



Towards Realizable Strategies

Alexandre David

Kim G. Larsen

Didier Lime



Center for Indlejrede Software Systemer



Overview

- Motivation
- Automatic generation of non-zero strategies
 - More general approach using timed games with Büchi control objectives
- Timed games with partial observability

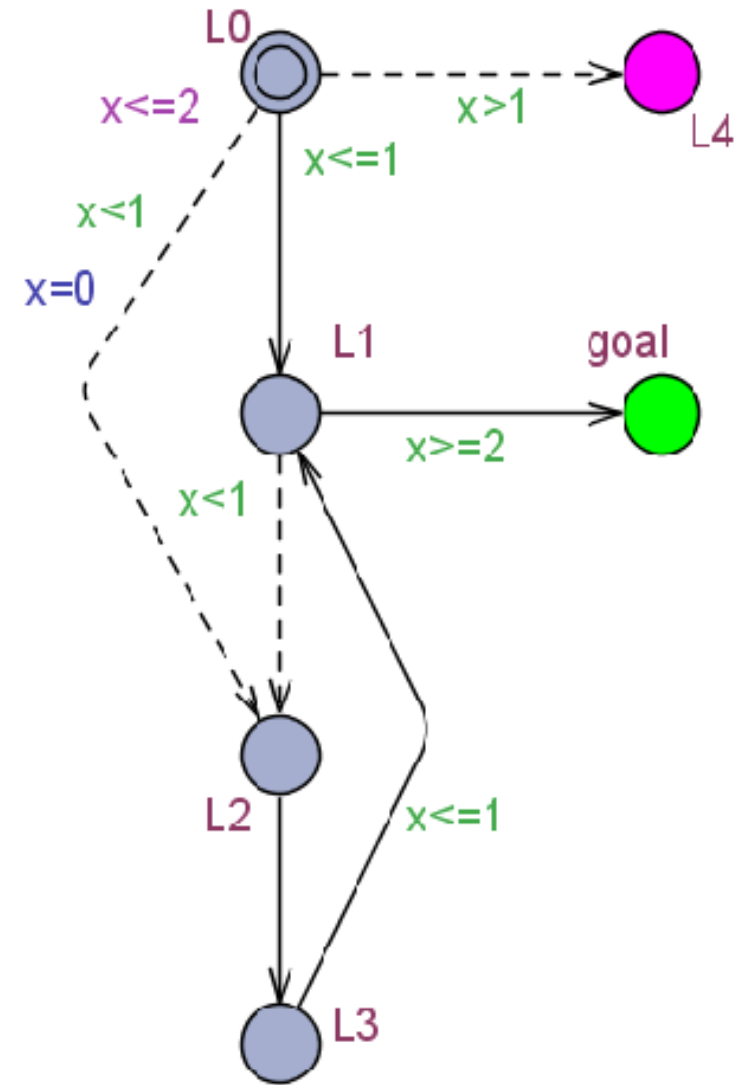


Motivation

- Goal: Generate controllers closer to reality
 - For safety: Good behaviors where time diverges. (Non-zero controllers).
 - Use a more general approach using games with Büchi control objective.
 - Use an observer to generate automatically non-zero strategies.
 - Real controllers have partial observability through limited sensors.
 - Take that partial observability into account.
 - ATVA'07

TGA in a Nutshell

- Timed automata with controllable and uncontrollable transitions.
- Reachability & safety games.
 - control: $A \langle \rangle$ TGA.goal
 - control: $A[]$ not TGA.L4
- Memoryless strategy:
 - state \rightarrow action.



Strategies in a Nutshell

- control: $A \leftrightarrow TGA.goal$

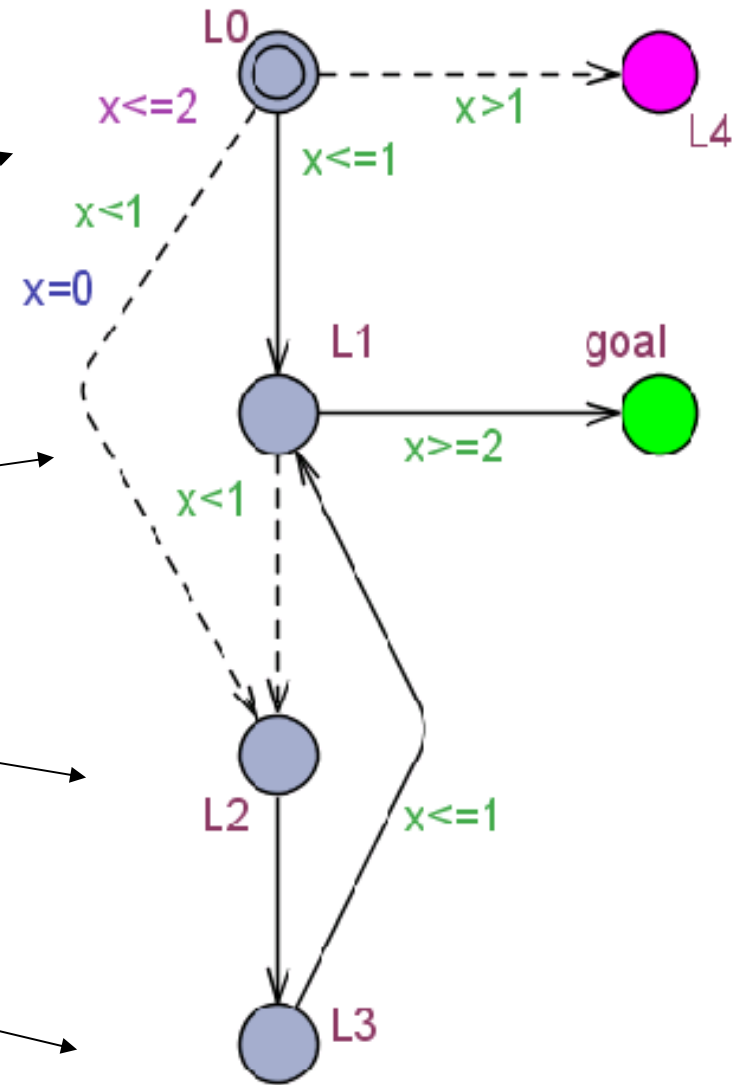
$x < 1$: λ
 $x == 1$: c

$x < 2$: λ
 $x \geq 2$: c

Strategy

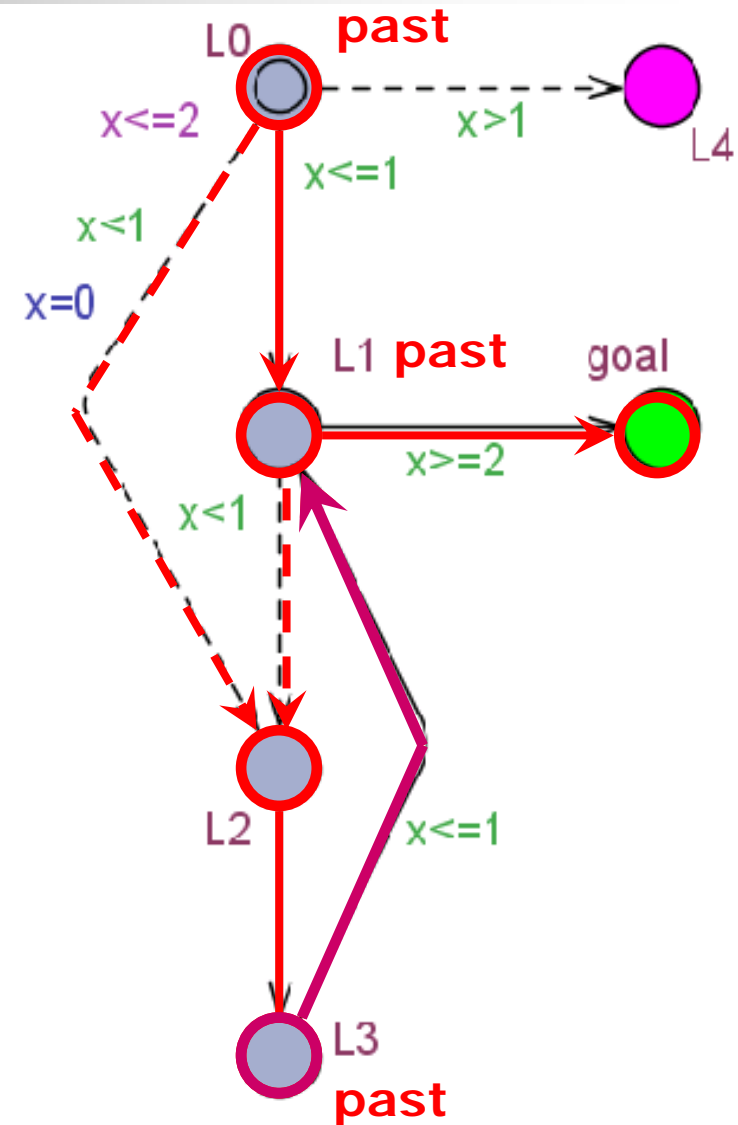
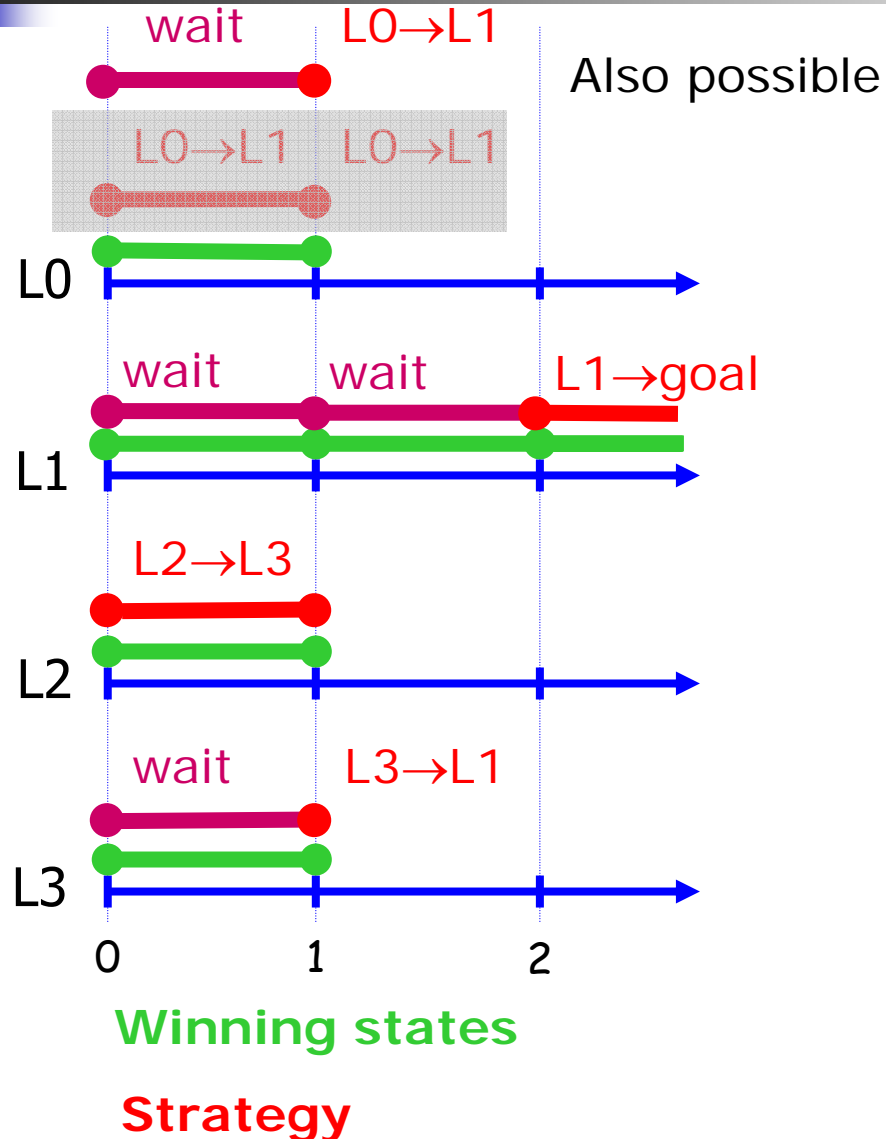
$x \leq 1$: c

$x < 1$: λ
 $x == 1$: c



Note: This is *one* strategy.
 There are other solutions.

Winning States → Strategy





TIGA in a Nutshell

- Editor + simulator + verifier – TGA
- Properties:
 - control: $A[p U q]$
 - control: $A \langle \rangle q \Leftrightarrow \text{control: } A[\text{true} U q]$
 - control: $A[p W q]$
 - control: $A[] p \Leftrightarrow \text{control: } A[p W \text{false}]$
 - control_t*(u,g): $A[p U q]$
 - $E \langle \rangle \text{control: } \varphi$
 - control: $A[] (p \ \&\& \ A \langle \rangle q)$
 - control: $A[] A \langle \rangle q \Leftrightarrow \text{control: } A[] (\text{true} \ \&\& \ A \langle \rangle q)$
 - $\{ \text{obs1, obs2 ...} \} \text{control: } \text{reachability} \mid \text{safety}$
 - $\{ A, B ... \} \leq \{ C, D ... \}$



Today

- Games with Buchi accepting states.
 - control: $A[] (p \ \&\& \ A<> \ q)$
 - control: $A[] A<> q$
- Partial observable systems:
 - $\{ \text{obs1, obs2 ...} \}$ control: reachability | safety
 - Generate a controller based on partial observations of the system (obs1, obs2...).
 - [Cassez & al. '07] Efficient on-the-fly algorithm, **EXPTIME**.



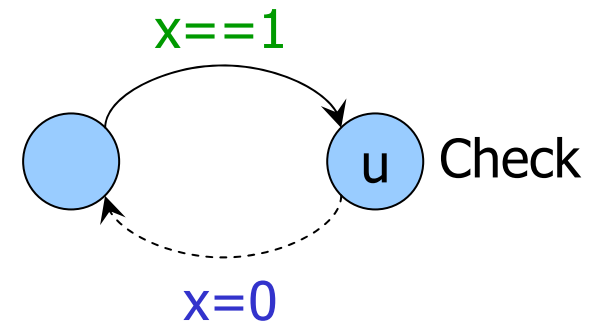
TGA with Buchi Accepting States

- Use TIGA's algorithm as an intermediate steps in a fixpoint, but modified:
 - winning states are states that can reach goals
 - goal states defined by queries, not necessarily winning
 - Fixpoint:
 $Win = SOTF(goal)$
while $(Win \cap goal) \neq goal$
 $goal = Win \cap goal$
 $Win = SOTF(goal)$
done
return $S_0 \in Win$

Simple algorithm.
Complexity in line
with other known
algorithms.

Application: Non-zero Strategies

- Add a monitor with the rest of the system.
- Ask
control: $A[]$ ($p \ \&\& \ A \langle \rangle$
 Monitor.Check)



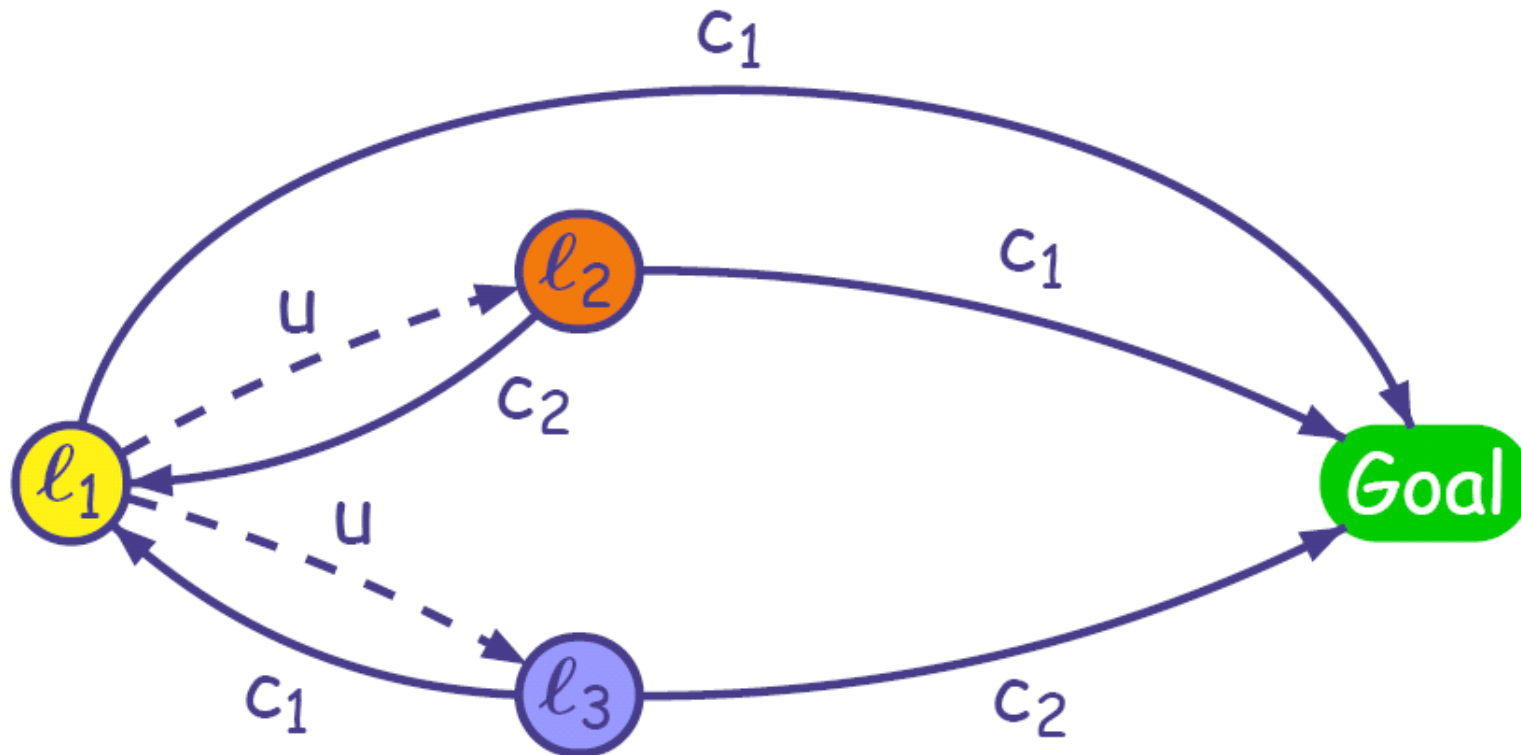


Timed Games

with Partial Observability

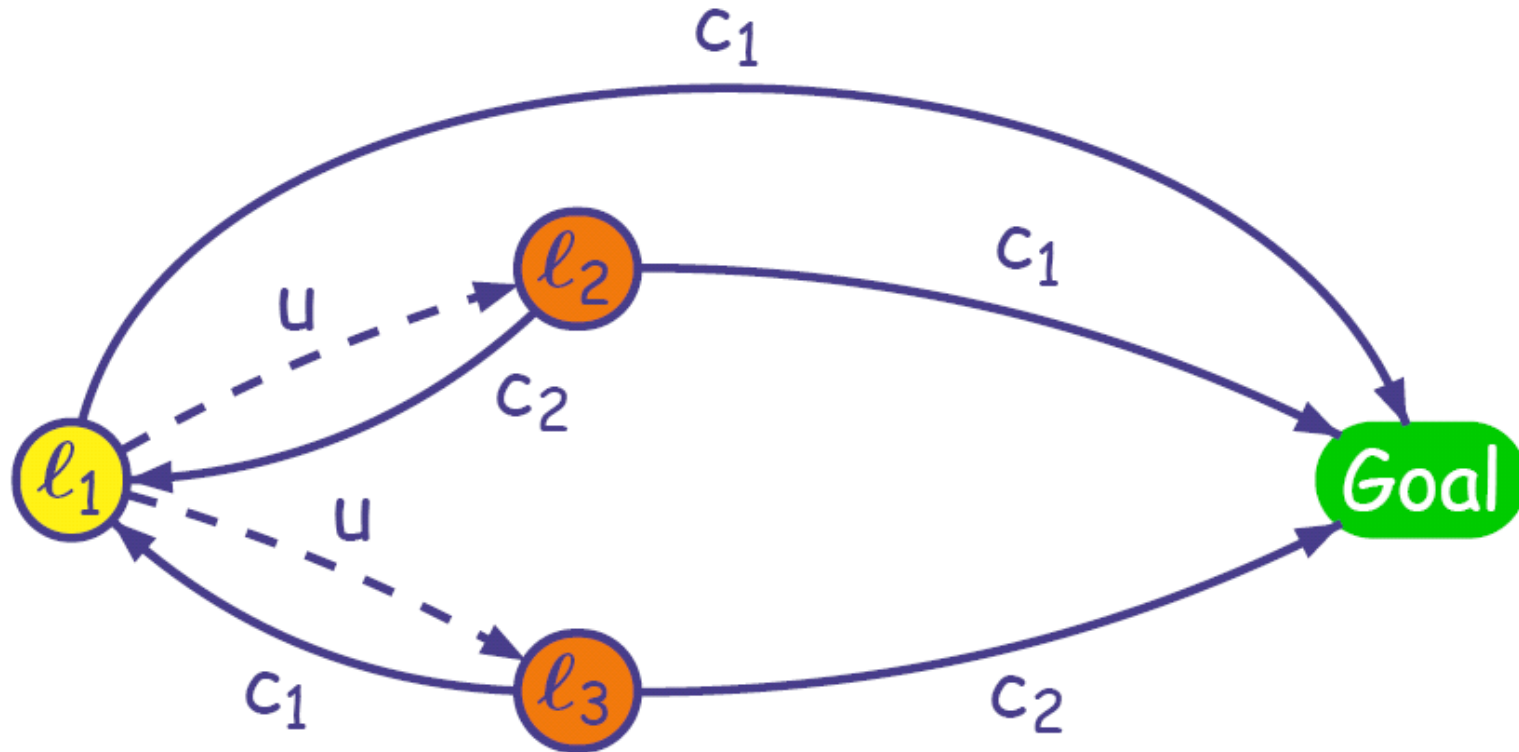
- Previous: Perfect information.
 - Not always suitable for controllers.
- Partial observation.
 - States or events, here states.
 - Distinguish states w.r.t. observations.
 - Strategy keeps track of states w.r.t. observations.
 - Observations = predicates over states.

State Based Full Observation



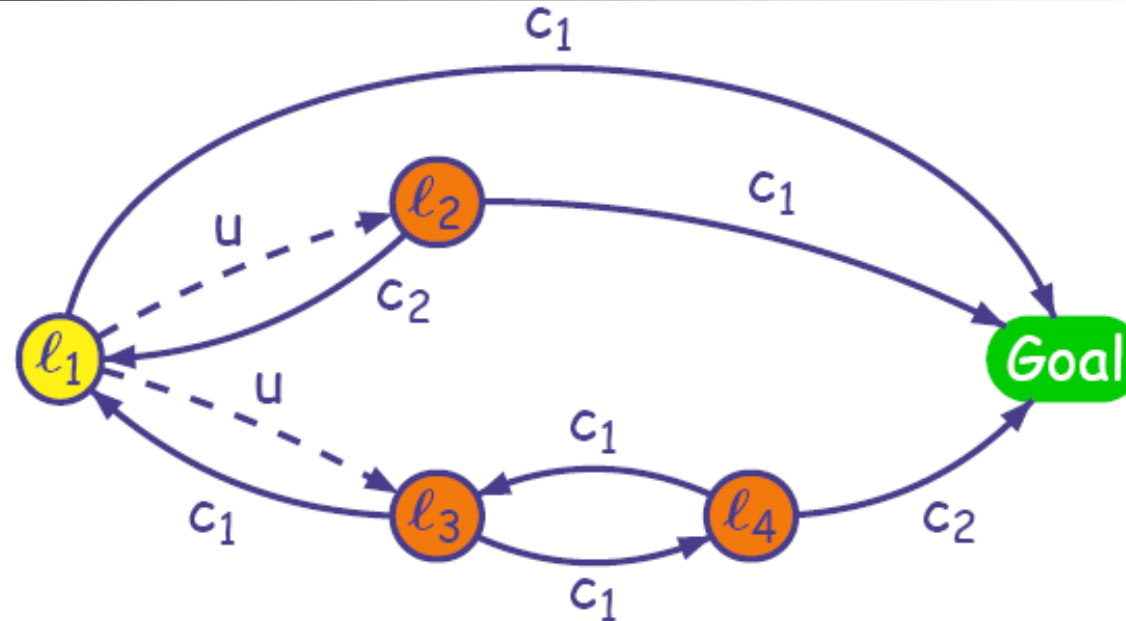
- 2-player reachability game, controllable + uncontrollable actions.
- Full observation: in l_2 do c_1 , in l_3 do c_2 .

State Based Partial Observation



- Partition the state-space $l_2=l_3$.
- Can't win here.

State Based Partial Observation



Winning Strategy:

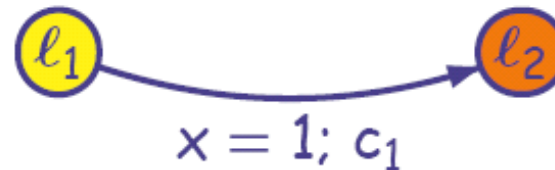
- after: ● play c_1
- after: ● ● play c_1
- after: ● ● ● play c_2

The controller **can observe** each state's change

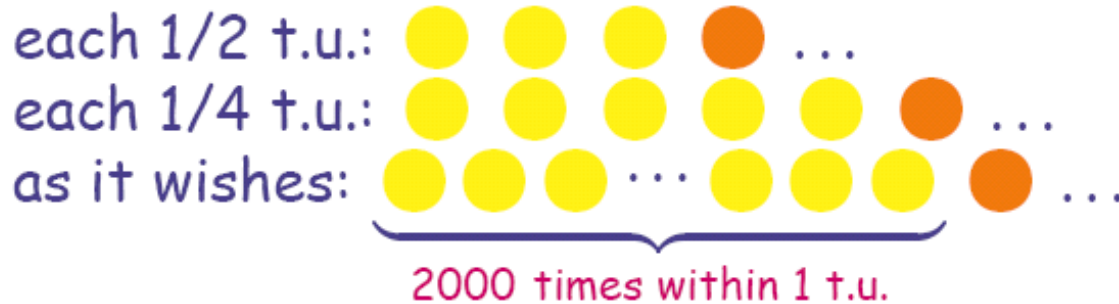
Observation For Timed Systems

In **Continuous** Timed Systems, "next state" is reached by:

- ▶ either a **discrete** step
- ▶ or a **continuous** time-step



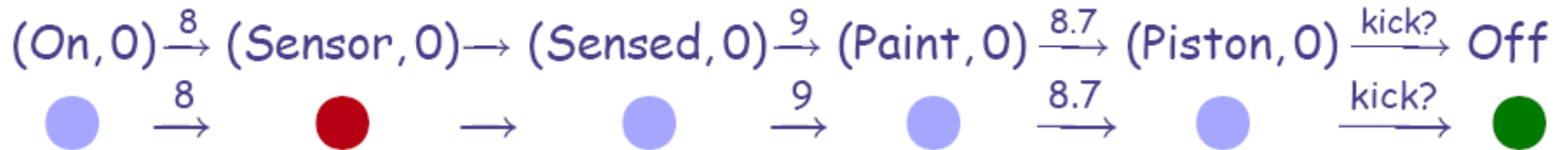
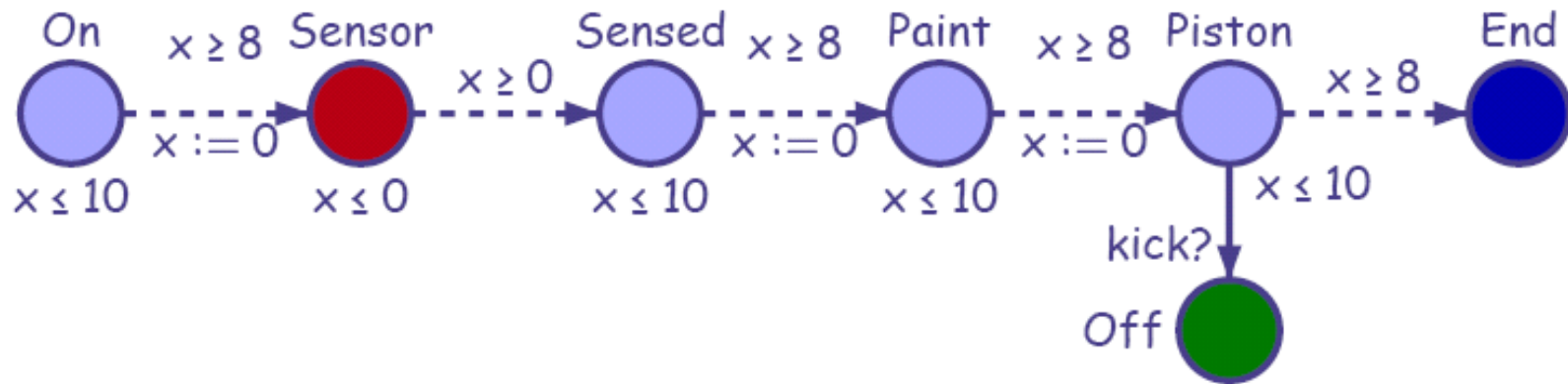
- ▶ Possible Observations:



- ▶ the controller **cannot observe** each state's change

Issue: **When** does the controller **observe** the system ?

Stuttering-Free Invariant Observations



Assumption: the controller can only see **changes** of observations

Stuttering-free observation: ● ● ● ●

Must play based on **stuttering-free** observations



TGA with PO: Rules

- If player 1 wants to take an action c , then
 - player 2 can **choose to play** any of his actions or c as long as the observation stays the same, or
 - player 2 can **delay** as long as c is not enabled and the observation stays the same.
 - $\Rightarrow c$ is urgent.
- If player 1 wants to delay, then
 - player 2 can delay or take any of his actions as long as the observation stays the same.
- The turn is back to player 1 as soon as the observation changes.

On-the-Fly Algorithm

Initialization:

```

Passed ← {{s0}};
Waiting ← {({s0}, α, W') | α ∈ Σ1, o ∈ O, W' = Nextα({s0}) ∩ o ∧ W' ≠ ∅};
Win[{s0}] ← ({s0} ⊆ γ(Goal) ? 1 : 0);
Losing[{s0}] ← ({s0} ⊈ γ(Goal) ∧ (Waiting = ∅ ∨ ∀α ∈ Σ1, Sinkα(s0) ≠ ∅) ? 1 : 0);
Depend[{s0}] ← ∅;

```

Main:

```

while ((Waiting ≠ ∅) ∧ Win[{s0}] ≠ 1 ∧ Losing[{s0}] ≠ 1) do

```

```

  e = (W, α, W') ← pop(Waiting);

```

```

  if s' ∉ Passed then

```

```

    Passed ← Passed ∪ {W'};

```

```

    Depend[W'] ← {(W, α, W')};

```

```

    Win[W'] ← (W' ⊆ γ(Goal) ? 1 : 0);

```

```

    Losing[W'] ← (W' ⊈ γ(Goal) ∧ Sinkα(W') ≠ ∅ ? 1 : 0);

```

```

    if (Losing[W'] ≠ 1) then (* if losing it is a deadlock state *)

```

```

      NewTrans ← {(W', α, W'') | α ∈ Σ, o ∈ O, W' = Nextα(W) ∩ o ∧ W' ≠ ∅};

```

```

      if NewTrans = ∅ ∧ Win[W'] = 0 then Losing[W'] ← 1;

```

```

      if (Win[W'] ∨ Losing[W']) then Waiting ← Waiting ∪ {e};

```

```

      Waiting ← Waiting ∪ NewTrans;

```

```

    else (* reevaluate *)

```

```

      Win* ← ∨c ∈ Enabled(W) ∧W →c W'' Win[W''];

```

```

      if Win* then

```

```

        Waiting ← Waiting ∪ Depend[W]; Win[W] ← 1;

```

```

      Losing* ← ∧c ∈ Enabled(W) ∨W →c W'' Losing[W''];

```

```

      if Losing* then

```

```

        Waiting ← Waiting ∪ Depend[W]; Losing[W] ← 1;

```

```

      if (Win[W'] = 0 ∧ Losing[W'] = 0) then Depend[W'] ← Depend[W'] ∪ {e};

```

```

    endif

```

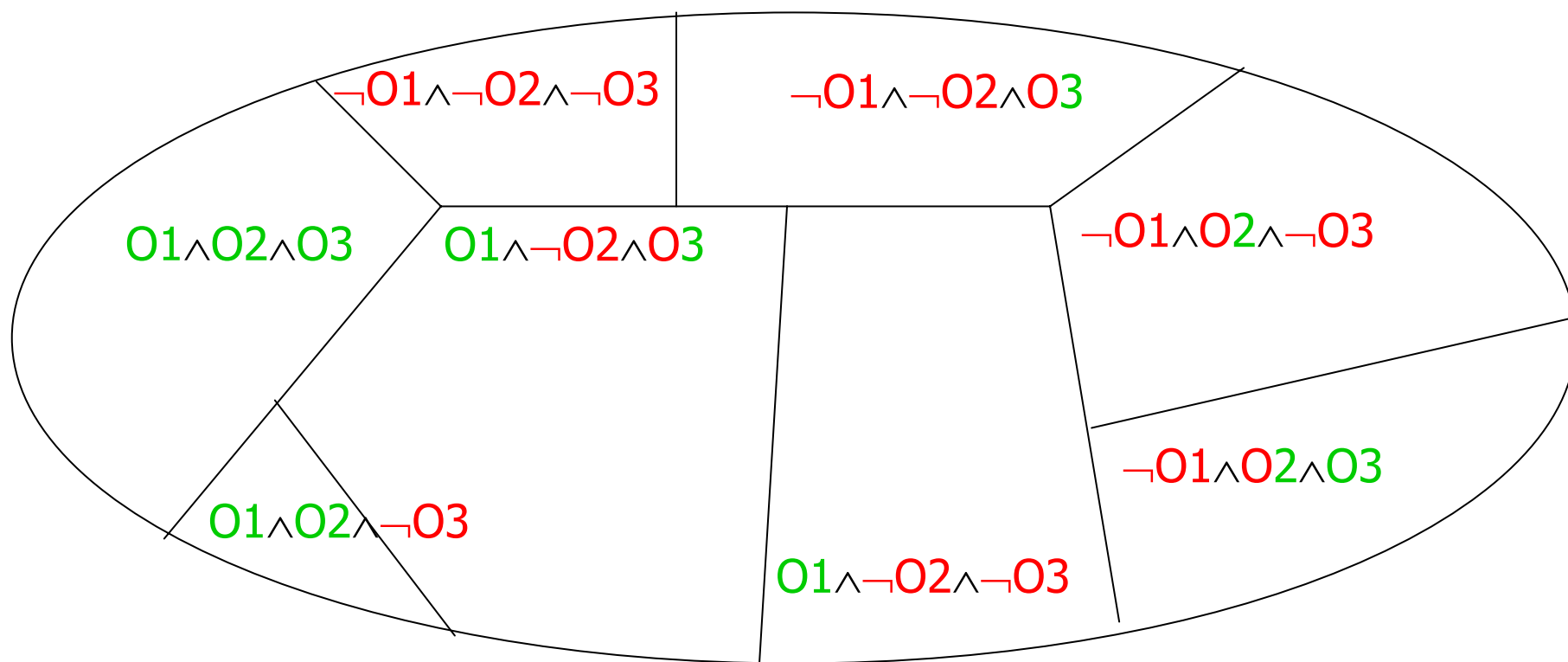
```

endwhile

```

Algorithm

Partition the state-space w.r.t. observations.
Observations $O1$ $O2$ $O3$.
Winning/losing is observable.

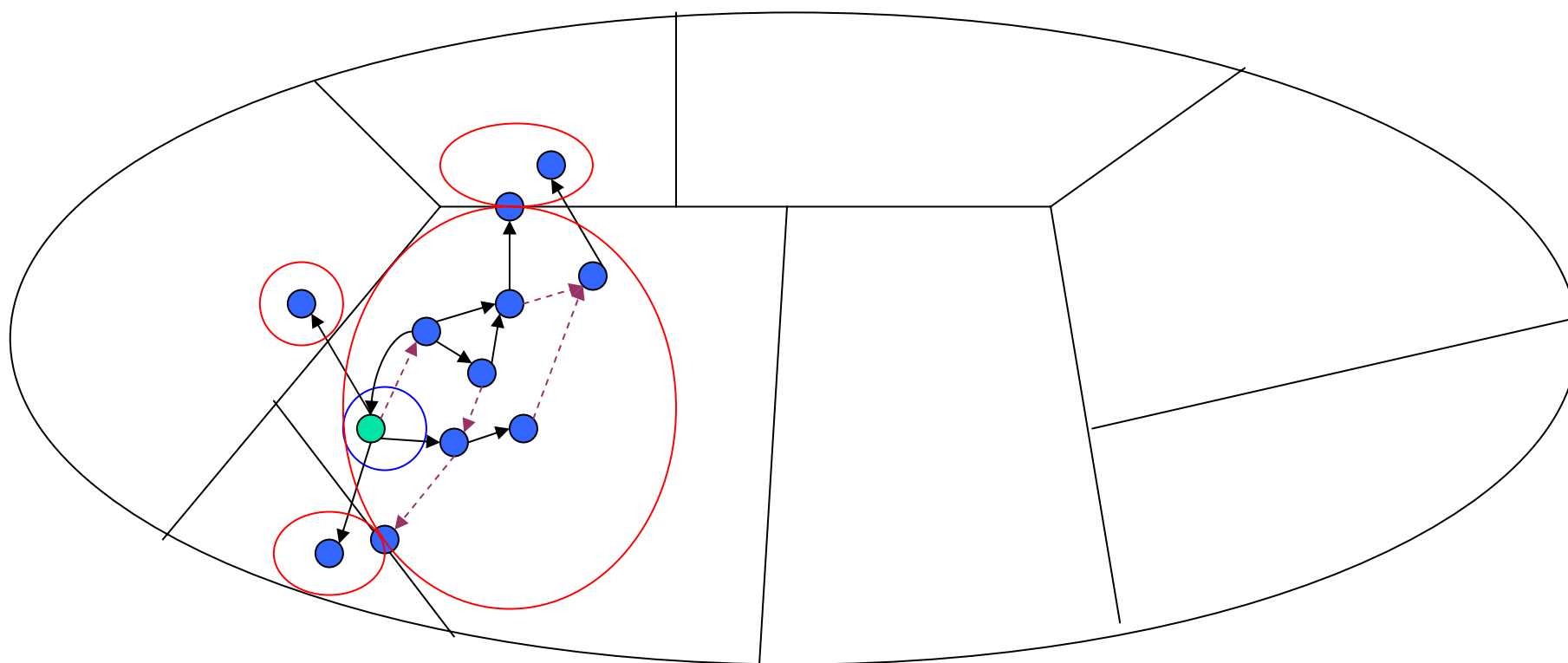


Algorithm

Initial state in some partition.

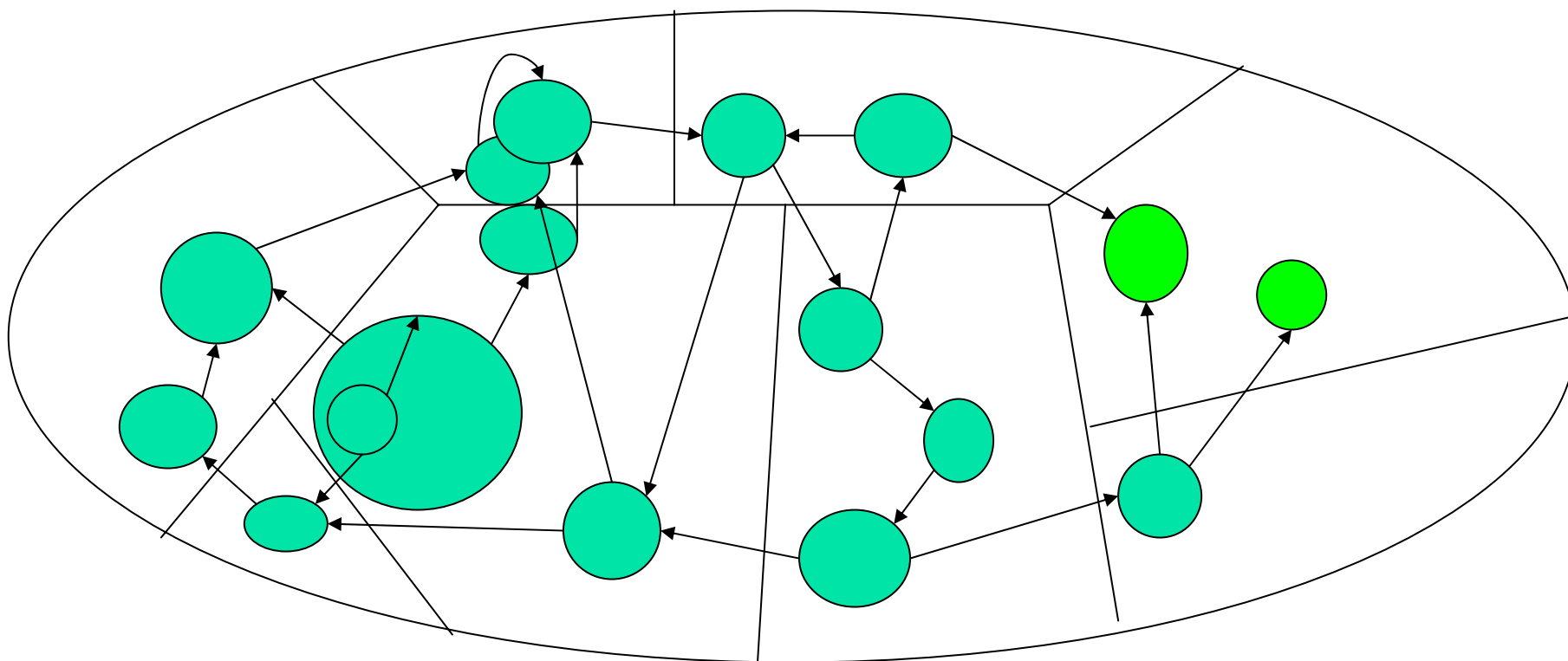
Compute successors { set of states } w.r.t. a controllable action.

Successors distinguished by observations.



Algorithm

Construct the graph of sets of symbolic states.
Back-propagate winning/losing states.





Notes

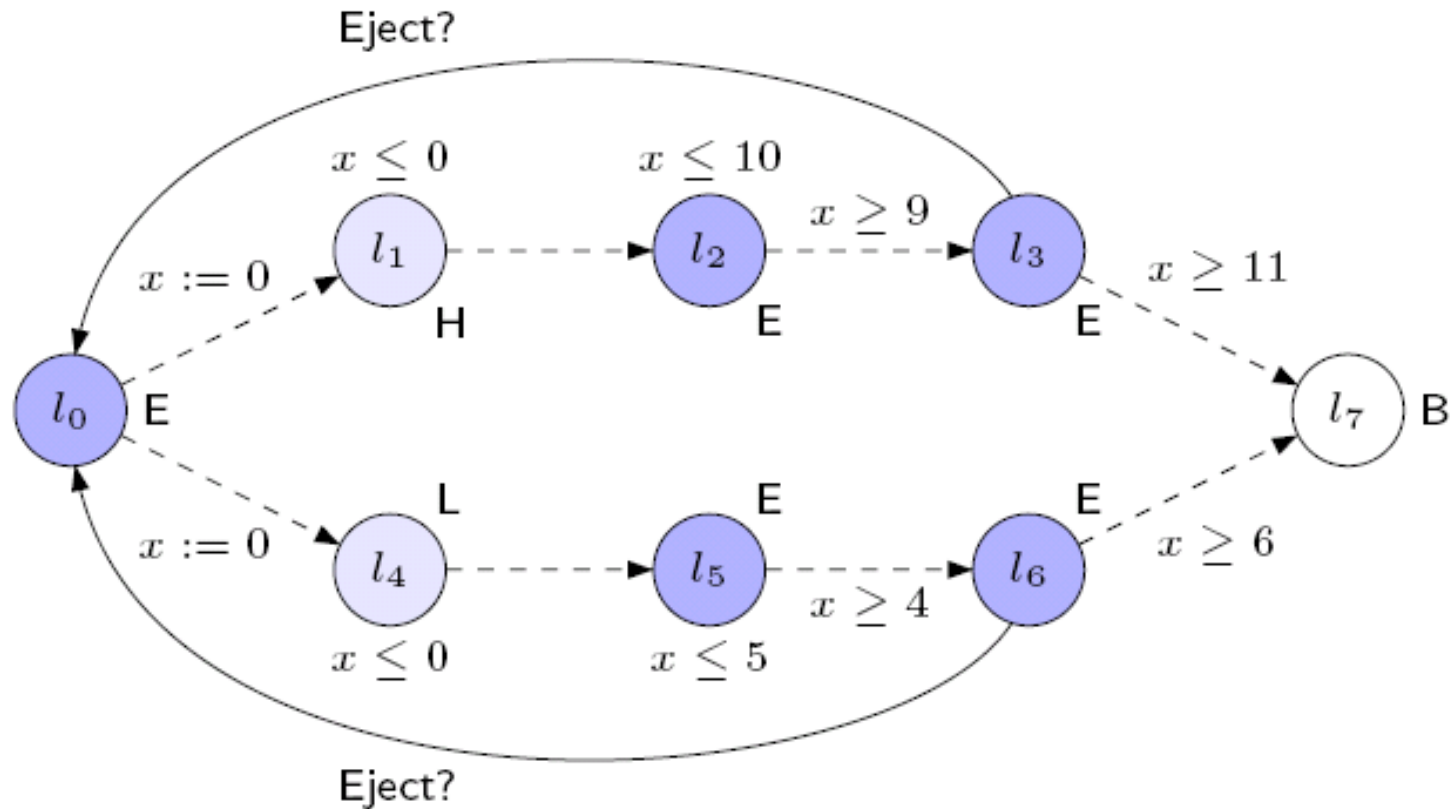
- Stuttering-free invariant observations
 - sinks possible in observations (deadlock/livelock/loop)
- Actions are urgent
 - delay until actions are enabled (or observation changes)



Algorithm

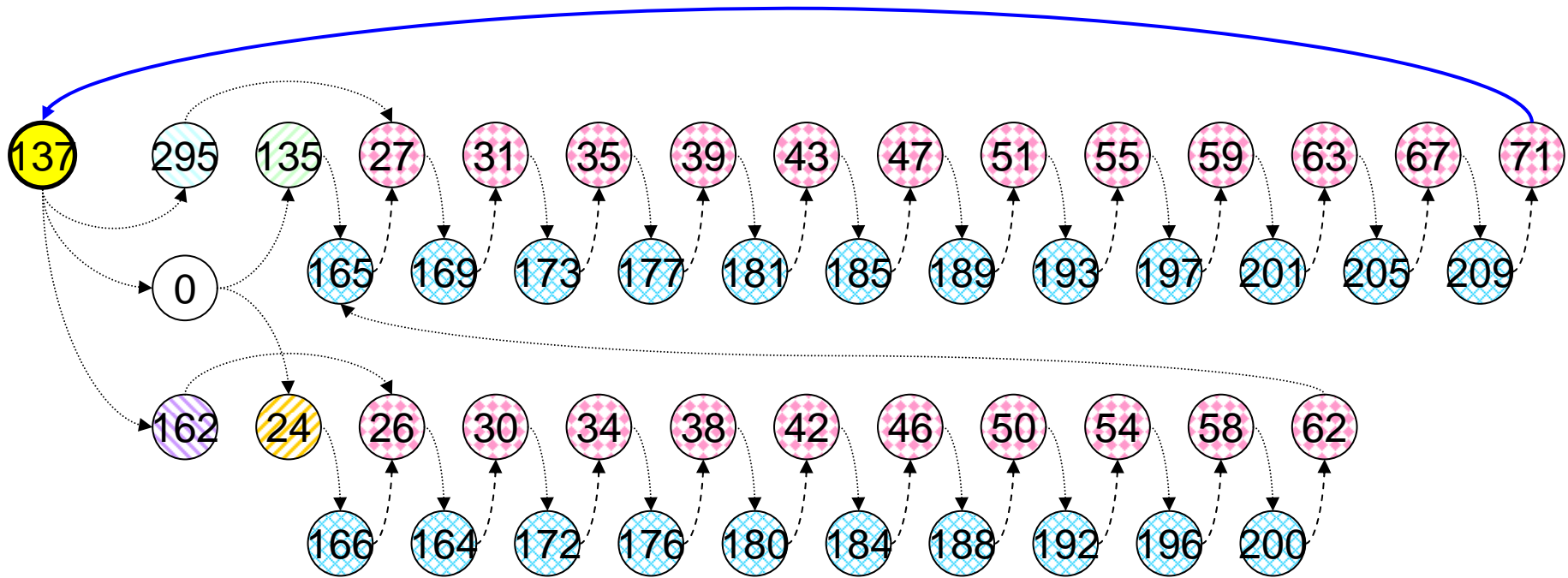
- Forward exploration
 - constrained by action + observations
 - delay special
- Back-propagation.
 - If all successors^{*a*} are winning, declare current state winning, strategy: take action *a*.
 - If one successor^{*a*} is losing, avoid action *a*.
If no action is winning the current state is losing.

Example



Observations: L, H, E, B, y in $[0,1[$

Example



Partition:



Actions:

