# Generalised Rabin(1) synthesis[1]

Rüdiger Ehlers

Saarland University, Reactive Systems Group

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# Overview of the talk

1. Synthesis of reactive systems - an overview
2. GR(1) - Generalised reactivity(1) synthesis
3. GRabin(1) - Generalised Rabin(1) synthesis
4. There's no room beyond GRabin(1)
5. What can we use GRabin(1) synthesis for?

# Synthesis of reactive systems - overview

## Problem description

Given ...

- a set of input atomic propositions $\mathsf{AP}_I$,
- a set of output atomic propositions $\mathsf{AP}_O$,
- a temporal logic formula $\psi$ over $\mathsf{AP}_I \uplus \mathsf{AP}_O$

... does there exist a Mealy/Moore automaton reading $\mathsf{AP}_I$ and writing $\mathsf{AP}_O$ that satisfies $\psi$?

## Properties of this problem

Church's problem is known to be 2EXPTIME-complete [PR89] for LTL specifications.

# Incomplete overview of past synthesis approaches

## Fast methods for subsets of LTL

- Safety synthesis
- Generalised reactivity(1) synthesis [PPS06]

## Methods for full LTL

- Parity game based methods
  - Generalised parity game based methods [CHP07, SS09]
- Safraless synthesis [KV05]
  - Bounded synthesis [SF07]
- Rabin game based methods

# GR(1) synthesis [PPS06] - Description

## Form of the specification (f.t.p. of [KHB09])

$$(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \ldots \wedge g_m)$$

with:

- a set $A$ of assumption $a_1, \ldots, a_n$
- a set $G$ of guarantees $g_1, \ldots, g_m$

such that:

- all elements in $A \cup G$ are deterministic Büchi automata
- all elements in $A \cup G$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

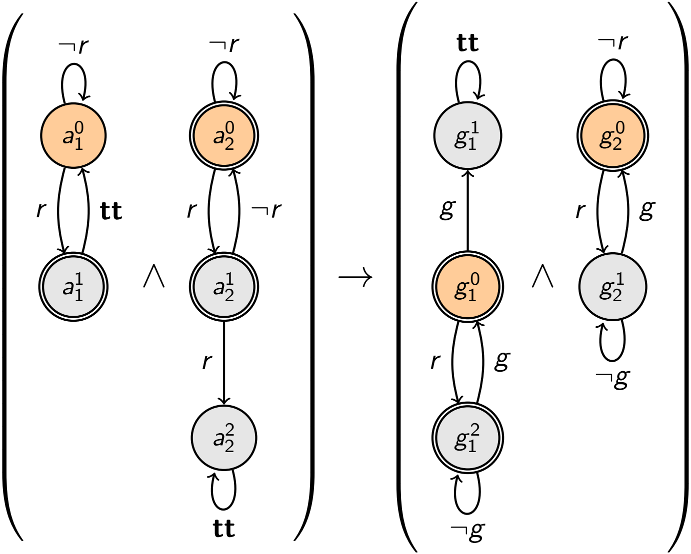
## Explanation by example: a mutex

- Set of inputs: $\{r\}$ (a request)
- Set of outputs: $\{g\}$ (a grant)
- Assumptions:
  - $\mathsf{GF}\, r$
  - $\mathsf{G}(r \rightarrow \mathsf{X}\neg r)$
- Guarantees:
  - A grant is only issued after a request has been issued (since the last grant)
  - $\mathsf{GF}\, g$

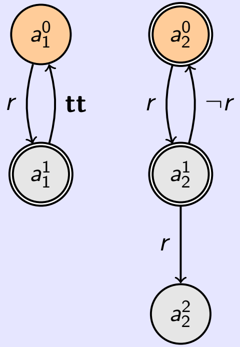
## Basic idea of the algorithm

- Reduce the problem to solving a parity game with 3 colours : $0, 1, 2$
- The system player wins if the highest colour occurring infinitely often in the play is 0 or 2

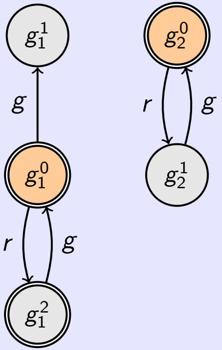
# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

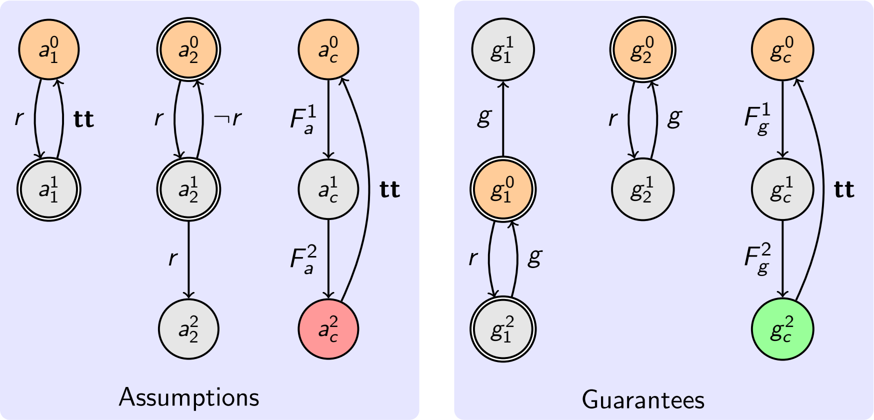



Assumptions




Guarantees

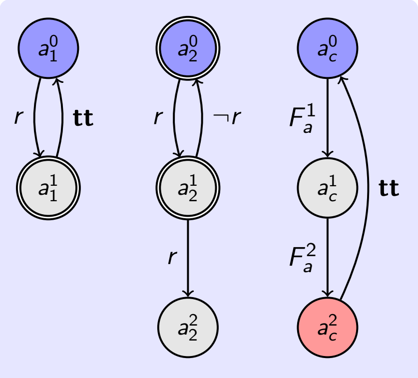
# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



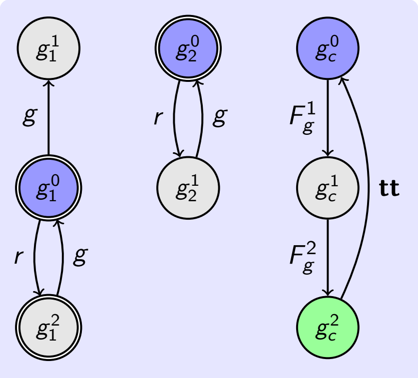

## Main idea

Make a parity game over the product state space with 3 colours.

# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])




Assumptions

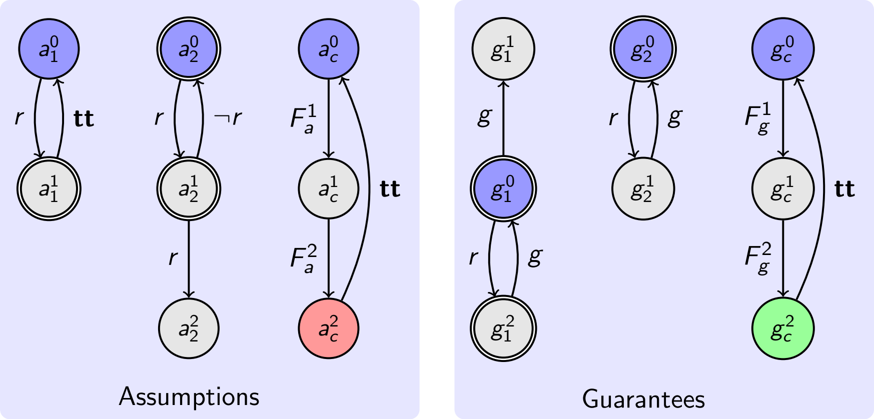



Guarantees

## Example run

| | | | |
|---|---|---|---|
| Request: | | | |
| Grant: | | | |
| Colour: | | | |

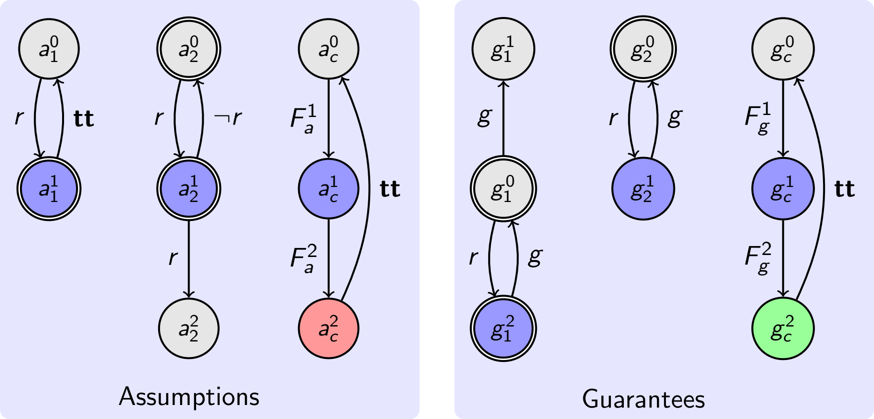
# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])




## Example run

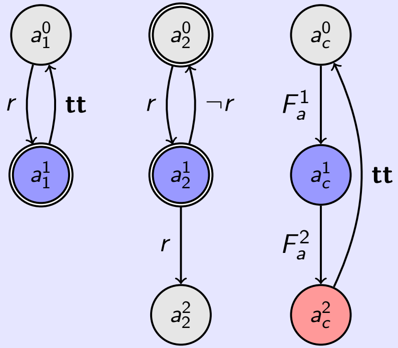
| | | | |
|---|---|---|---|
| Request: | 1 | | |
| Grant: | | | |
| Colour: | | | |

# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

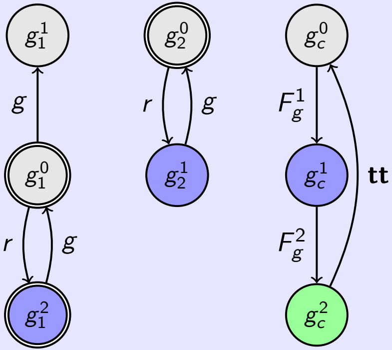



## Example run

| | | | |
|---|---|---|---|
| Request: | 1 | | |
| Grant: | 0 | | |
| Colour: | 0 | | |




Assumptions

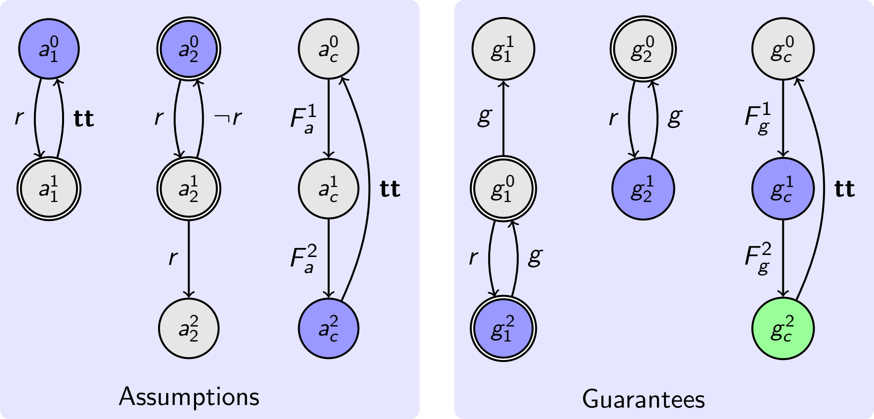



Guarantees

## Example run

| | | | |
|---|---|---|---|
| Request: | 1 | 0 | |
| Grant: | 0 | | |
| Colour: | 0 | | |

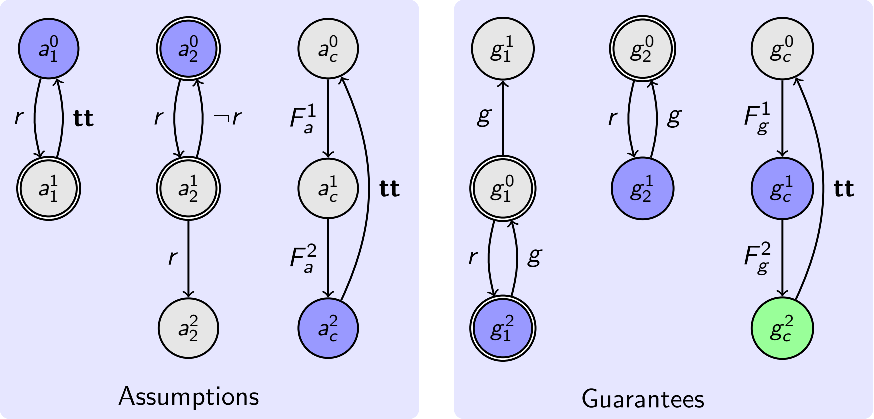
# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])




## Example run

| | | |
|---|---|---|
| Request: | 1 | 0 |
| Grant: | 0 | 0 |
| Colour: | 0 | 1 |

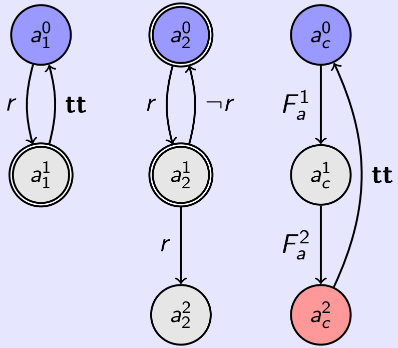
# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



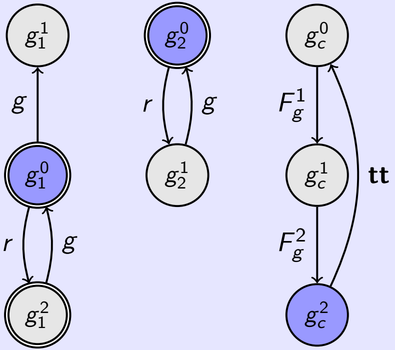

## Example run

| | | | |
|---|---|---|---|
| Request: | 1 | 0 | 0 |
| Grant: | 0 | 0 | |
| Colour: | 0 | 1 | |

# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])




Assumptions

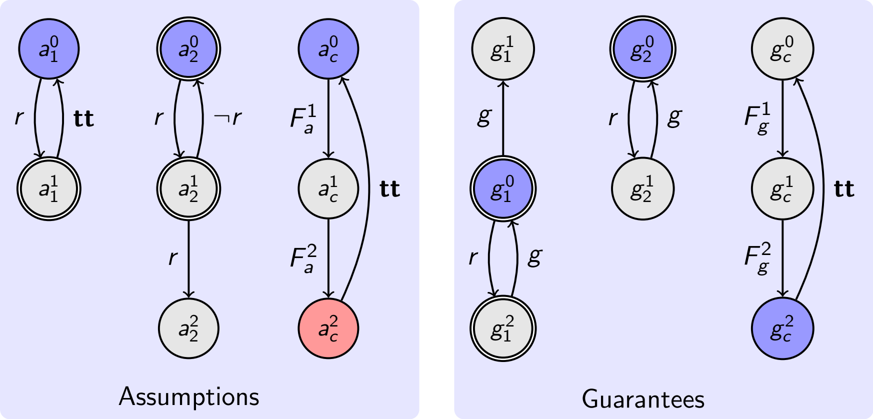



Guarantees

## Example run

| | | | |
|---|---|---|---|
| Request: | 1 | 0 | 0 |
| Grant: | 0 | 0 | 1 |
| Colour: | 0 | 1 | 2 |

# GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])




## Example run

| | | | |
|---|---|---|---|
| Request: | 1 | 0 | 0 |
| Grant: | 0 | 0 | 1 |
| Colour: | 0 | 1 | 2 |

...

# GR(1) synthesis - summary

## The highest colour occurring infinitely often

- Colour 0 - Neither the assumptions nor the guarantees are fulfilled
- Colour 1 - The assumptions are fulfilled but not the guarantees
- Colour 2 - The guarantees are fulfilled

## The nice properties of this approach

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- The parity game we obtain has a constant number of colours

$\Rightarrow$ Amenable to symbolic implementations, as confirmed by two case studies [BGJ$^{+}$07a, BGJ$^{+}$07b]

# On extending GR(1)

## Main question of this work

**How far can we push the expressivity of GR(1) without losing its nice properties?:**

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- We have a constant number of colours, independent of the number of assumptions and guarantees

## Intuition on why we want this

- Many properties in practice cannot be expressed yet, e.g., FG(*ready*)

## Answer to the question raised

**A little bit further, but that's it then (assuming P≠NP)**

# Generalised Rabin(1) synthesis

## Form of the specification

$$(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \ldots \wedge g_m)$$

with:

- a set $A$ of assumption $a_1, \ldots, a_n$
- a set $G$ of guarantees $g_1, \ldots, g_m$

such that:

- all elements in $A \cup G$ are deterministic **Rabin automata with one acceptance pair**
- all elements in $A \cup G$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

# Rabin automata

## Formal definition

A **one-pair** det. Rabin automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ with:

- a state set $Q$
- an alphabet $\Sigma$ (here, $\Sigma = 2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$)
- a transition relation $Q \times \Sigma \to Q$
- an initial state $q_0 \in Q$
- an acceptance component $F = (F_1, F_2) \in 2^Q \times 2^Q$

We say that $\mathcal{A}$ accepts a run $\pi = \pi_0\pi_1\ldots$ if $\pi_0 = q_0$ and $\inf(\pi) \cap F_1 = \emptyset$ and $\inf(\pi) \cap F_2 \neq \emptyset$

## A special property of one-pair Rabin automata

A run is accepted by a one-pair Rabin automaton $(Q, \Sigma, \delta, q_0, (F_1, F_2))$ iff it is accepted by the co-Büchi automaton $(Q, \Sigma, \delta, q_0, F_1)$ and the Büchi automaton $(Q, \Sigma, \delta, q_0, F_2)$.

# Generalised Rabin(1) synthesis - simplified

## Form of the specification

$$\begin{aligned} & (A_1 \wedge A_2 \wedge \ldots \wedge A_{|A|} \wedge B_1 \wedge B_2 \wedge \ldots \wedge B_{|B|}) \\ \rightarrow \quad & (C_1 \wedge C_2 \wedge \ldots \wedge C_{|C|} \wedge D_1 \wedge D_2 \wedge \ldots \wedge D_{|D|}) \end{aligned}$$

with:

- a set $A$ of Büchi assumptions $A_1, \ldots, A_{|A|}$
- a set $B$ of co-Büchi assumptions $B_1, \ldots, B_{|B|}$
- a set $C$ of Büchi guarantees $C_1, \ldots, C_{|C|}$
- a set $D$ of co-Büchi guarantees $D_1, \ldots, D_{|D|}$

such that:

- all elements in $A \cup C$ are deterministic Büchi automata
- all elements in $B \cup D$ are deterministic co-Büchi automata
- all elements in $A \cup B \cup C \cup D$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

# GRabin(1) synthesis - solution idea

## (Still flawed) reduction to a parity game

- State space is the product of the automata state spaces and the two Büchi assumption- and guarantee-checker automata $A_C$ and $G_C$
- 5 colours:
  - Colour 4 - Some co-Büchi assumption is violated
  - Colour 3 - Some co-Büchi guarantee is violated
  - Colour 2 - The Büchi guarantee automata have “recently” all visited their accepting states
  - Colour 1 - The Büchi assumption automata have “recently” all visited their accepting states
  - Colour 0 - none of the above

# GRabin(1) synthesis - solution idea analysis

**Maximal colour occurring inf. often on a play**

| | | co-B. Ass. | co-B. Ass. | | |
|---|---|---|---|---|---|
| | | | B. Ass. | B. Ass. | |
| co-B. Gua. | | 0 | 1 | 4 | 4 |
| co-B. Gua. | B. Gua. | 2 | 2 | 4 | 4 |
| | B. Gua. | 3 | 3 | 4 | 4 |
| | | 3 | 3 | 4 | 4 |

# GRabin(1) synthesis - solution idea analysis

## Maximal colour occurring inf. often on a play

| | co-B. Ass. | co-B. Ass. B. Ass. | B. Ass. | |
|---|---|---|---|---|
| co-B. Gua. | 0 | 1 | 4 | 4 |
| co-B. Gua. B. Gua. | 2 | 2 | 4 | 4 |
| B. Gua. | 3 | 3 | 4 | 4 |
| | 3 | 3 | 4 | 4 |

## A problem

Currently we lose on too many plays!

## The solution

Add storage bit to the game tracking whether the counter automaton for $C$ has “recently” completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

# GRabin(1) synthesis - solution idea analysis

## Maximal colour occurring inf. often on a play (fixed)

| | co-B. Ass. | co-B. Ass. B. Ass. | B. Ass. | |
|---|---|---|---|---|
| co-B. Gua. | 0 | 1 | 4 | 4 |
| co-B. Gua. B. Gua. | 2 | 2 | 4 | 4 |
| B. Gua. | 2 | 3 | 4 | 4 |
| | 0 | 3 | 4 | 4 |

## A problem

Currently we lose on too many plays!

## The solution

Add storage bit to the game tracking whether the counter automaton for $C$ has "recently" completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

# Can we generalise even further?

### Answer: No!

Generalised Streett(1) synthesis cannot work in the way described:

- Game arena is the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees)
- Parity game with a constant number of colours

### Reason

Generalised parity game solving is NP-hard for certain cases

# Generalised parity games [CHP07]

## The conjunctive version

- A game graph with $k$ parity functions is given
- Player 0 needs to win for all of these functions at the same time

## A hard case

For parity functions with the co-domain $\{0, 1, 2\}$, solving (the conjunctive version of) generalised parity games is NP-hard

## A reduction of the hard case to generalised Streett(1) games

Convert such a game with $k$ parity functions to the specification $(\mathbf{tt}) \rightarrow (G_1 \wedge \ldots \wedge G_k)$ for one-pair Streett guarantees $G_i$

# The reduction (continued)

## Examining the specification

$(\mathbf{tt}) \rightarrow (G_1 \wedge \ldots \wedge G_k)$ has a special property: $G_1, \ldots, G_k$ have the same transition structure.

## Consequence

If the product game arena was the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees) and the game had a constant number of colours, we could solve an NP-hard problem in polynomial time.

# An application: synthesis of robust systems

## Example specification - A processing machine (base version)

$$\left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\ \text{incoming}\end{array}\right) \wedge \mathsf{G}\left(\begin{array}{c}\text{no over-}\\ \text{sized parts}\end{array}\right) \wedge \ldots\right)$$

$$\rightarrow \left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\ \text{processed}\end{array}\right) \wedge \mathsf{G}\left(\begin{array}{c}\text{no}\\ \text{jam}\end{array}\right) \wedge \ldots\right)$$

## Example specification - A processing machine (robust)

$$\left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\ \text{incoming}\end{array}\right) \wedge \mathsf{FG}\left(\begin{array}{c}\text{no over-}\\ \text{sized parts}\end{array}\right) \wedge \ldots\right)$$

$$\rightarrow \left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\ \text{processed}\end{array}\right) \wedge \mathsf{FG}\left(\begin{array}{c}\text{no}\\ \text{jam}\end{array}\right) \wedge \ldots\right)$$

# Summary

## Generalised Rabin(1) synthesis is ...

- a symbolically implementable synthesis method
- in some sense the best we can get (in this line of research)
- a practically relevant fragment of LTL

## Important questions beyond the scope of this talk

- How to get from logic to Rabin(1) automata
- How to encode these automata into BDDs
- How to solve the resulting games efficiently

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