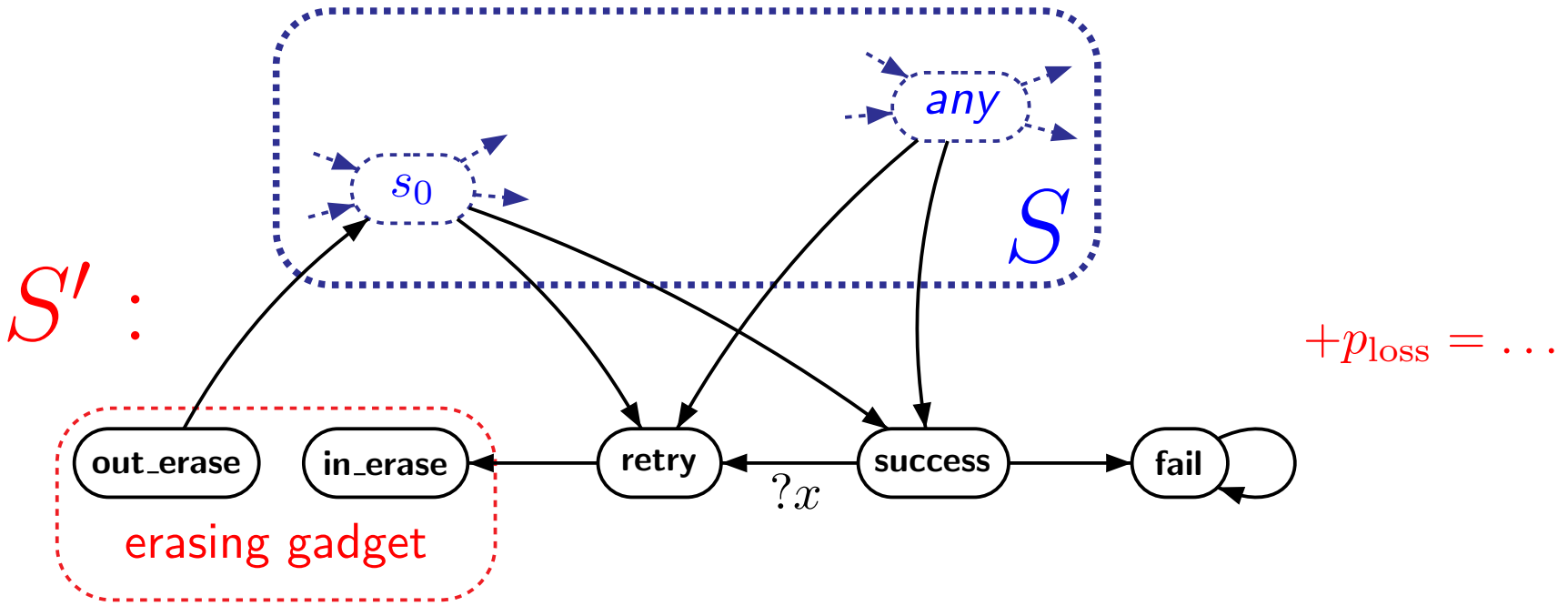

Question: is there a scheduler $\mathcal{U}$ that makes S' visit **success** infinitely often with > 0 probability?

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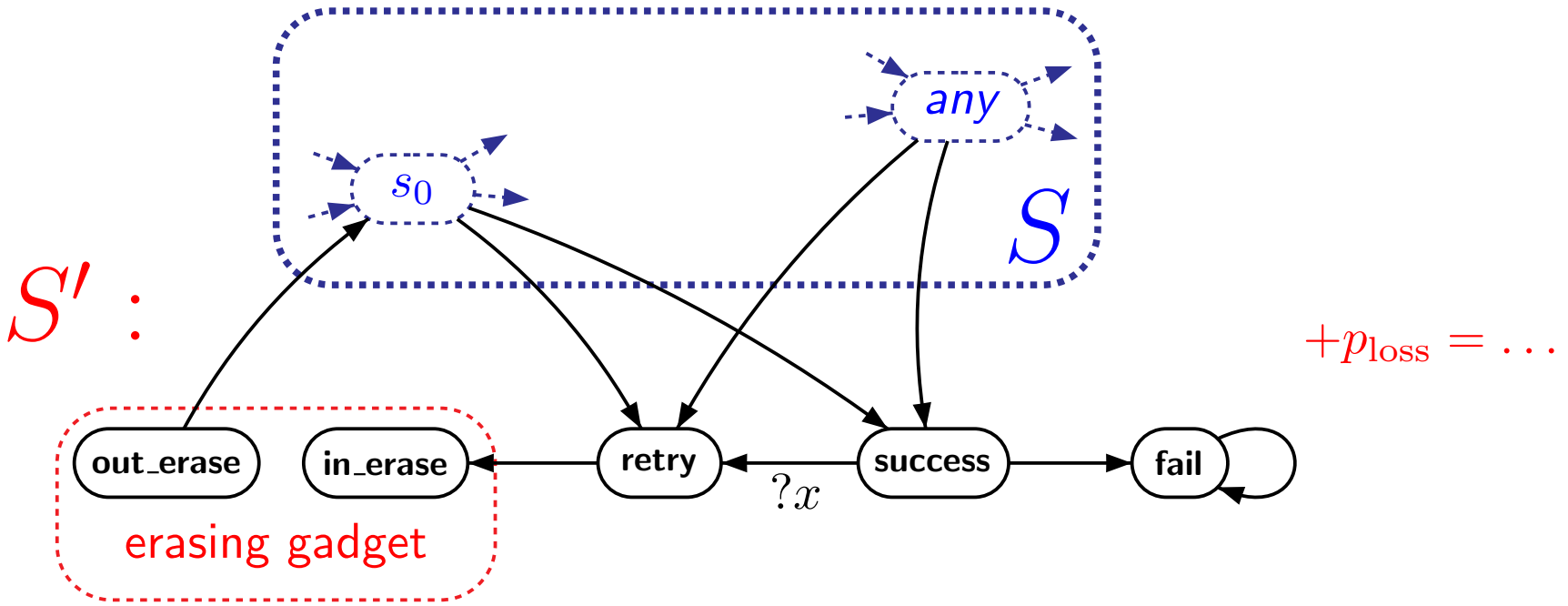

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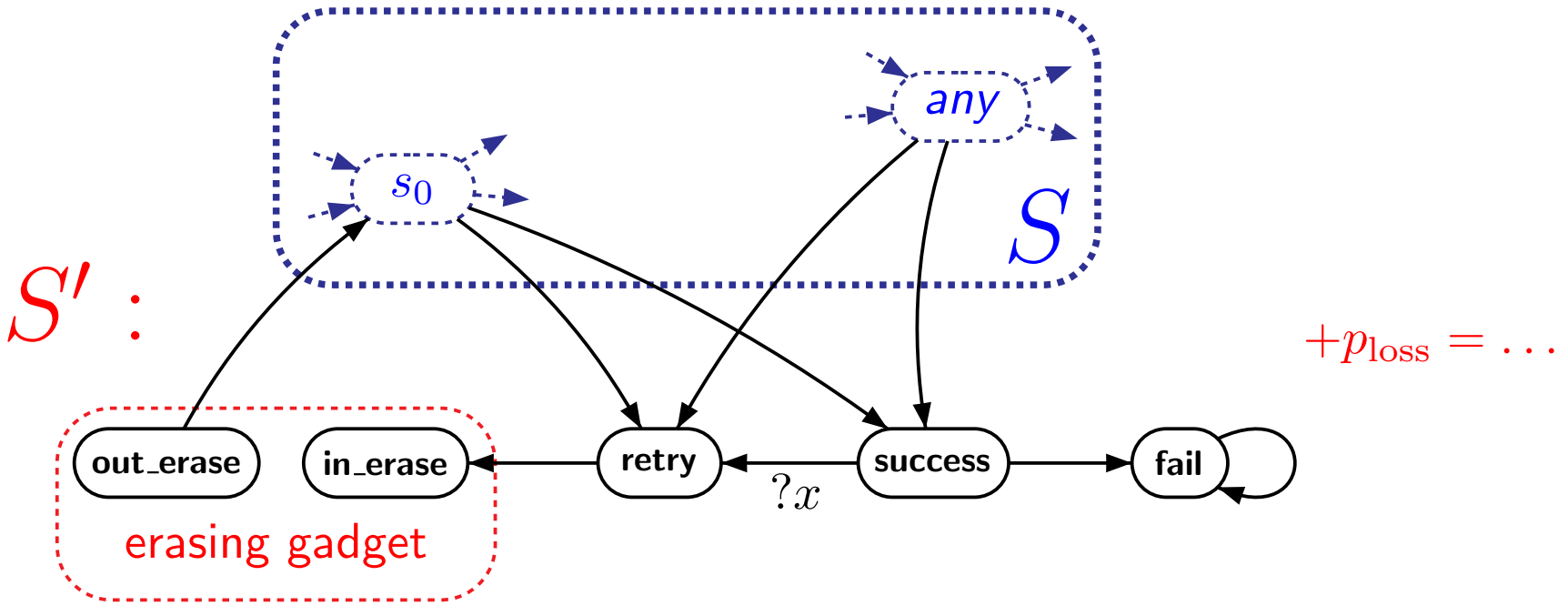

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Corollary: model checking qualitative properties under all schedulers is undecidable.

Reducing to Reachability

Thm. 1. There is a scheduler $\mathcal{U}$ s.t. $\mathbb{P}_{\mathcal{U}}(s \models \Box A) = 1$ iff $s \models \mathit{Safe}(A)$
2. There is a scheduler $\mathcal{U}$ s.t. $\mathbb{P}_{\mathcal{U}}(s \models \Box A) > 0$ iff $s \models \exists(A \text{ Until } \mathit{Safe}(A))$

with $\mathit{Safe}(A) \stackrel{\text{def}}{=} \nu X.A \wedge \bigvee_{\delta \in \Delta} \big(\langle \delta \rangle \top \wedge [\delta]\, X\big)$

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Used to verify $\forall \mathcal{U}.\mathbb{P}_{\mathcal{U}}(s \models \Diamond \overline{A}) > 0$ and $\forall \mathcal{U}.\mathbb{P}_{\mathcal{U}}(s \models \Diamond \overline{A}) = 1$

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Thm. 3. There is a finite-memory scheduler $\mathcal{U}$ s.t. $\mathbb{P}_{\mathcal{U}}(s \models \Box\Diamond A \wedge \Diamond\Box B) = 1$ iff $s \models \exists \mathsf{F}\mathit{Prom}_B(A)$

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NB. When verifying liveness, sets A and B cannot just be defined in terms of control locations. One has to take channel contents into account!

Symbolic Computation of Reachability Sets

Prefixed closure $\quad \theta, \sigma \ ::= \ \alpha \uparrow u \qquad \alpha \in \mathsf{M} \cup \{\varepsilon\}, \ \ u \in \mathsf{M}^*$

Symbolic set $\quad \gamma \ ::= \ \sum_i \langle q_i, \theta_i - \sigma_{i,1} - \cdots - \sigma_{i,n-i} \rangle$

A region is any set of configurations that can be denoted symbolically.

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A region is any set of configurations that can be denoted symbolically.

Thm. Regions are closed under Boolean operations, Pre-image, $\exists(\_ \ \mathsf{Until} \ \_)$, $Safe()$, $Prom_{(\_)}(\_)$.

Termination of fixpoint symbolic computations requires careful analysis of how restrictions appear in symbolic sets.

Prototype implementation performs encouragingly well!

Concluding remarks

A model that combines two hard features: probabilities and infinite state space.

Qualitative verification "*for all finite-memory schedulers*" is decidable.

Positive result requires many ingredients. Hard to see what can be stated in a more generalized framework.

Many open questions remain. The priority should be on assessing the model and strengthening its foundations.