

Games with Finitary Parity and Finitary Streett Objectives

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Model Checking and Synthesis

- Model checking: given a model (of a system) and a desired set of behaviors (specification) whether the model satisfies the specification.

$$\boxed{M} \models \psi$$

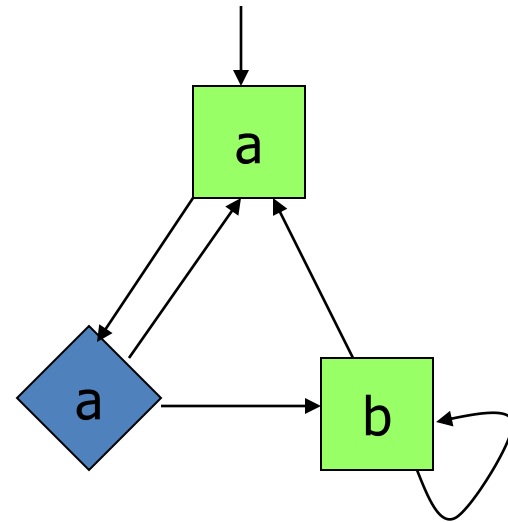
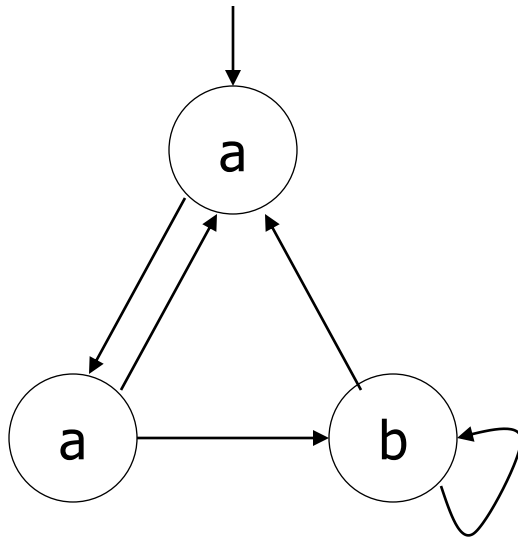
- Synthesis: the model in open environment; system interacts with the environment.

$$\boxed{M} \parallel \boxed{E} \models \psi$$

Verification and Synthesis

- Model for verification (model checking): labeled transition systems.
- Model for synthesis: games on graphs.
- Specification: an ω -regular set of paths on the graphs.

Labeled Graphs vs. Labeled Games



ω -Regular Objectives

- ω -regular objectives

- Extension of classical regular languages to infinite strings.

- Regular: Union (\cup), Concatenation (\cdot) and Kleene star (finite repetition ($*$)).

- Extension to infinite string by infinite repetition (ω).

$$r ::= \sigma \mid r \cup r \mid r \cdot r \mid (r)^* \mid (r)^\omega \quad \sigma \in \Sigma$$

- Non-deterministic Buchi automata as compared to finite-automata.

- Deterministic parity and Streett automata are canonical forms to express ω -regular languages.

ω -Regular Objectives

- ω -regular objectives
 - Extension of classical regular languages to infinite strings: infinite behaviors of non-terminating systems.
 - Express liveness, fairness and most commonly used specifications in verification.
 - Robust and expressive language for specifications.

ω -Regular Objectives

- ω -regular objectives
 - All ω -regular objectives can be decomposed as a safety part and a liveness part [Alpern-Schneider 85].
 - Liveness: something “good” eventually happens.

Strengths of Classical Formulation

- Classical infinitary formulation
 - Strengths
 - Robustness: Independence of granularity of transitions.
 - Simplicity: Abstraction of complicated time bounds.

Weakness of Classical Formulation

- Classical infinitary formulation
 - Weakness
 - Something “good” happens eventually and the time something “good” happens may be unbounded.
 - Everything happens eventually!!!
 - Elevator controller: every request must be eventually granted.



Finitary Formulation

- Finitary fairness [Alur-Henzinger 94]
 - There exists unknown fixed bound b such that something “good” happens within b transitions.
 - Retains simplicity and robustness of ω -regular specifications.
 - Rules out undesired behaviors.
- Synthesis of finitary specifications gives games with finitary objectives.
 - This talk: Games with finitary parity and Streett objectives.

Synthesis of Finitary ω -Regular Conditions

- Game graphs
 - Two-player perfect-information games.
- Winning conditions
 - Finitary parity conditions: canonical form for ω -regular objectives.
 - Finitary Streett conditions: strong fairness.

Game Graphs

Components for Games on Graphs

- Where: the game arena (perfect-information turn-based games)
- Why: objectives (finitary parity and finitary Streett)
- How: strategies (pure strategies).

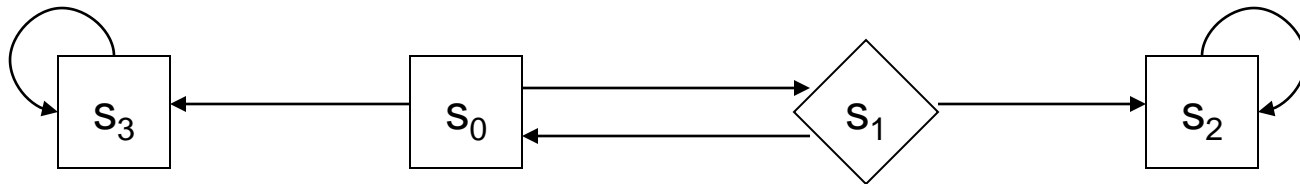
Turn-based Games

- Turn-based deterministic or Perfect-information deterministic Games
 - Game graph $G = ((V, E), (V_1, V_2))$.
 - V : finite set of states (vertices).
 - $E \subseteq V \times V$.
 - (V_1, V_2) : a partition of the state space.
- The game
 - Played by moving token along edges of the graph.

Play and Strategies

- Play or outcome of a game
 - Infinite sequence of states (s_1, s_2, \dots) such that for all $i \geq 1$, we have $(s_i, s_{i+1}) \in E$.
 - We denote by Ω the set of all plays.
- Strategies
 - For player 1 given a history of play specifies how to extend the play; $\sigma : V^* \cdot V_1 \rightarrow V$ such that for all $x \in V^*$ and $v \in V_1$ we have $(v, \sigma(x \cdot v)) \in E$.
 - Similar definition of strategy π for player 2.

Example: a game graph



- Example of a play: $s_0 s_1 s_2^{(\omega)}$
- Strategy:
 - Player 1 : $s_0 \rightarrow s_1$
 - Player 2 : $s_1 \rightarrow s_2$

Play and Strategies

- Strategies
 - Memoryless strategies: Independent of history of the play, i.e., $\sigma : V_1 \rightarrow V$.
 - Finite memory strategies: depends only on finite amount of information about the past history, i.e., can be played with a finite automaton.
- Play
 - Unique play: given strategy σ, π and a starting state s there is a unique play denoted as $\Omega_{\sigma, \pi}(s)$.

Objectives

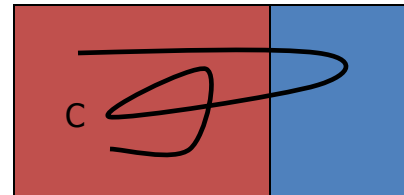
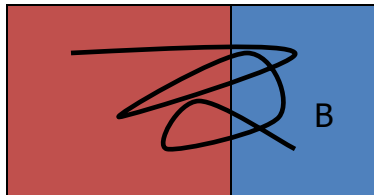
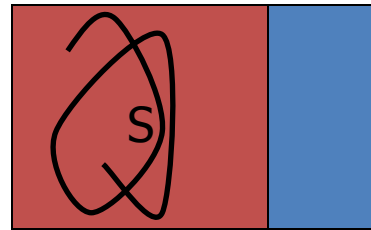
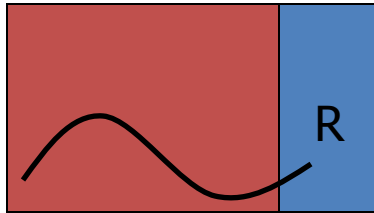
Objectives

- **Plays**: infinite sequence of states.
- **Objectives**: subset of plays, $\Psi_1 \subseteq V^\omega$.
- Play is winning for player 1 if it is in Ψ_1 .
- Zero-sum game: $\Psi_2 = V^\omega \setminus \Psi_1$.

Reachability, Safety, Buechi and coBuechi Objectives

- **Reachability and Safety objectives:**
 - Reachability: given a set $R \subseteq V$, the objective requires to visit a state in R .
 - Safety: given a set $F \subseteq V$, the objective requires to remain always in F .
- **Buechi and coBuechi Objectives:**
 - Buechi: given a set $B \subseteq V$, the objective requires to visit B infinitely often.
 - coBuechi: given a set $C \subseteq V$, the objective requires to visit $V \setminus C$ finitely often.

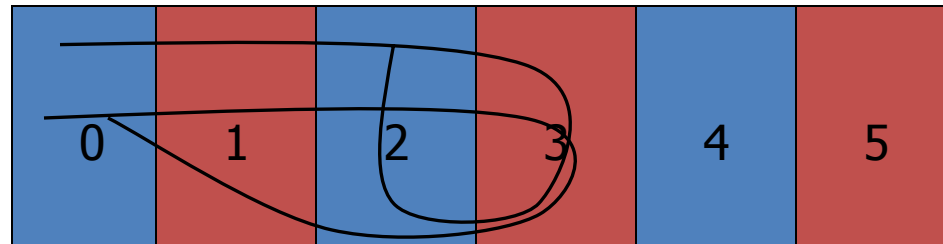
Objectives



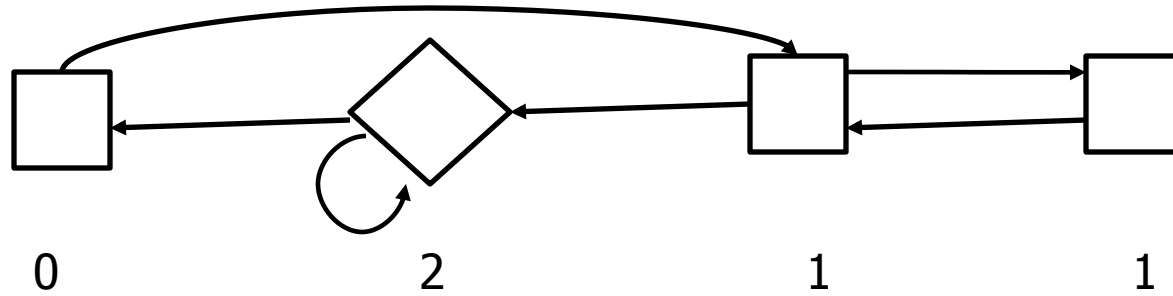
Parity Objectives

- Parity objectives:
 - Consider a priority function $p: V \rightarrow \{0,1,2,\dots,d\}$.
 - The parity objective requires that the minimum priority visited infinitely often along a play is even.
 - Canonical for ω -regular objectives.

Objectives



Example



Box is Player 1 and winning strategy is to go left.

Streett Objectives

- Let $\{(E_1, F_1), (E_2, F_2), \dots, (E_d, F_d)\}$ set of state set pairs.
 - Streett: requires for every pair (E_j, F_j) if F_j infinitely often then E_j infinitely often.
 - $\forall j. (F_j \text{ finitely} \vee E_j \text{ infinitely}) =$
 $\forall j. (F_j \text{ infinitely} \rightarrow E_j \text{ infinitely})$
 - Conjunction of fairness conditions.

Games with ω -Regular Objectives

- Objectives: Φ_1 for player 1 and $\Phi_2 = V^\omega \setminus \Phi_1$ for player 2:
 - Determinacy [Gurevich Harrington, Buechi Landweber]
For all states s one of the following holds:
 - $\exists \sigma \forall \pi. \Omega_{\sigma, \pi}(s) \in \Phi_1$;
 - $\exists \pi \forall \sigma. \Omega_{\sigma, \pi}(s) \in \Phi_2$.
 - Finite-memory determinacy: there always exist witness winning strategies that are finite-memory.

Parity and Streett Games

- Objectives: Φ_1 for player 1 and $\Phi_2 = V^\omega \setminus \Phi_1$ for player 2:
 - Parity objectives: memoryless winning strategies for both players in the respective winning set [Emerson-Jutla].
 - Streett objectives: finite-memory winning strategy for the player with Streett objective and the other player has memoryless winning strategy [Emerson-Jutla].
 - Streett objectives with d-pairs require $d!$ memory [Zielonka , Dziembowski-Jurdzinski-Walukiweicz]

Parity and Streett Games

- Objectives: Φ_1 for player 1 and $\Phi_2 = V^\omega \setminus \Phi_1$ for player 2:
 - Computational complexity:
 - Parity objectives lie in $NP \cap co\ NP$ [Emerson-Jutla]; also in $UP \cap coUP$ [Jurdzinski].
 - Streett objectives: coNP-complete [Emerson-Jutla].

Finitary Parity Objectives

Finitary Parity Objectives

- Parity objectives requires minimum priority visited infinitely often to be even.
- Finitary parity objectives distance to a better even priority, for odd priorities visited infinitely often, to be bounded.

Finitary Parity Objective

- Given a play $\omega=(s_0,s_1,s_2,\dots)$ consider the distance sequence as follows:
 - $\text{dist}_i(\omega,p)= 0$ if $p(s_i)$ is even; and otherwise $\inf\{ j - i : j > i, p(s_j) < p(s_i) \text{ and } p(s_j) \text{ is even} \}$.
 - Finitary parity objective requires the distance sequence be bounded in the limit:
 $\limsup_i \text{dist}_i(\omega,p)$ to be bounded.

Finitary Parity Objectives

- Finitary coBuechi objectives coincide with coBuechi objectives.
- Finitary Buechi objectives may be strict subset of Buechi objectives.
 - e.g., the distance sequence to Buechi states be $1,2,3,4,\dots$
 - Visit Buechi states infinitely often, but infinitely rarely.
- For a parity objective, the finitary parity objective is in general a strict subset of the plays defined by parity objectives.

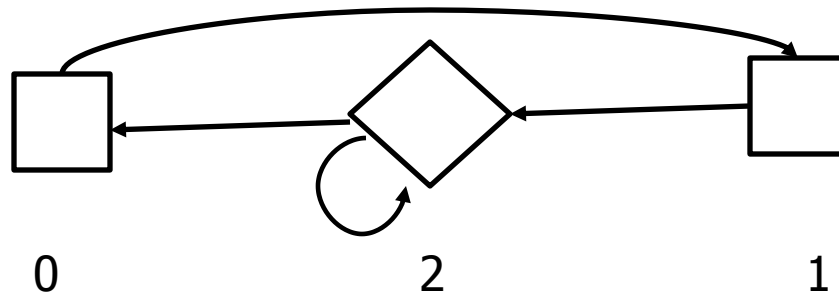
Finitary Parity Objectives

- $\text{CoBuechi} = \text{Finitary CoBuechi}$.
- $\text{Buechi} \subsetneq \text{Finitary Buechi}$.
- $\text{Parity} \subsetneq \text{Finitary Parity}$.

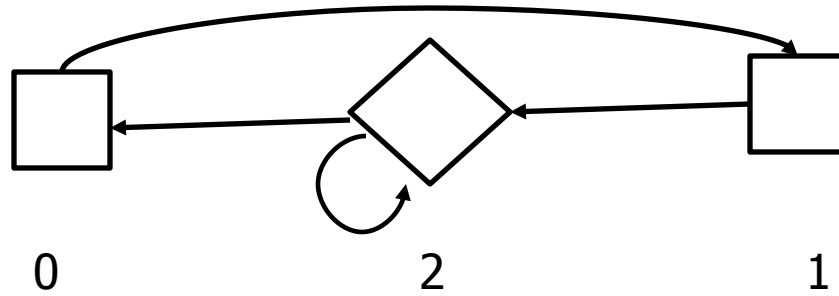
Finitary Parity Objectives

- Finitary coBuechi objectives coincide with coBuechi objectives: so winning sets coincide.
- Finitary Buechi objectives may be strict subset of Buechi objectives.
 - However, it can be shown for games winning sets of finitary Buechi and Buechi objectives coincide (mainly due to existence of memoryless strategies).
- Finitary parity objectives may be strict subset of parity objectives and the winning set may also be strict subset.

Example

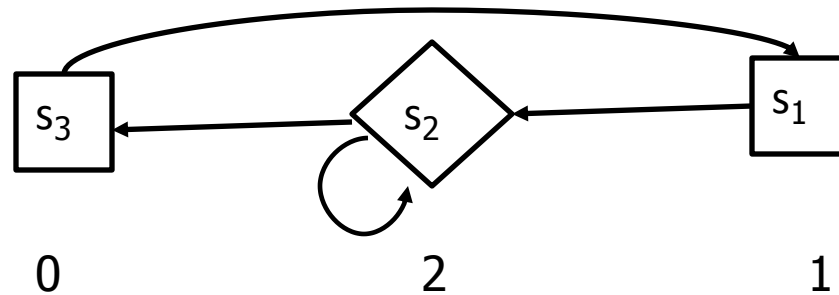


Example



Parity objective: player 1 wins.

Example



Parity objective: player 1 wins.

Finitary parity objective: player 2 play the following strategy in rounds.

In round k stay in s_2 for k times and move to s_3 and proceed to round $k+1$. The strategy is winning for player 2.

Player 2 winning strategy require infinite memory.

Finitary Parity Objectives

- Winning CoBuechi = Winning Finitary CoBuechi.
- Winning Buechi = Winning Finitary Buechi.
- Winning Parity \subsetneq Winning Finitary Parity.

Finitary Parity and Streett Games



- Winning sets different from classical winning set (generally can be a strict subset).
- Winning strategies very different: may require infinite memory.
- Even if classical and finitary winning set coincide the structure of winning strategies may be different.

Finitary Parity Games

Iterative Algorithm

- We presented an iterative algorithm to compute winning sets for both players for finitary parity objectives [C Henzinger Horn].
- The algorithm quite different from the classical iterative and recursive algorithm for infinitary parity objectives.
 - Recall player 2 requires infinite memory strategies.

Iterative Algorithm

- Three step algorithm.
 - Weak-parity objectives.

 - Bounded parity objectives.

 - Finitary parity objectives.

Idea of the Iterative Algorithm

- Weak-parity objectives: requires that the minimum priority that appears is even (contrast to classical parity which require the minimum priority to be even).
 - A linear time algorithm is known to solve games with weak parity objectives: basically iteratively applying solution for reachability games.
- Bounded parity objectives: requires the sequence to be bounded (not in the limit, but bounded).
 - An algorithm that iteratively applies the solution of weak parity games to solve bounded parity games is given in [C Henzinger Horn]: complexity is quadratic.

Idea of the Iterative Algorithm

- An iterative algorithm that uses solution of bounded parity games to solve finitary parity games is given in [C Henzinger Horn]
- It requires $O(n)$ iterations of the bounded parity games and the overall time complexity is $O(n^2 \cdot m)$

Idea of the Iterative Algorithm

- Bounded parity is stricter than finitary parity.
- We solve bounded parity for Player 1 and take attractor and remove from the game graph. When we stop we have a game graph where all states are winning for Player 2 for complement of bounded parity.
- We show if all states are winning for Player 2 for complement of bounded parity, then an infinite memory strategy can be constructed for Player 2 to violate finitary parity against all Player 1 strategies.

Iterative Algorithm

- The correctness of our algorithm also proves the determinacy result, i.e., for all finitary parity objectives Φ_1 , for all states s one of the following holds:
 - $\exists \sigma \forall \pi. \Omega_{\sigma, \pi}(s) \in \Phi_1$;
 - $\exists \pi \forall \sigma. \Omega_{\sigma, \pi}(s) \in V^\omega \setminus \Phi_1$.

Complexity of the Recursive Algorithm

- Time complexity: Recursive algorithm for finitary parity objectives
 - $O(n^2 \cdot m)$ for game graphs with n states, m edges.
- For infinitary parity objectives:
 - Classical recursive [McNaughton 93]: $O(n^{d-1} \cdot m)$.
 - Small-progress measure [Jurdzinski 00]: $O(n^{d/2} \cdot m)$.
 - Big-step-small-step algorithm [Schewe 07] : $O(n^{d/3} \cdot m)$
 - Sub-exponential algorithm [Jurdzinski et.al. 06] : $O(n^{n^{1/2}})$.

Strategy Complexity

- Memoryless winning strategy exist for player 1.
- Winning strategy for player 2 require infinite memory in general.
- Witnesses for winning strategies:
 - Linear size witnesses exist for winning strategies for player 1.
 - Polynomial (quadratic) size witnesses exist for winning strategies for player 2.
 - Mainly by the reachability and safety characterization.

Comparison Infinitary vs Finitary Parity Objectives

	Classical infinitary parity	Finitary parity
Computational Complexity	NP and coNP [EJ88] UP and coUP [Jur98]	PSPACE-complete
Algorithmic complexity	$O(n^{d/3} \cdot m)$, $n^{O(\sqrt{n})}$	$O(n^2 \cdot m)$
Strategy complexity	Memoryless for both players	Memoryless for finitary parity; opposing player needs infinite memory.

Finitary Streett Objectives

Finitary Streett Objective

- Natural extension in terms of distances
- Streett objective: F_j inf. often then E_j inf. often.
- Distances: from F_j states to E_j states.
- Finitary Streett: the distance sequences be bounded in the limit.

Service and Request Example

- Requests of two-types R_1 and R_2 .
- Requests generated by player 2.
- Once a request of a type is generated further requests are of the type are disabled unless the request is granted. Grants are denoted as G_1 and G_2 ., respectively.
- Streett objective: R_i inf often, then G_i inf often.

Service and Request Example

- Two strategies: queue service and stack service.
- Both strategies are Streett winning.
- Only queue strategy is finitary Streett winning.
 - Stack strategy is not finitary Streett winning as player 2 can generate a request R_1 , and then a sequence of R_2 before R_1 is served and this sequence can grow unbounded.

Finitary Streett Games

Finitary Streett Objectives

- The algorithm is an iterative algorithm on a game graph of exponential size. The iterative algorithm uses solution of generalized Buechi objectives. The solution needs the notion of bounded Streett objectives.
- Bounded Streett objectives: requires the distance sequence to be bounded

Finitary Streett Objectives

- Algorithm ideas:
 - For Parity: weak \rightarrow bounded \rightarrow finitary.
 - For Streett: bounded \rightarrow finitary.
- Winning for bounded Streett is same as winning for Request-response objectives.
- Request-response: every request is eventually granted within bounded steps.

Finitary Streett Objectives

- The solution of bounded Streett objectives: by solving generalized Buchi objectives on a game graph of size $O(2^d \cdot n \cdot m)$ [Wallmeier Hutten Thomas].
- An algorithm that uses $O(n)$ iterations of bounded Streett objectives is given in [C Henzinger Horn].

Finitary Streett Objectives

- The algorithm gives an EXPTIME upper bound.
- Complexity bounds for finitary Streett and Request-response are very related (lower bound of the later gives lower bound for the other).

Request-Response and Finitary Streett Games: Complexity

- The complexity is EXPTIME-complete.
 - Parity (no polynomial) whereas finitary parity is cubic.
 - Streett (coNP-complete) whereas finitary Streett is EXPTIME-complete.
- Two natural extension of Streett objectives gives the high complexity.
 - Reduction from APTM and the objective is used to tackle the exponential number of configurations succinctly.

Request-Response and Finitary Streett Games: Complexity

- Streett:
 - Conjunction (Always Eventually F_i implies Always Eventually E_i).
 - coNP-complete
- Request-response
 - Conjunction (Always (F_i implies Eventually E_i))
 - EXPTIME-complete.

Request-Response and Finitary Streett Games: Complexity

- Computational, algorithmic and strategy complexity.
- Computational complexity: bad news!
- Algorithmic complexity: good news!

Request-Response and Finitary Streett Games: Complexity

- Algorithmic complexity
 - Best known algorithm for Streett: $O(n^d \cdot d! \cdot m)$.
 - Request-response games: $O(n \cdot 2^d \cdot m)$.
 - Finitary Streett games: $O(n^2 \cdot 2^d \cdot m)$.
 - Complexity in terms of game size is polynomial and the exponent is only the number of pairs.

Strategy Complexity

	Player 1	Player 2
Request-response	Upper bound: $d \cdot 2^d$ Lower bound: $2^d - 1$	2^d
Finitary Streett	Upper bound: $d \cdot 2^d$ Lower bound: $2^d - 1$	Infinite memory
Classical Streett	$d!$	Memoryless

Comparison Infinitary vs Finitary Streett Objectives

	Classical infinitary Streett	Finitary Streett
Computational Complexity	coNP-complete	EXPTIME-complete
Algorithmic complexity	$O(n^d \cdot d! \cdot m)$	$O(n^2 \cdot 2^d \cdot m)$
Strategy complexity	$d!$ for Streett; Memoryless for the opposing player	$2^d \cdot d$ for finitary Streett; opposing player needs infinite memory.

Conclusion

- Conclusion
 - Finitary ω -regular conditions
 - Finitary parity and finitary Streett conditions.
 - Retains simplicity and robustness of classical specifications and rules out undesired behaviors.
 - Synthesis: Games and stochastic games with finitary parity and Streett conditions.
 - Algorithmic and strategy characterization of games with finitary parity and Streett objectives.

Interesting Directions

- Interesting questions
 - Logical and topological characterization of finitary objectives.
 - Improved algorithm for finitary parity games ($O(n^{1.5} m)$ or $O(n m)$).
 - More general class of games graphs: extension to turn-based stochastic games already done in [C Henzinger Horn MFCS 09].

Thanks

<http://www.ist.ac.at/research/research-groups/chatterjee-group/>



Questions ???