

Controller Synthesis Based on On-Demand Partial Observations



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Overview

- Time games with full observability.
- Time games with partial observability.
- Example and motivation.
- Time games with lattice of observations.



Controller Synthesis/TGA

- Given
 - System moves S ,
 - Controller moves C ,
 - and a property ϕ ,
- find
 - a strategy S_c s.t. $S_c || S \models \phi$,
 - or prove there is no such strategy.



Previous Work

- The controller continuously observes the system (all delays & moves are observable).
 - [CONCUR'05] (Franck Cassez, Alexandre David, Emmanuel Fleury, Kim G. Larsen, Didier Lime).
- The controller observes the system through one set of predicates (observations). Actions are urgent.
 - [ATVA'07] (Franck Cassez, Alexandre David, Kim G. Larsen, Didier Lime, Jean-François Raskin).

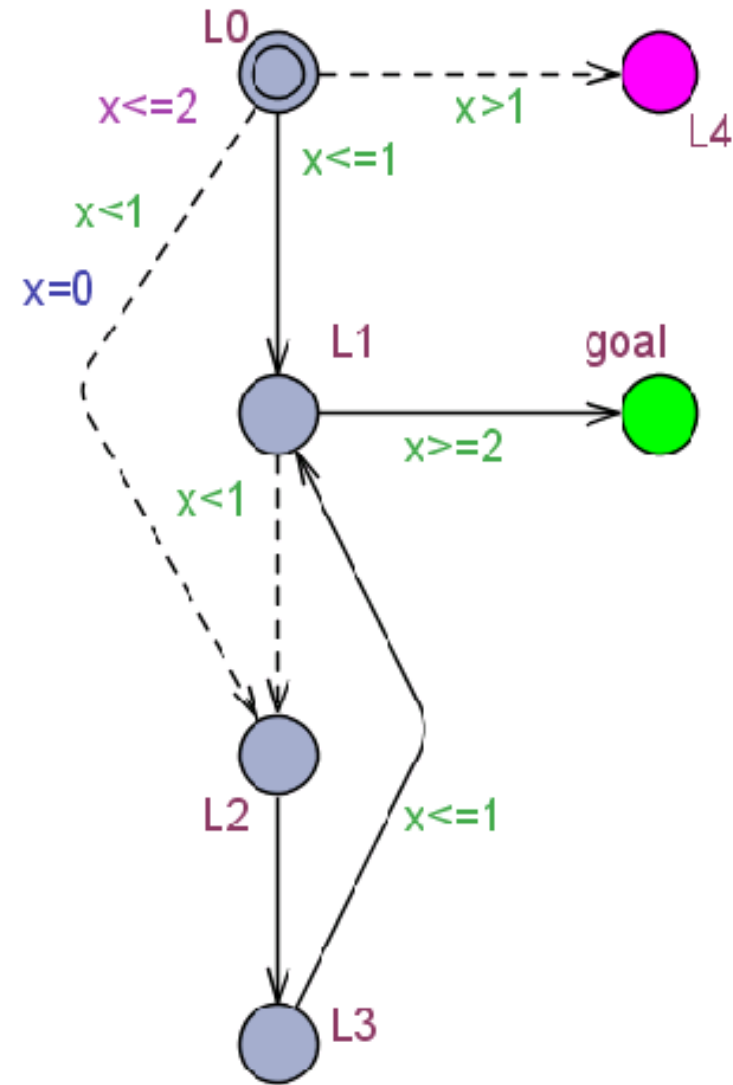


Case-Studies

- [FORMATS'07] Guided Controller Synthesis Using UPPAAL-TIGA. Jan Jakob Jessen, Jacob Illum Rasmussen, Kim G. Larsen, Alexandre David.
- [HSCC'09] Automatic Synthesis of Robust and Optimal Controllers – An Industrial Case Study. Franck Cassez, J.J. Jessen, Kim G. Larsen, Jean-François Raskin, Pierre-Alain Reynier. (Hydac)

Timed Game Automata

- Timed automata with controllable and uncontrollable transitions.
- Reachability & safety games.
 - control: $A \leftrightarrow \text{TGA.goal}$
 - control: $A[\]$ not TGA.L4
- Memoryless strategy:
 - state \rightarrow action.



One Strategy

- control: $A \leftrightarrow \text{TGA.goal}$

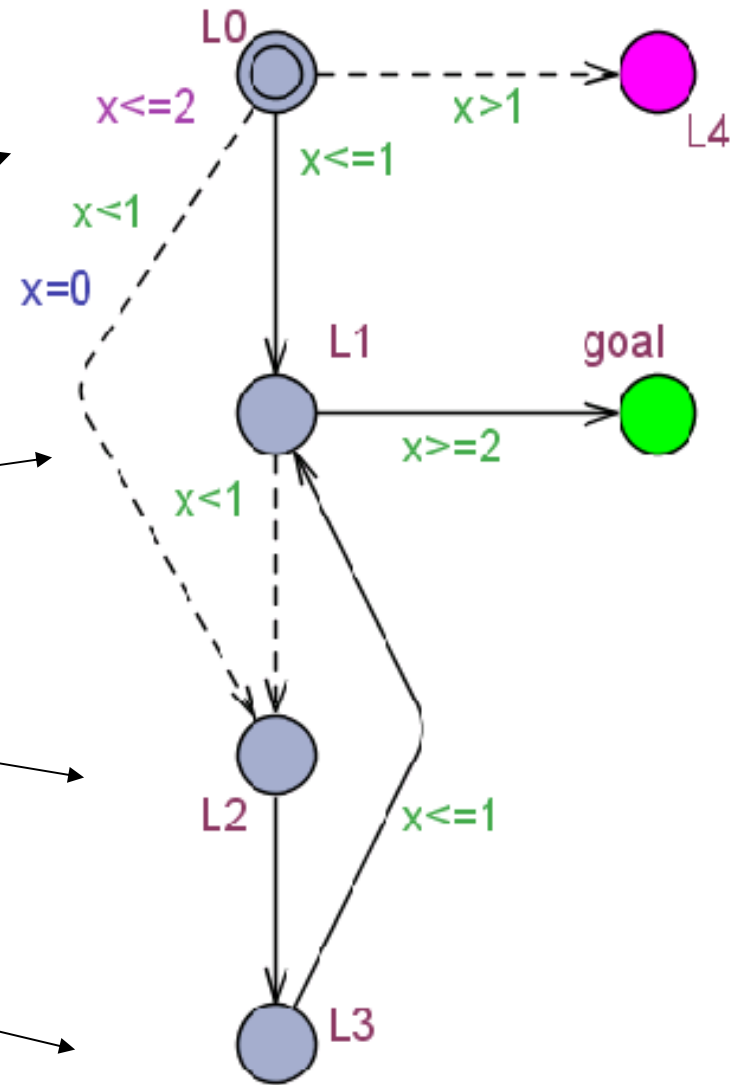
Strategy

$x < 1$: λ
 $x == 1$: c

$x < 2$: λ
 $x \geq 2$: c

$x \leq 1$: c

$x < 1$: λ
 $x == 1$: c

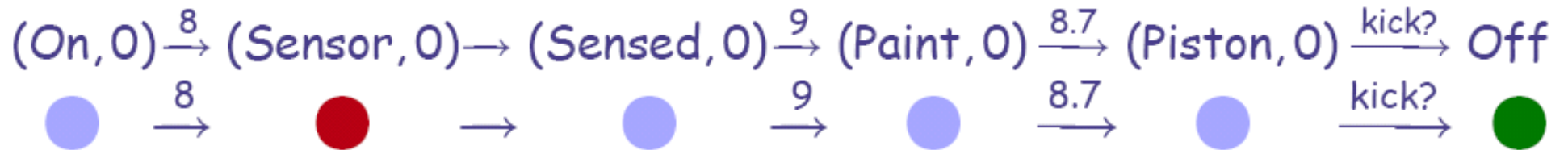
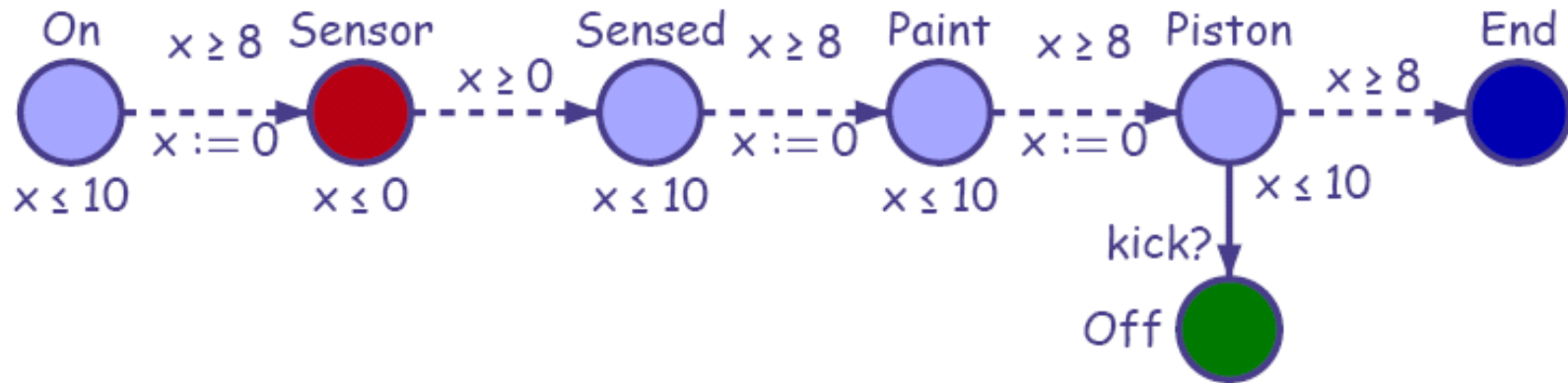




Game with Partial Observability

- State-based observations
 - locations, variables
 - clock intervals of the form [lower,upper[
- What is observable:
 - change of observation,
 - *not* every state change.
 - → stuttering free observations.

Example



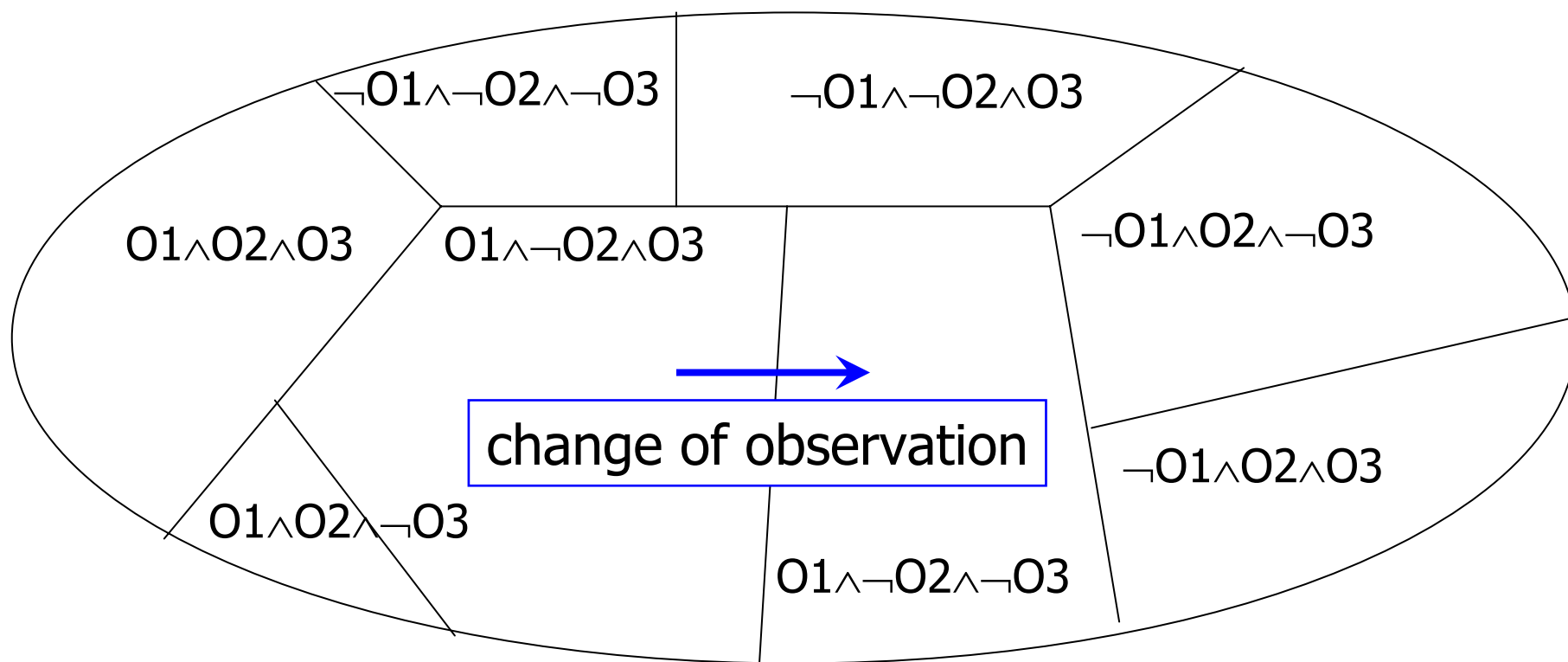
Assumption: the controller can only see **changes** of observations

Stuttering-free observation: ● ● ● ●

Must play based on **stuttering-free** observations

Algorithm

Partition the state-space w.r.t. observations.
Observations $O_1 O_2 O_3$.
Winning/losing is observable.



Algorithm with Reachability Objective

Initialization:

1

```

Passed ←  $\{\{s_0\}\}$ ;
Waiting ←  $\{(\{s_0\}, \alpha, W') \mid \alpha \in \Sigma_1, o \in \mathcal{O}, W' = \text{Next}_\alpha(\{s_0\}) \cap o \wedge W' \neq \emptyset\}$ ;
Win $[\{s_0\}]$  ←  $(\{s_0\} \subseteq \gamma(\text{Goal}) ? 1 : 0)$ ;
Losing $[\{s_0\}]$  ←  $(\{s_0\} \not\subseteq \gamma(\text{Goal}) \wedge (\text{Waiting} = \emptyset \vee \forall \alpha \in \Sigma_1, \text{Sink}_\alpha(s_0) \neq \emptyset) ? 1 : 0)$ ;
Depend $[\{s_0\}]$  ←  $\emptyset$ ;
    
```

Main:

while $((\text{Waiting} \neq \emptyset) \wedge \text{Win}[\{s_0\}] \neq 1 \wedge \text{Losing}[\{s_0\}] \neq 1)$ **do**

$e = (W, \alpha, W') \leftarrow \text{pop}(\text{Waiting})$;

if $W' \notin \text{Passed}$ **then**

2

```

    Passed ← Passed  $\cup \{W'\}$ ;
    Depend $[W']$  ←  $\{(W, \alpha, W')\}$ ;
    Win $[W']$  ←  $(W' \subseteq \gamma(\text{Goal}) ? 1 : 0)$ ;
    Losing $[W']$  ←  $(W' \not\subseteq \gamma(\text{Goal}) \wedge \text{Sink}_\alpha(W') \neq \emptyset ? 1 : 0)$ ;
    if  $(\text{Losing}[W'] \neq 1)$  then (* if losing it is a deadlock state *)
      NewTrans ←  $\{(W', \alpha, W'') \mid \alpha \in \Sigma, o \in \mathcal{O}, W' = \text{Next}_\alpha(W) \cap o \wedge W' \neq \emptyset\}$ ;
      if  $\text{NewTrans} = \emptyset \wedge \text{Win}[W'] = 0$  then Losing $[W'] \leftarrow 1$ ;
      if  $(\text{Win}[W'] \vee \text{Losing}[W'])$  then Waiting ← Waiting  $\cup \{e\}$ ;
      Waiting ← Waiting  $\cup \text{NewTrans}$ ;
    
```

else (* reevaluate *)

3

```

    Win $^* \leftarrow \bigvee_{c \in \text{Enabled}(W)} \bigwedge_{W \xrightarrow{c} W''} \text{Win}[W'']$ ;
    if Win $^*$  then
      Waiting ← Waiting  $\cup \text{Depend}[W]$ ; Win $[W] \leftarrow 1$ ;
    Losing $^* \leftarrow \bigwedge_{c \in \text{Enabled}(W)} \bigvee_{W \xrightarrow{c} W''} \text{Losing}[W'']$ ;
    if Losing $^*$  then
      Waiting ← Waiting  $\cup \text{Depend}[W]$ ; Losing $[W] \leftarrow 1$ ;
    if  $(\text{Win}[W'] = 0 \wedge \text{Losing}[W'] = 0)$  then Depend $[W'] \leftarrow \text{Depend}[W'] \cup \{e\}$ ;
    
```

endif

endwhile



Initialization

$Passed \leftarrow \{\{s_0\}\};$

$Waiting \leftarrow \{(\{s_0\}, \alpha, W') \mid \alpha \in \Sigma_1, o \in \mathcal{O}, W' = \text{Next}_\alpha(\{s_0\}) \cap o \wedge W' \neq \emptyset\};$

$Win[\{s_0\}] \leftarrow (\{s_0\} \subseteq \gamma(\text{Goal}) ? 1 : 0);$

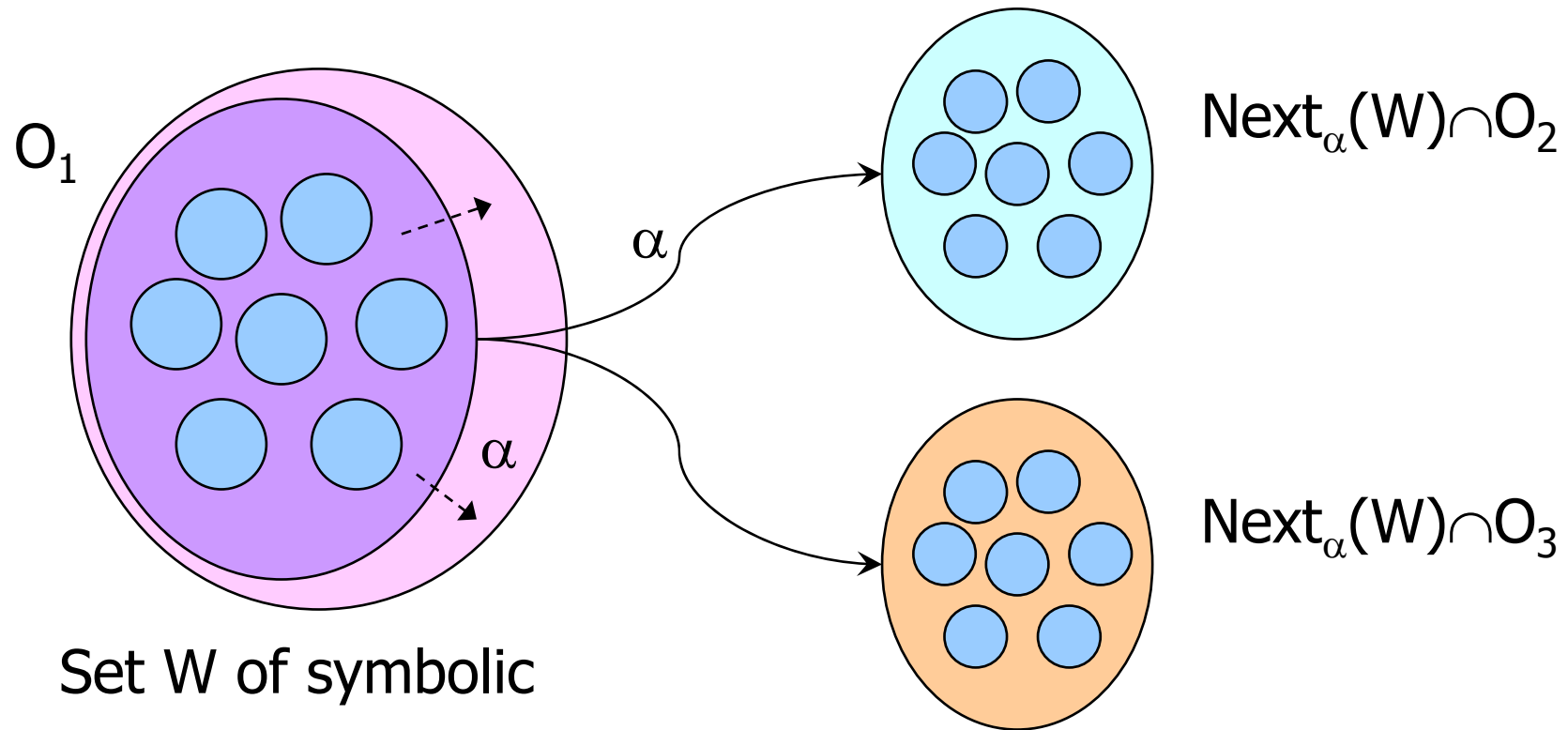
$Losing[\{s_0\}] \leftarrow (\{s_0\} \not\subseteq \gamma(\text{Goal}) \wedge (Waiting = \emptyset \vee \forall \alpha \in \Sigma_1, \text{Sink}_\alpha(s_0) \neq \emptyset) ? 1 : 0);$

$Depend[\{s_0\}] \leftarrow \emptyset;$

- Passed list { sets of W }.
- Waiting list { tuples (W, α, W') }.
- $Win[W]$ maps to 1 (winning) or 0 (unknown).
- Similar $Losing[W]$.
- $Depend[W]$ records the graph.
- Sink_α : sink states by doing α .

Sets of Symbolic States & Successors

$\text{Next}_\alpha(W)$: sets of symbolic successors for some action α .



Set W of symbolic states within one observation.

Next_α : do $\{\alpha$ asap and wait for $\alpha\}$ until change of observation.

Forward Phase

```
Passed ← Passed ∪ {W'};  
Depend[W'] ← {(W,  $\alpha$ , W')};  
Win[W'] ← (W' ⊆  $\gamma$ (Goal) ? 1 : 0);  
Losing[W'] ← (W' ⊈  $\gamma$ (Goal) ∧ Sink $_{\alpha}$ (W') ≠ ∅ ? 1 : 0);
```

Update Win,
Losing, graph.

```
if (Losing[W'] ≠ 1) then (* if losing it is a deadlock state *)
```

```
  NewTrans ← {(W',  $\alpha$ , W'') |  $\alpha \in \Sigma$ ,  $o \in \mathcal{O}$ , W' = Next $_{\alpha}$ (W) ∩ o ∧ W' ≠ ∅};
```

```
  if NewTrans = ∅ ∧ Win[W'] = 0 then Losing[W'] ← 1;
```

```
  if (Win[W'] ∨ Losing[W']) then Waiting ← Waiting ∪ {e};
```

```
  Waiting ← Waiting ∪ NewTrans;
```

Continue forward.

Back-propagate if necessary.

Detect deadlocks.

Partition successors.



Backward Phase

If there is a c whose successors are all winning

$$Win^* \leftarrow \bigvee_{c \in Enabled(W)} \bigwedge_{W \xrightarrow{c} W''} Win[W''];$$

if Win^* then

$$Waiting \leftarrow Waiting \cup Depend[W]; Win[W] \leftarrow 1;$$

then W is winning and back-propagate.

If every c has a losing successor

$$Losing^* \leftarrow \bigwedge_{c \in Enabled(W)} \bigvee_{W \xrightarrow{c} W''} Losing[W''];$$

if $Losing^*$ then

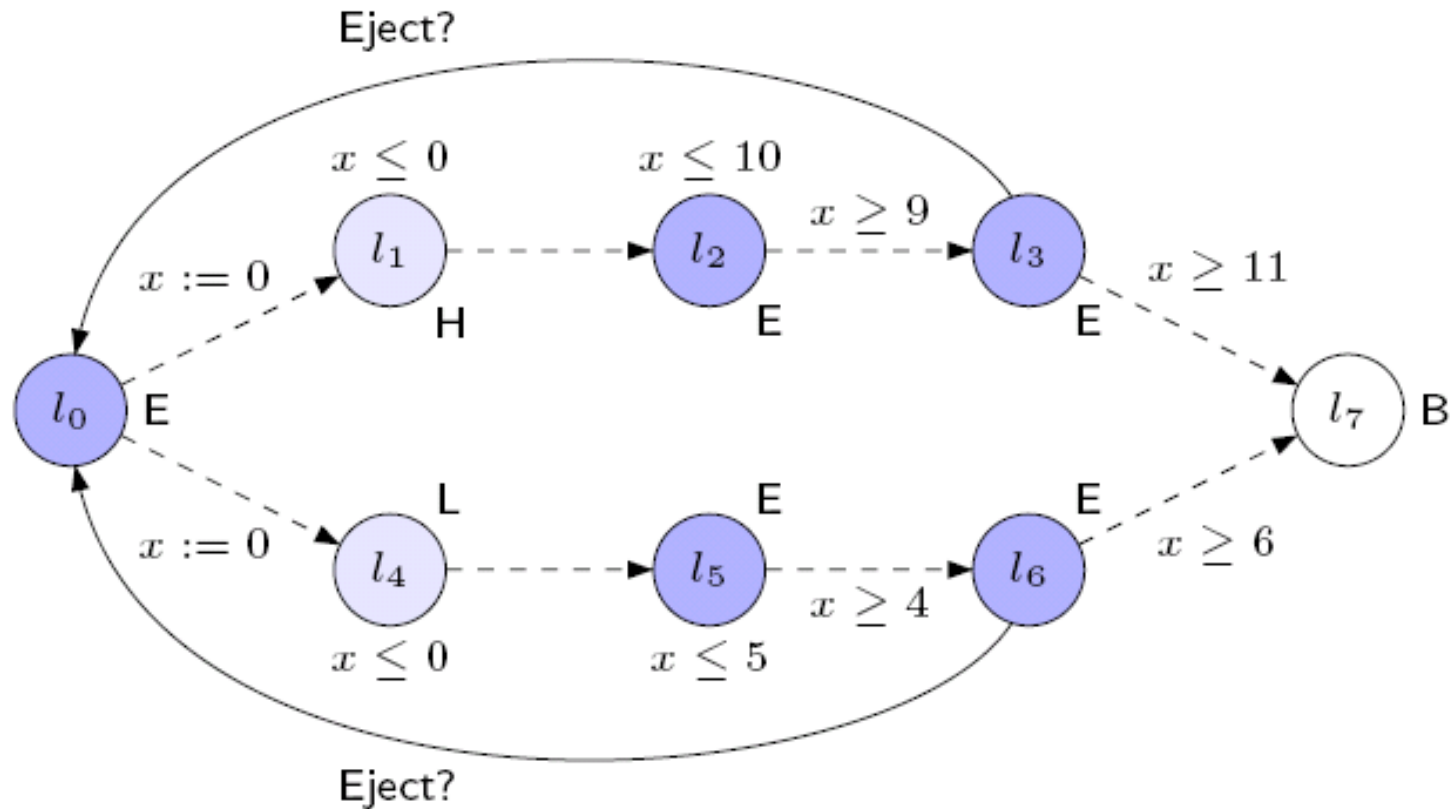
$$Waiting \leftarrow Waiting \cup Depend[W]; Losing[W] \leftarrow 1;$$

then W is losing and back-propagate.

Update graph in case of new paths to passed states.

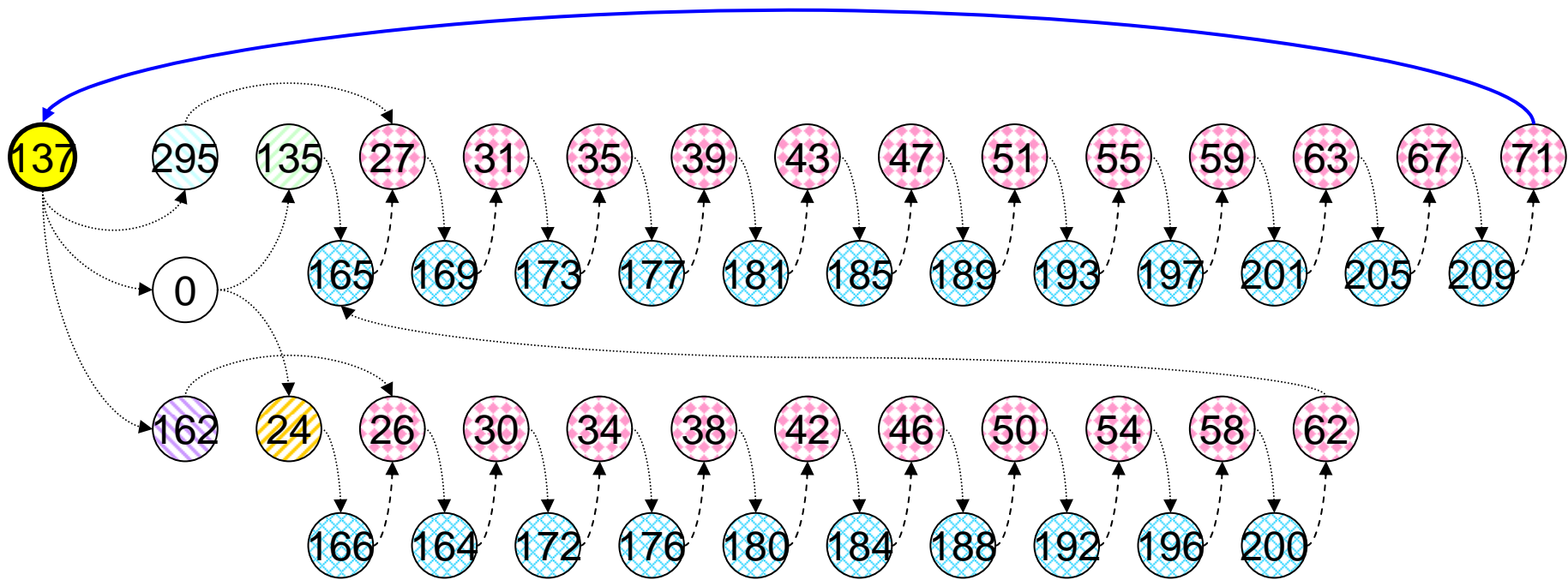
$$\text{if } (Win[W'] = 0 \wedge Losing[W'] = 0) \text{ then } Depend[W'] \leftarrow Depend[W'] \cup \{e\};$$

Example - Model



Observations: L, H, E, B, y in $[0, 1/2[$

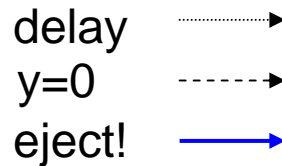
Example – Memoryful Strategy



Partition:

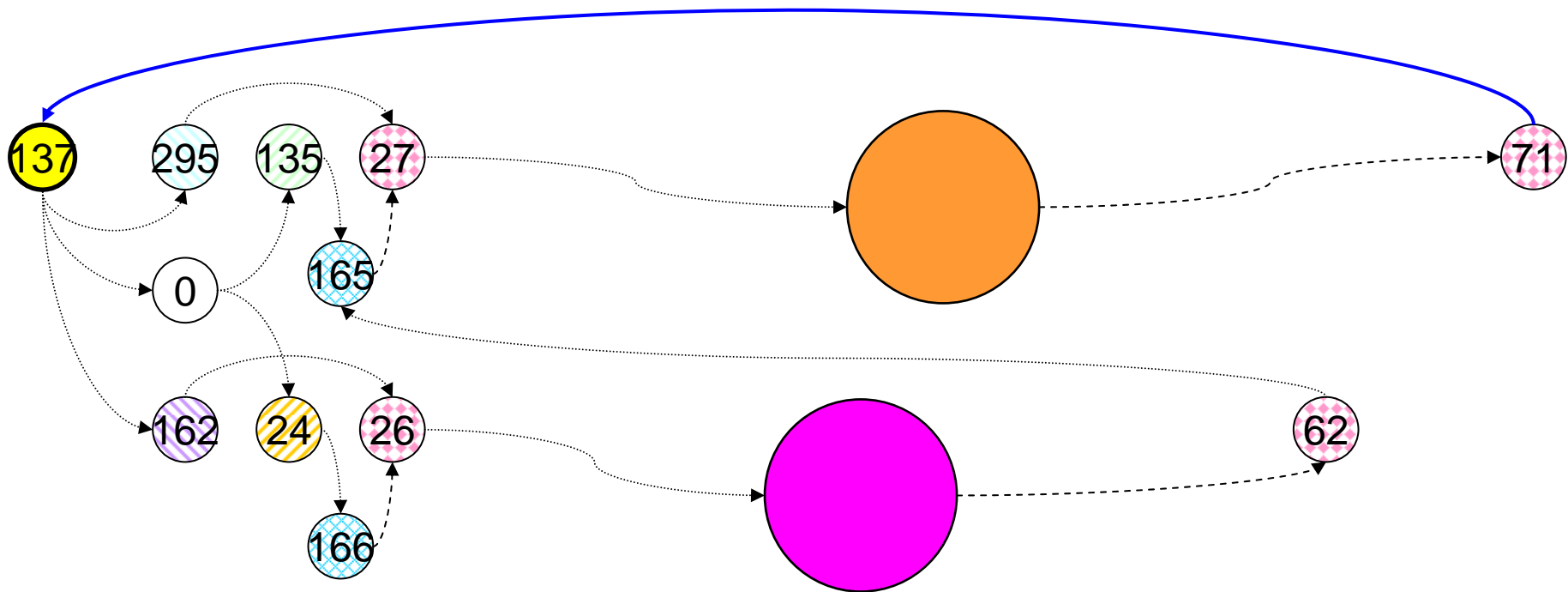


Actions:



Controllable with $y \in [0, 1/2[$,
not with $y \in [0, 1[$.

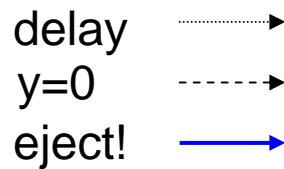
Motivations



Partition:



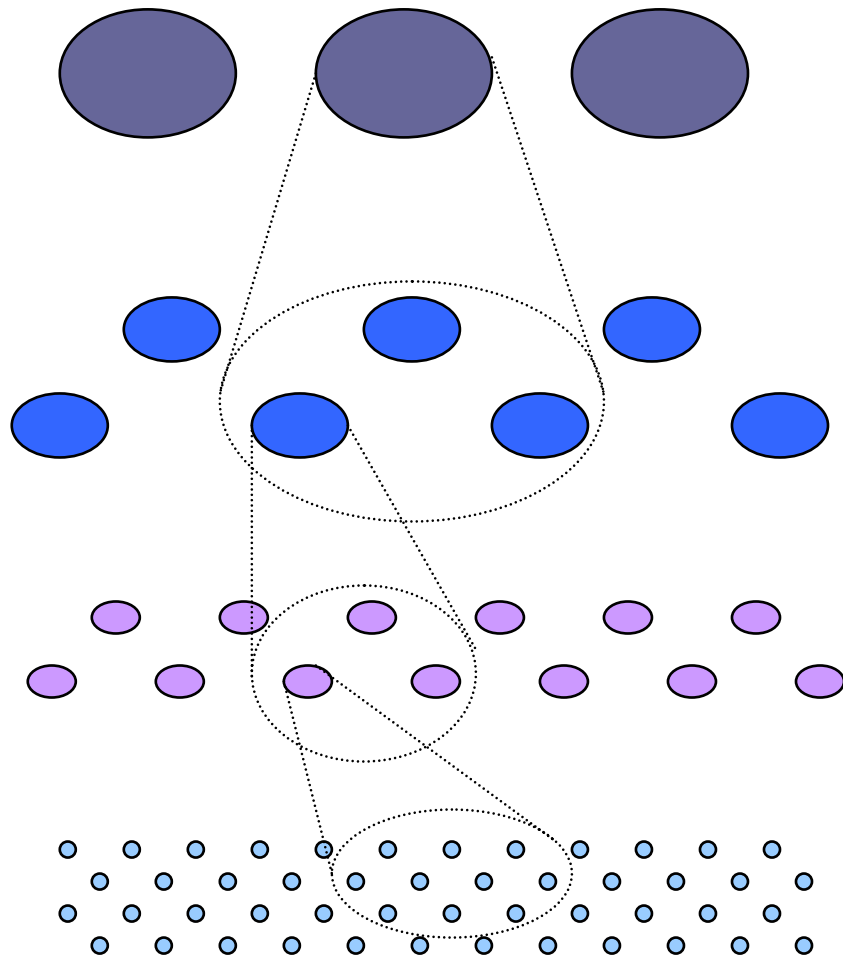
Actions:



Use coarser observations
where possible

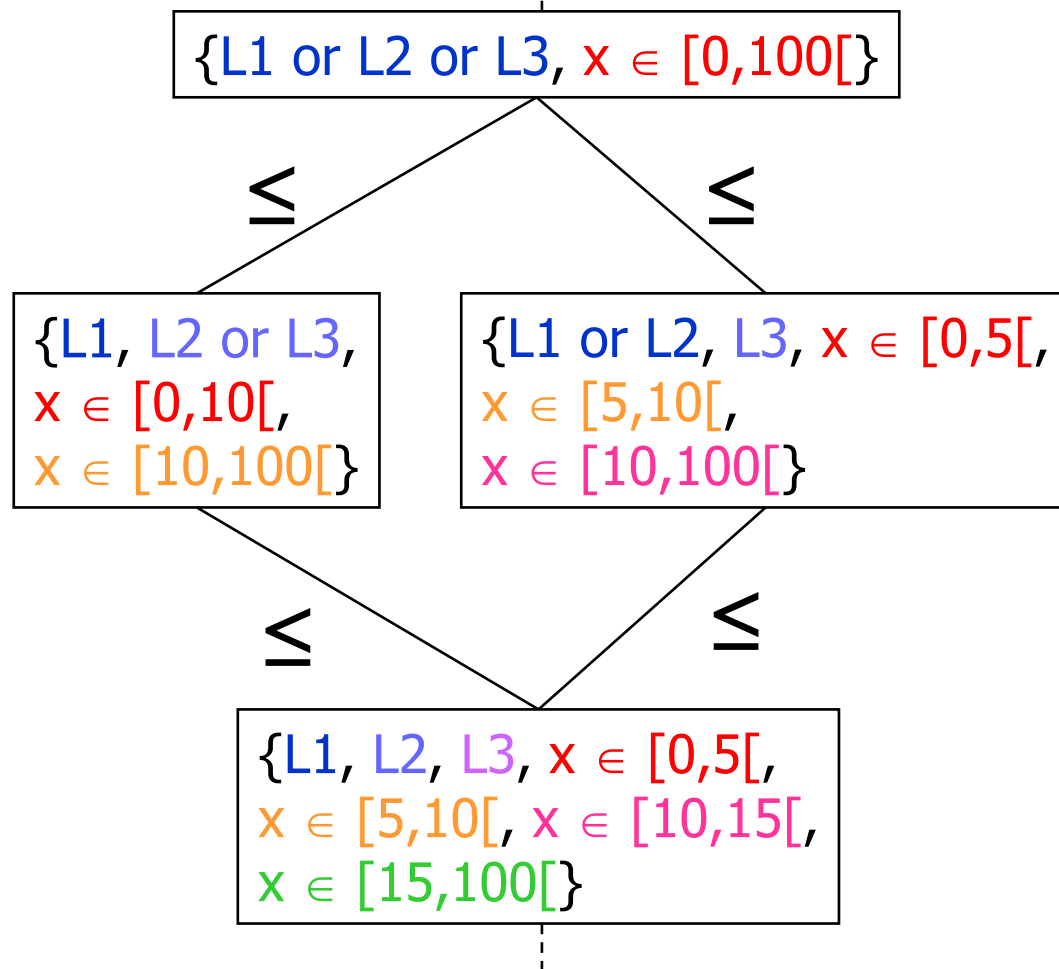
- shorter strategies
- cheaper sensors

Algorithm: Intuition



- Use the algorithm for partial observability at a coarse level.
- Refine the observation and run the algorithm at the finer level.
- When to refine?
- Relationship between observations?

Lattice of Observations



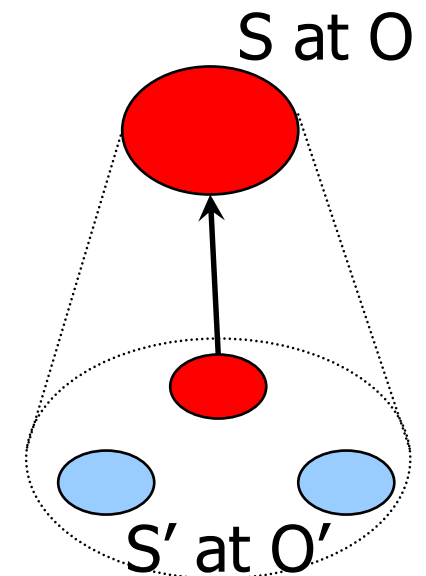
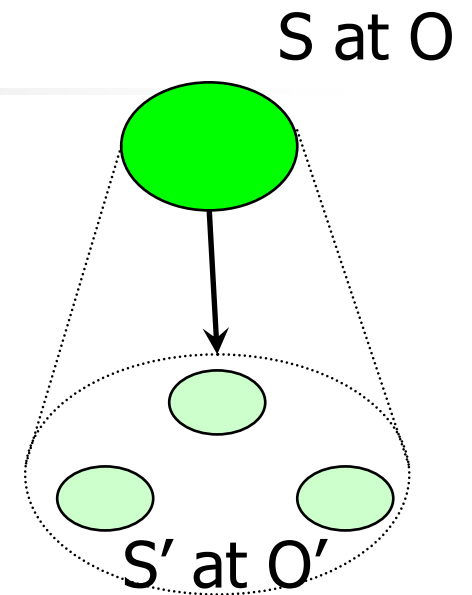
- Ordering relation: (Obs1 refines Obs2) $\text{Obs1} \leq \text{Obs2}$ iff states observable by Obs2 are observable by Obs1.
 - Finer clock constraints.
 - Finer sets of locations
- Top: coarse observations. Supremum observes nothing (blind).
- Bottom: fine observation. Infimum: full observability.

Locations L1,L2,L3,L4, clock x.

Lattice of Observations

■ Why?

- Notion of refinement of observations consistent with inclusion of sets of states.
- If S at O is winning then S' at O' is also winning for O' finer than O and S' included in S .
- If S' at O' is losing then S at O is also losing for O coarser than O' and S including S' .





Algorithm for Safety Games

- Duplicate structures for each set of observations O .
 - $Waiting_O$, $Passed_O$, $Depend_O$
 - Win_O is different
 - $0 \rightarrow$ lose
 - $? \rightarrow$ unknown
 - $1 \rightarrow$ win(put together previous Win and Losing)
 - No $Losing_O$.
 - $Refine_O$ maps S to its refined states.



Algorithm

- Several actions that can be taken non-deterministically.
 - Initialize Win_0 to ?.
 - ACT1: main exploration.
 - ACT2: propagate losing “up” (more abstract).
 - ACT3: propagate winning “down” (finer).
 - ACT4: conclude winning/losing at some O .
 - ACT5: change to winning because of refinement.

Algorithm (1)

ACT1

Pick (S, a, S') in some $Waiting_O$

if $S' \notin Passed_O$

add S' to $Passed_O$

add (S, a, S') to $Depend_O$

add successors of S' to $Waiting_O$

compute $Win_O[S']$

else

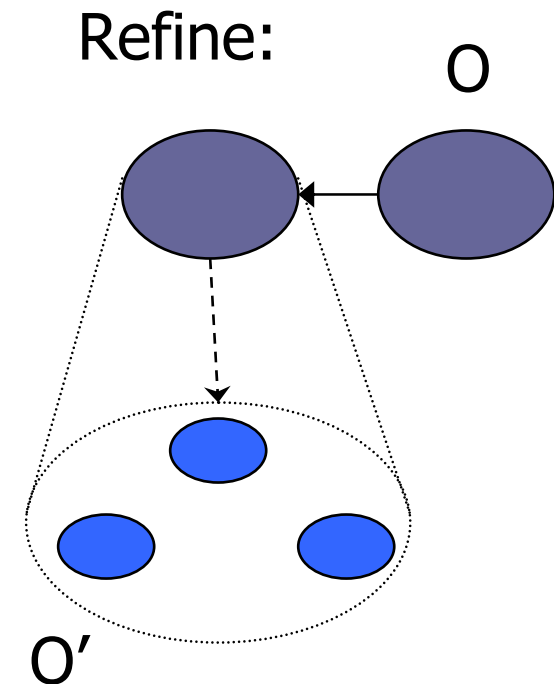
choose

:: Re-evaluate some $Win_O[S]$

:: Refine S' for some finer O'

and add the successors to $Waiting_{O'}$

endif





Algorithm (2,3)

ACT2

if $\text{Win}_O[S'] = 0$ then
 $\text{Win}_O[S] = 0$ for any coarser O and larger S
endif

ACT3

if $\text{Win}_O[S'] = 1$ then
 $\text{Win}_O[S] = 1$ for any finer O and smaller S
endif



Algorithm (4)

Safety only

ACT4

if $\text{Waiting}_O = \emptyset$ then

$\text{Win}_O[S] = 1$ for any S in Passed_O where $\text{Win}_O[S] = ?$
endif

Reachability only

ACT4

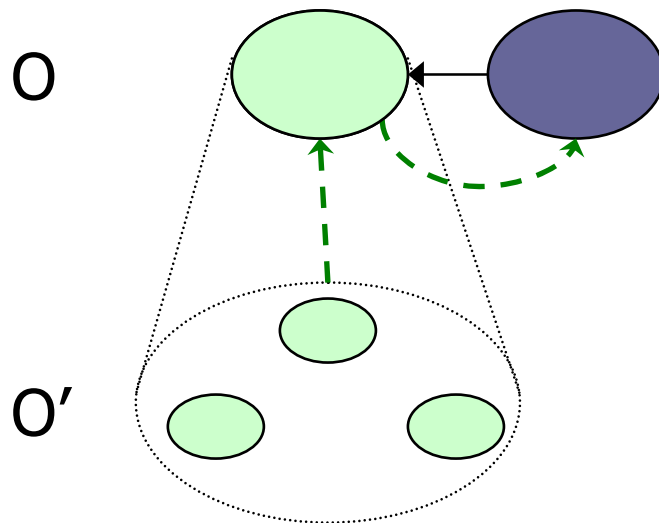
if $\text{Waiting}_O = \emptyset$ then

$\text{Win}_O[S] = 0$ for any S in Passed_O where $\text{Win}_O[S] = ?$
endif

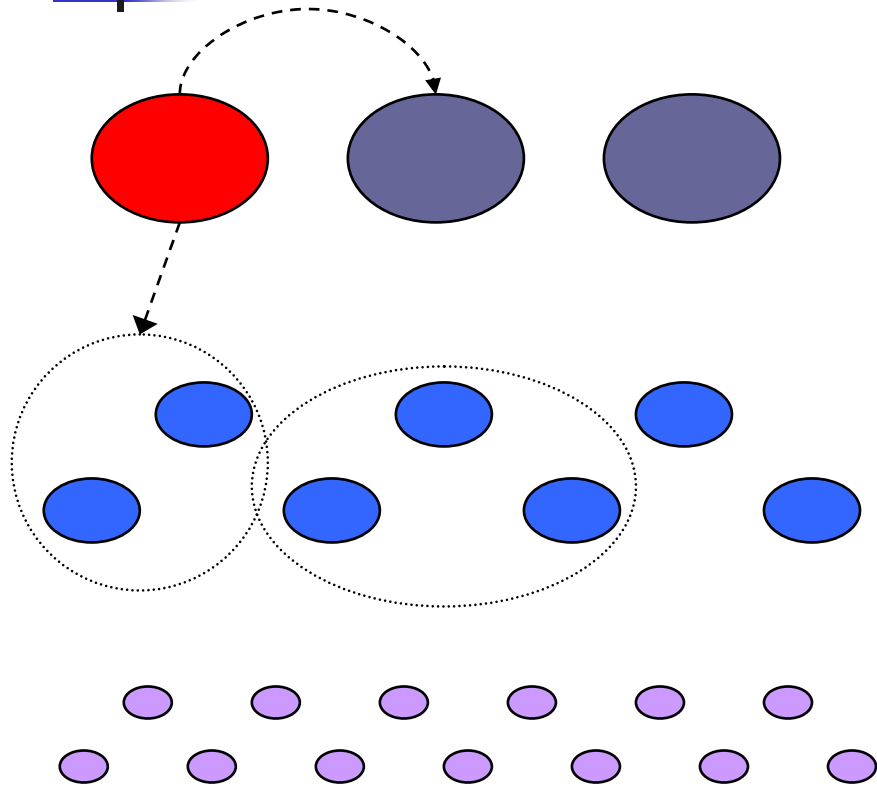
Algorithm (5)

ACT5

if $\text{Win}_O[S] \neq 1$ and
 $\forall S' \in \text{Refine}_O[S]. \text{Win}_O[S'] = 1$ then
 $\text{Win}_O[S] = 1$
 add $\text{Depend}_O[S]$ to Waiting_O
endif



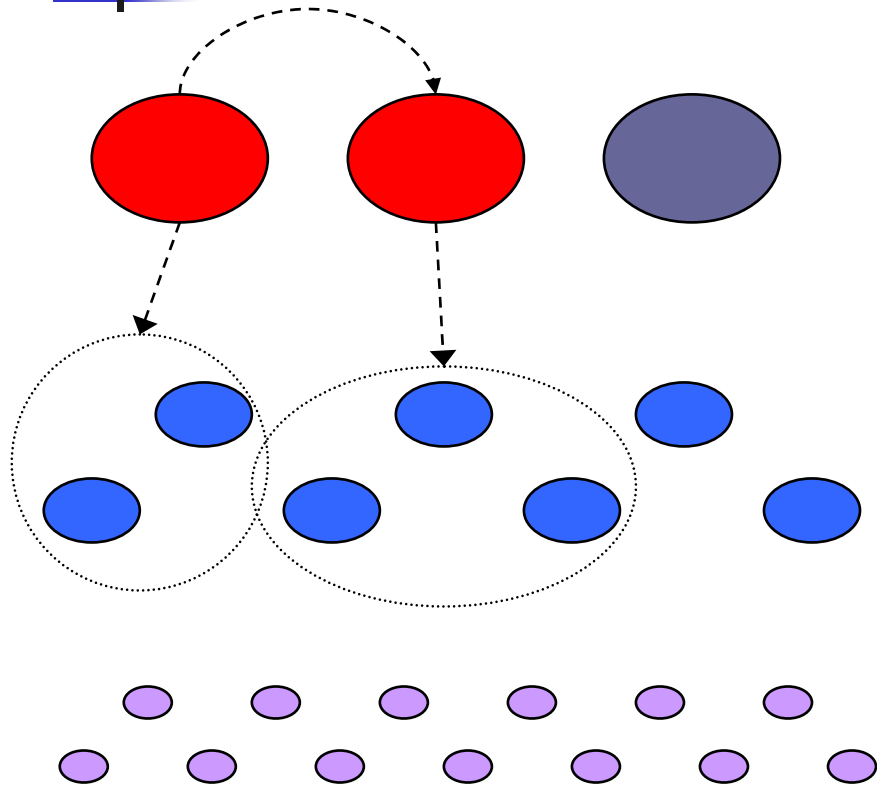
Issue with Strategies



1. Explore forward.
2. Find losing.
3. Back-propagate.

Suppose we refine
when detecting
losing.

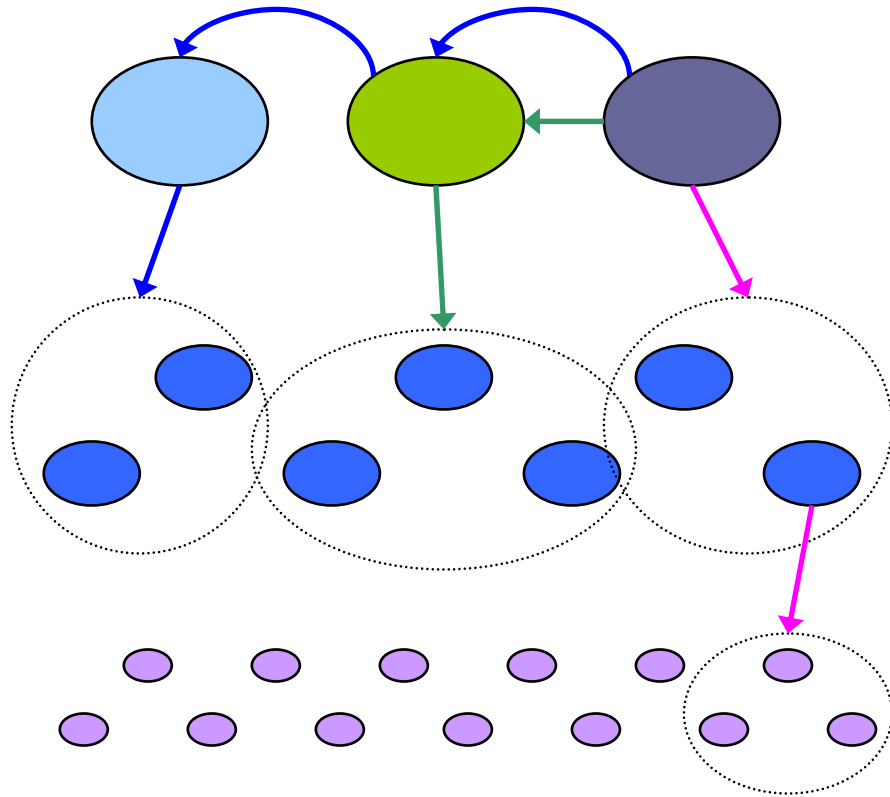
Issue with Strategies



1. Explore forward.
2. Find losing.
3. Back-propagate.

Information is not there at the right place to extract the strategy.

Issue with Strategies



- Different possible solutions.
- “Good” path is not known.
- 2nd issue to ensure progress → no loop.
 - Fix-point of winning \neq how to win.
 - Solved for full observability, different problem here.



Work in Progress

- Experiments done with a Ruby prototype implementation – no strategy.
- Post-processing of refinement graph can solve the problem.
- On-the-fly problem: How to choose good actions.