



# Generalised Rabin(1) synthesis<sup>1</sup>

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# Overview of the talk

- 1 Synthesis of reactive systems - an overview
- 2 GR(1) - Generalised reactivity(1) synthesis
- 3 GRabin(1) - Generalised Rabin(1) synthesis
- 4 There's no room beyond GRabin(1)
- 5 What can we use GRabin(1) synthesis for?

# Synthesis of reactive systems - overview

## Problem description

Given ...

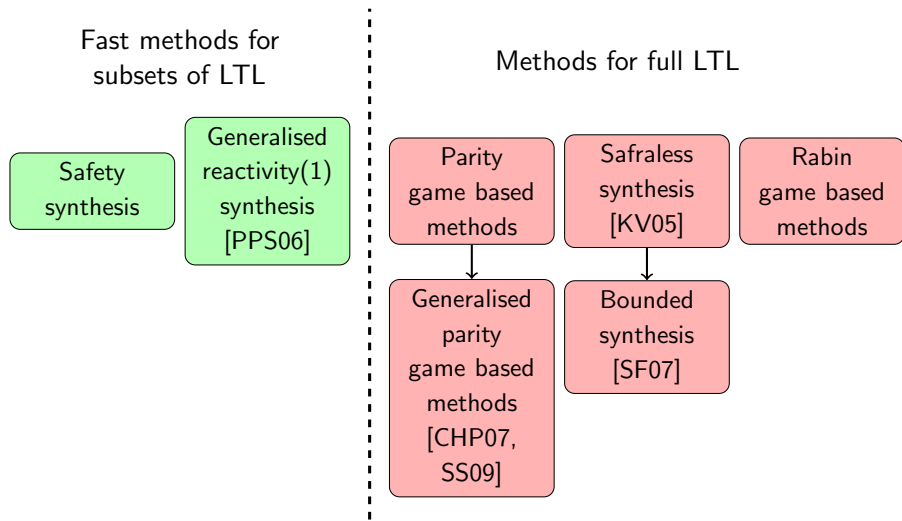
- a set of input atomic propositions  $AP_I$ ,
- a set of output atomic propositions  $AP_O$ ,
- a temporal logic formula  $\psi$  over  $AP_I \uplus AP_O$

... does there exist a Mealy/Moore automaton reading  $AP_I$  and writing  $AP_O$  that satisfies  $\psi$ ?

## Properties of this problem

Church's problem is known to be 2EXPTIME-complete [PR89] for LTL specifications.

# Incomplete overview of past synthesis approaches



Form of the specification (f.t.p. of [KHB09])

$$(a_1 \wedge a_2 \wedge \dots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \dots \wedge g_m)$$

with:

- a set  $A$  of assumption  $a_1, \dots, a_n$
- a set  $G$  of guarantees  $g_1, \dots, g_m$

such that:

- all elements in  $A \cup G$  are deterministic Büchi automata
- all elements in  $A \cup G$  run over  $2^{AP_I \uplus AP_O}$

# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])

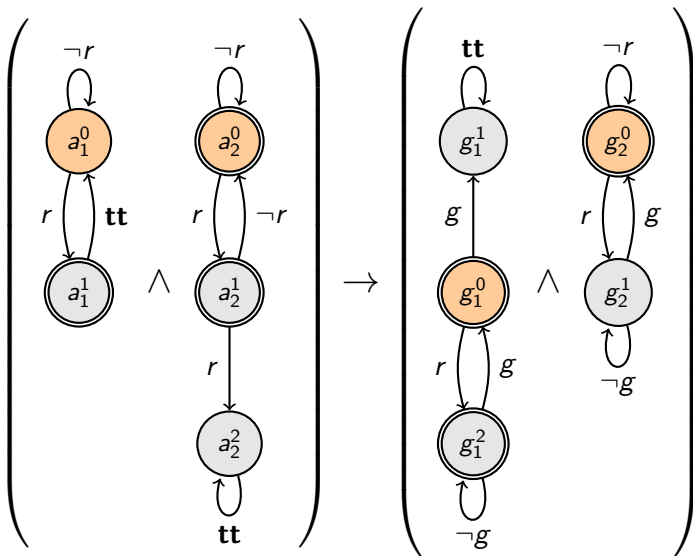
## Explanation by example: a mutex

- Set of inputs:  $\{r\}$  (a request)
- Set of outputs:  $\{g\}$  (a grant)
- Assumptions:
  - $GF r$
  - $G(r \rightarrow X\neg r)$
- Guarantees:
  - A grant is only issued after a request has been issued (since the last grant)
  - $GF g$

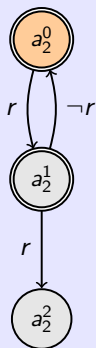
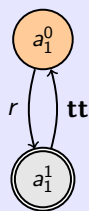
## Basic idea of the algorithm

- Reduce the problem to solving a parity game with 3 colours : 0, 1, 2
- The system player wins if the highest colour occurring infinitely often in the play is 0 or 2

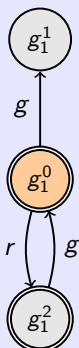
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



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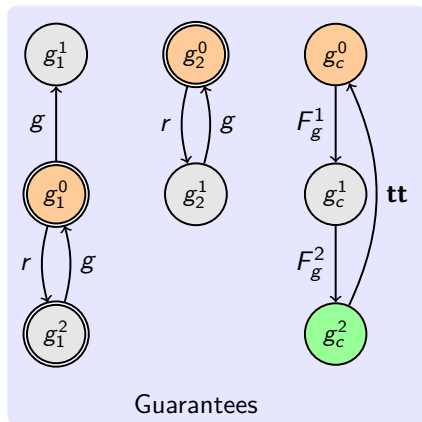
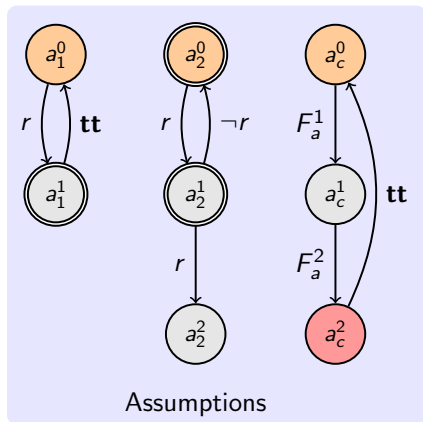
Assumptions



Guarantees



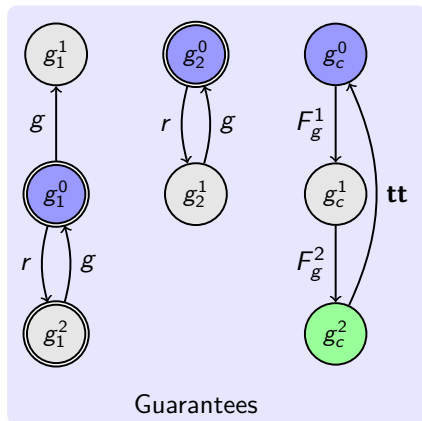
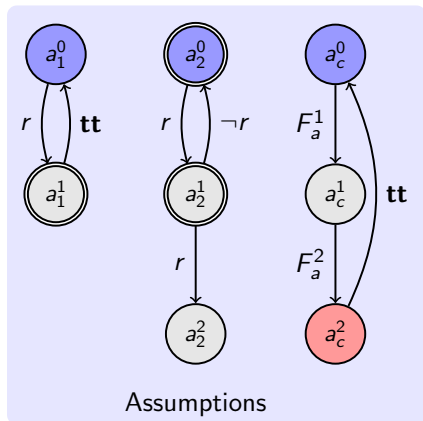
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Main idea

Make a parity game over the product state space with 3 colours.

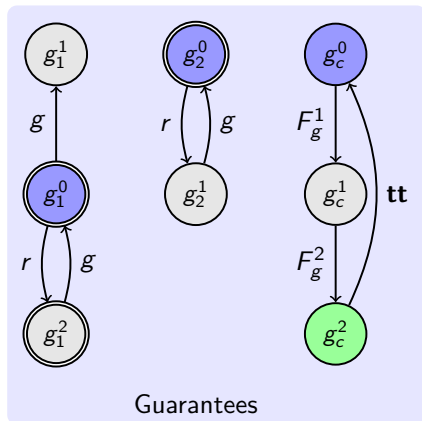
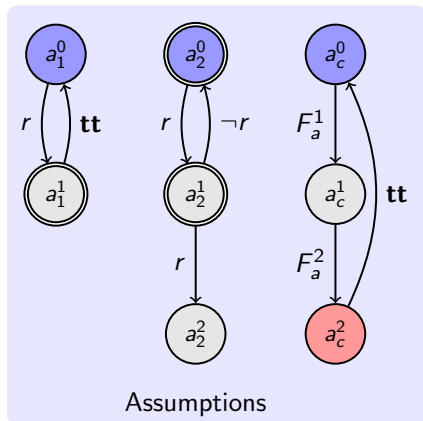
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:			
Grant:			
Colour:			

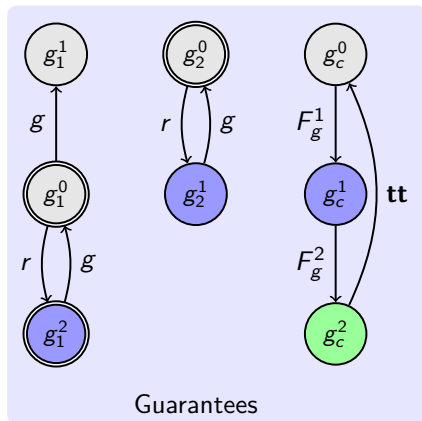
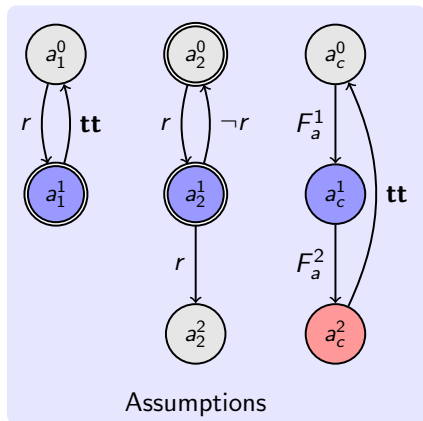
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1		
Grant:			
Colour:			

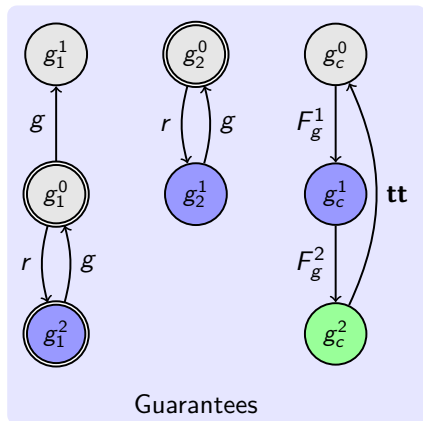
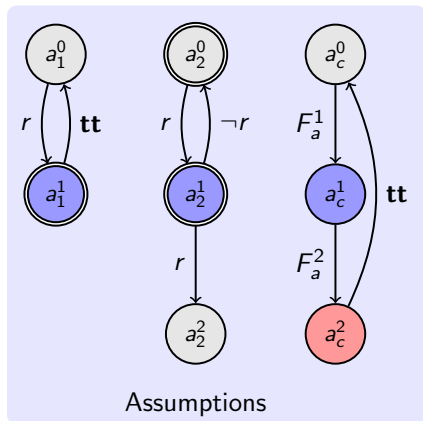
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1		
Grant:	0		
Colour:	0		

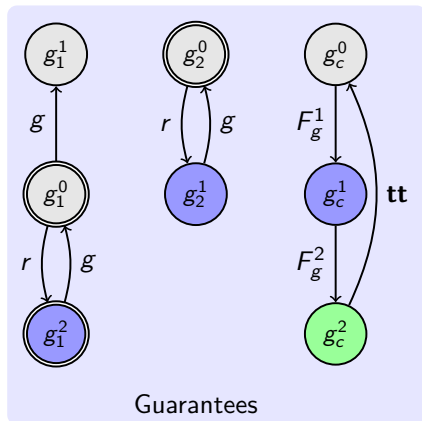
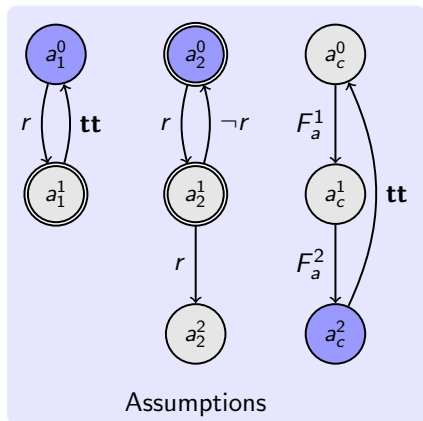
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1	0
Grant:	0	
Colour:	0	

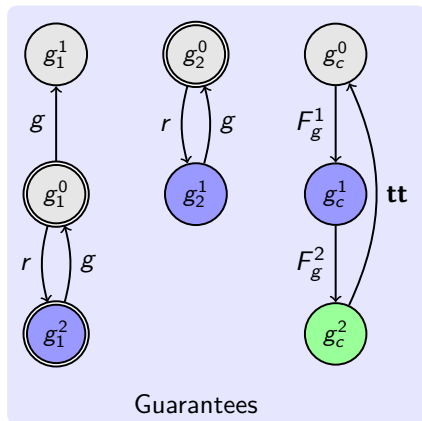
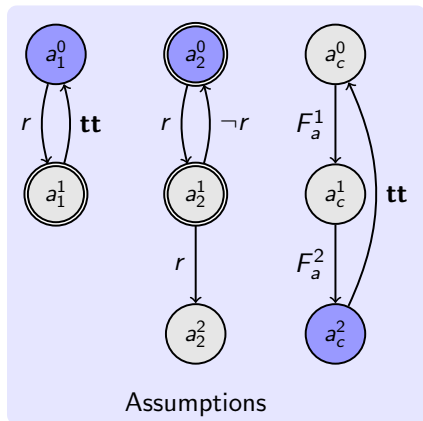
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1	0
Grant:	0	0
Colour:	0	1

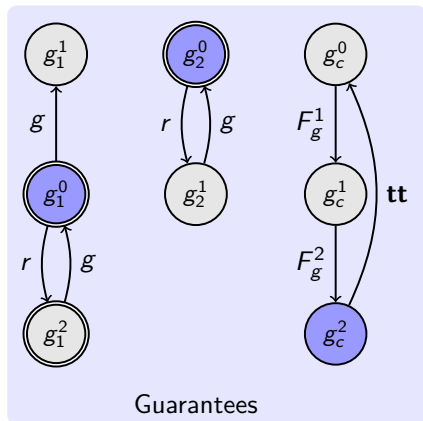
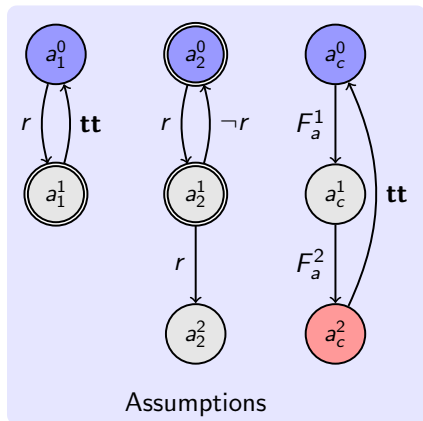
# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1	0	0
Grant:	0	0	
Colour:	0	1	

# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])

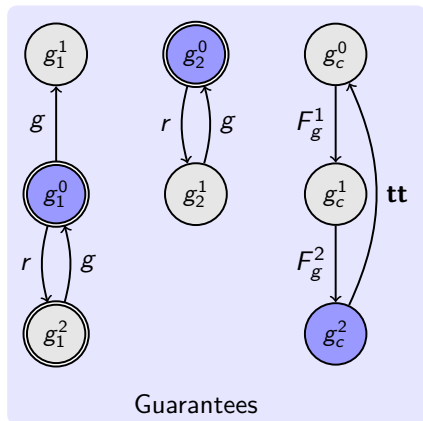
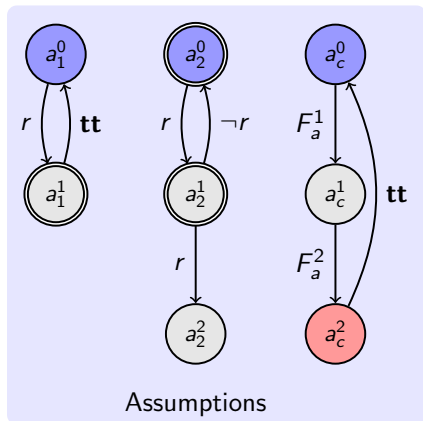


## Example run

Request:	1	0	0
Grant:	0	0	1
Colour:	0	1	2



# GR(1) synthesis - Algorithm (f.t.p. of [BCG<sup>+</sup>10])



## Example run

Request:	1	0	0	
Grant:	0	0	1	...
Colour:	0	1	2	

## The highest colour occurring infinitely often

- Colour 0 - Neither the assumptions nor the guarantees are fulfilled
- Colour 1 - The assumptions are fulfilled but not the guarantees
- Colour 2 - The guarantees are fulfilled

## The nice properties of this approach

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- The parity game we obtain has a constant number of colours

⇒ Amenable to symbolic implementations, as confirmed by two case studies [BGJ<sup>+</sup>07a, BGJ<sup>+</sup>07b]

# On extending GR(1)

## Main question of this work

**How far can we push the expressivity of GR(1) without losing its nice properties?:**

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- We have a constant number of colours, independent of the number of assumptions and guarantees

## Intuition on why we want this

- Many properties in practice cannot be expressed yet, e.g., FG(*ready*)

## Answer to the question raised

**A little bit further, but that's it then (assuming  $P \neq NP$ )**

## Form of the specification

$$(a_1 \wedge a_2 \wedge \dots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \dots \wedge g_m)$$

with:

- a set  $A$  of assumption  $a_1, \dots, a_n$
- a set  $G$  of guarantees  $g_1, \dots, g_m$

such that:

- all elements in  $A \cup G$  are deterministic **Rabin automata with one acceptance pair**
- all elements in  $A \cup G$  run over  $2^{\text{AP}_I \uplus \text{AP}_O}$

## Formal definition

A **one-pair** det. Rabin automaton is a tuple  $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$  with:

- a state set  $Q$
- an alphabet  $\Sigma$  (here,  $\Sigma = 2^{AP_I \uplus AP_O}$ )
- a transition relation  $Q \times \Sigma \rightarrow Q$
- an initial state  $q_0 \in Q$
- an acceptance component  $F = (F_1, F_2) \in 2^Q \times 2^Q$

We say that  $\mathcal{A}$  accepts a run  $\pi = \pi_0\pi_1\dots$  if  $\pi_0 = q_0$  and  $\text{inf}(\pi) \cap F_1 = \emptyset$  and  $\text{inf}(\pi) \cap F_2 \neq \emptyset$

## A special property of one-pair Rabin automata

A run is accepted by a one-pair Rabin automaton  $(Q, \Sigma, \delta, q_0, (F_1, F_2))$  iff it is accepted by the co-Büchi automaton  $(Q, \Sigma, \delta, q_0, F_1)$  and the Büchi automaton  $(Q, \Sigma, \delta, q_0, F_2)$ .

## Form of the specification

$$(A_1 \wedge A_2 \wedge \dots \wedge A_{|A|} \wedge B_1 \wedge B_2 \wedge \dots \wedge B_{|B|}) \\ \rightarrow (C_1 \wedge C_2 \wedge \dots \wedge C_{|C|} \wedge D_1 \wedge D_2 \wedge \dots \wedge D_{|D|})$$

with:

- a set  $A$  of Büchi assumptions  $A_1, \dots, A_{|A|}$
- a set  $B$  of co-Büchi assumptions  $B_1, \dots, B_{|B|}$
- a set  $C$  of Büchi guarantees  $C_1, \dots, C_{|C|}$
- a set  $D$  of co-Büchi guarantees  $D_1, \dots, D_{|D|}$

such that:

- all elements in  $A \cup C$  are deterministic Büchi automata
- all elements in  $B \cup D$  are deterministic co-Büchi automata
- all elements in  $A \cup B \cup C \cup D$  run over  $2^{AP_I \uplus AP_O}$

## (Still flawed) reduction to a parity game

- State space is the product of the automata state spaces and the two Büchi assumption- and guarantee-checker automata  $A_C$  and  $G_C$
- 5 colours:
  - Colour 4 - Some co-Büchi assumption is violated
  - Colour 3 - Some co-Büchi guarantee is violated
  - Colour 2 - The Büchi guarantee automata have “recently” all visited their accepting states
  - Colour 1 - The Büchi assumption automata have “recently” all visited their accepting states
  - Colour 0 - none of the above

# GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play

	co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.	0	1	4	4
co-B. B. Gua. Gua.	2	2	4	4
B. Gua.	3	3	4	4
	3	3	4	4



# GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play

	co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.	0	1	4	4
co-B. B. Gua. Gua.	2	2	4	4
B. Gua.	3	3	4	4
	3	3	4	4

## A problem

Currently we lose on too many plays!

## The solution

Add storage bit to the game tracking whether the counter automaton for  $C$  has “recently” completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

# GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play (fixed)

	co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.	0	1	4	4
co-B. B. Gua. Gua.	2	2	4	4
B. Gua.	2	3	4	4
	0	3	4	4

## A problem

Currently we lose on too many plays!

## The solution

Add storage bit to the game tracking whether the counter automaton for  $C$  has “recently” completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

# Can we generalise even further?

Answer: No!

Generalised Streett(1) synthesis cannot work in the way described:

- Game arena is the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees)
- Parity game with a constant number of colours

Reason

Generalised parity game solving is NP-hard for certain cases

# Generalised parity games [CHP07]

## The conjunctive version

- A game graph with  $k$  parity functions is given
- Player 0 needs to win for all of these functions at the same time

## A hard case

For parity functions with the co-domain  $\{0, 1, 2\}$ , solving (the conjunctive version of) generalised parity games is NP-hard

## A reduction of the hard case to generalised Streett(1) games

Convert such a game with  $k$  parity functions to the specification  $(\mathbf{tt}) \rightarrow (G_1 \wedge \dots \wedge G_k)$  for one-pair Streett guarantees  $G_i$

# The reduction (continued)

## Examining the specification

$(\mathbf{tt}) \rightarrow (G_1 \wedge \dots \wedge G_k)$  has a special property:  $G_1, \dots, G_k$  have the same transition structure.

## Consequence

If the product game arena was the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees) and the game had a constant number of colours, we could solve an NP-hard problem in polynomial time.

# An application: synthesis of robust systems

## Example specification - A processing machine (base version)

$$\begin{aligned} & \left( GF \left( \begin{array}{c} \text{part} \\ \text{incoming} \end{array} \right) \wedge G \left( \begin{array}{c} \text{no over-} \\ \text{sized parts} \end{array} \right) \wedge \dots \right) \\ \rightarrow & \left( GF \left( \begin{array}{c} \text{part} \\ \text{processed} \end{array} \right) \wedge G \left( \begin{array}{c} \text{no} \\ \text{jam} \end{array} \right) \wedge \dots \right) \end{aligned}$$

## Example specification - A processing machine (robust)

$$\begin{aligned} & \left( GF \left( \begin{array}{c} \text{part} \\ \text{incoming} \end{array} \right) \wedge FG \left( \begin{array}{c} \text{no over-} \\ \text{sized parts} \end{array} \right) \wedge \dots \right) \\ \rightarrow & \left( GF \left( \begin{array}{c} \text{part} \\ \text{processed} \end{array} \right) \wedge FG \left( \begin{array}{c} \text{no} \\ \text{jam} \end{array} \right) \wedge \dots \right) \end{aligned}$$

## Generalised Rabin(1) synthesis is . . .

- a symbolically implementable synthesis method
- in some sense the best we can get (in this line of research)
- a practically relevant fragment of LTL

## Important questions beyond the scope of this talk

- How to get from logic to Rabin(1) automata
- How to encode these automata into BDDs
- How to solve the resulting games efficiently

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