# Games on Concurrent Processes: Epistemic Strategies

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# Outline

- Introduction: problems with traditional process algebra.
- Games and strategies
- Epistemic restrictions strategies
- Conclusions

# Goals

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- Understand and control information flow.

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- The scheduler is assumed to be omniscient.
- It is hard to **require** it to respect independence constraints without controlling it somehow.

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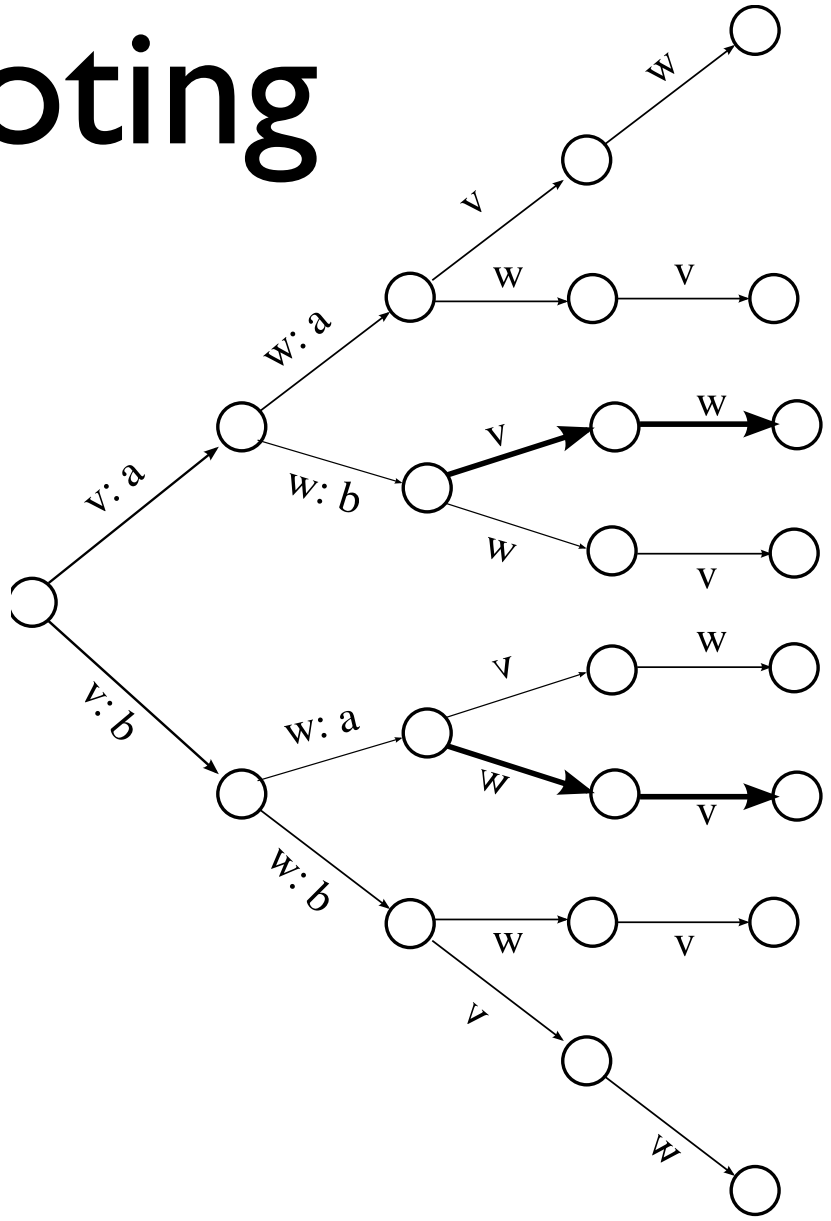
- Safety properties are usually required to hold with **universal quantification** over the possible schedulers,
- so it is impossible to prove any reasonable security properties in this setting.

# Example: voting

- Two candidates: a,b
- Two electors: v,w
- Must output who voted
- But not for whom they voted
- Thick arrows show scheduler that violates anonymity.



# Processes with labels

| | |
|---|---|
| $a, b$ | actions |
| $\bar{a}, \bar{b}$ | co-actions |
| $\tau$ | silent action |
| $\alpha, \beta$ | generic actions, co-actions, or silent action |

$$P, Q ::= 0 \mid l : \alpha.P \mid P|Q \mid P + Q \mid (\nu a)P \mid l : \{P\}$$

# Operational Semantics

ACT $\dfrac{}{l{:}\alpha.P \xrightarrow{\alpha} P}$

RES $\dfrac{P \xrightarrow{\alpha} P' \qquad \alpha \neq a, \bar{a}}{(\nu a)P \xrightarrow{\alpha} (\nu a)P'}$

SUM1 $\dfrac{P \xrightarrow{\alpha} P'}{P+Q \xrightarrow{\alpha} P'}$

SUM2 $\dfrac{Q \xrightarrow{\alpha} Q'}{P+Q \xrightarrow{\alpha} Q'}$

PAR1 $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$

PAR2 $\dfrac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$

COM $\dfrac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$

SWITCH $\dfrac{P \xrightarrow{\tau} P'}{l{:}\{P\} \xrightarrow{\tau} P'}$

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Required to do a silent action because otherwise the outcome of the protected choice would be visible to the scheduler.

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- Players are independent and act according to their strategies.
- Players interact to determine how process will execute.

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- A string of allowable moves is called a valid position.
- A valid position is like a trace, but with labels instead of actions.

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$$P = l_1 : \{k_1 : \tau.l_3 : a + k_2 : \tau.l_3 : b\} + l_2 : c$$

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The set of valid positions is prefix closed.

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**Definition**: In the game for $P$, a *strategy for player* $Z$ is a set $S$ of valid positions such that $\varepsilon \in S$ and if $s.m \in S$, then $m$ is a $Z$ move and every prefix of $s$ ending with a $Z$ move is in $S$.

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A strategy tells the player what move to make in a possible partial execution of the process.

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# Restrictions 1

**Definition** A strategy $S$ is *deterministic* if for all sequences $s$, $s.m_1 \in S$ and $s.m_2 \in S$ implies $m_1 = m_2$.

# Complete strategies

- We want some way of ensuring that the strategy tells the player what to do in every possible situation.
- This is formalized by the definition of complete strategy.
- The details are in the paper in the proceedings.

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- We are saying that strategies can only be based on what players know.
- One can design different equivalences to "engineer" the appropriate epistemic concept.

# Introspection

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- The player knows his own history and what moves were available to him at every point in the past.

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- they are equivalent to the syntactic schedulers of Chatzikokolakis and Palamidessi.

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- A framework that can be used for other such limitations.
- A better conceptual understanding of knowledge and interaction.