Partial-Observation Stochastic Games: How to Win when Belief Fails

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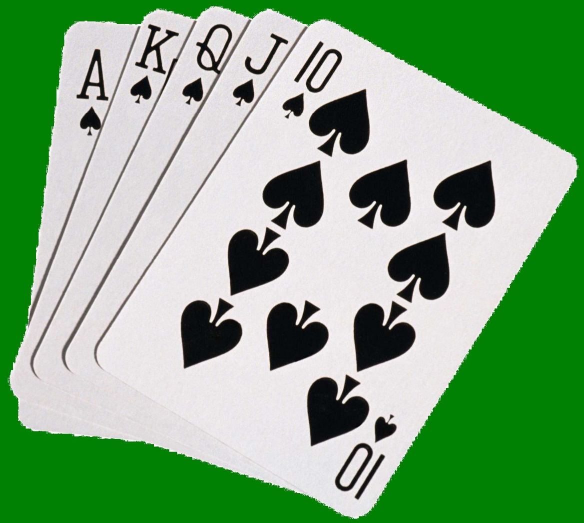
Outline

- Game model: example
- Challenges & Results: examples
- Solution insights: examples

Examples

- Poker
  - partial-observation
  - stochastic



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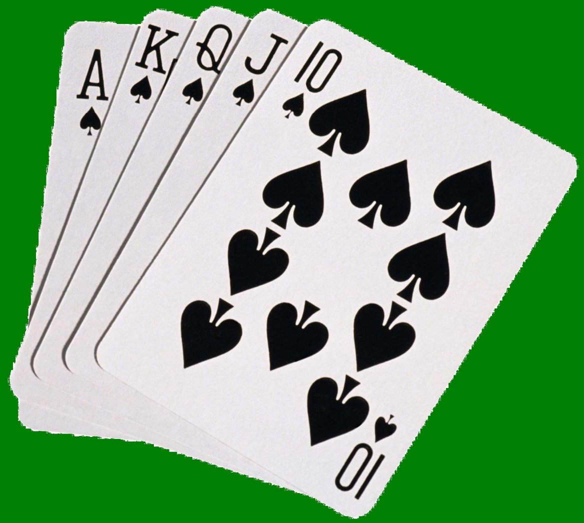
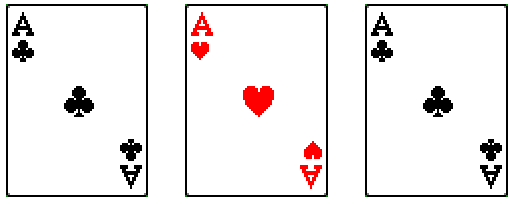


- Bonneteau

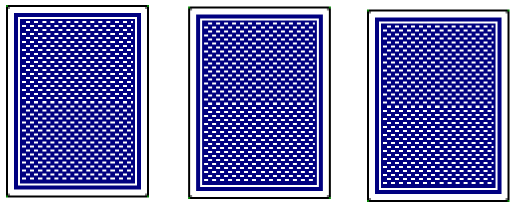
Bonneteau

2 black card, 1 red card



Initially, all are face down

Goal: find the red card



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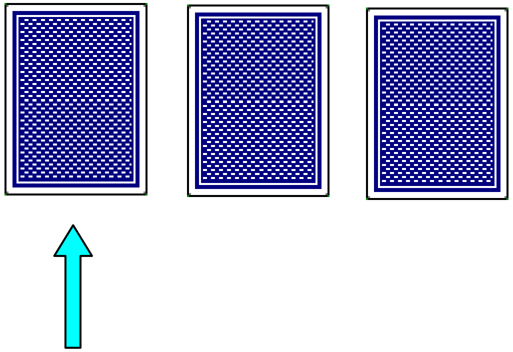

Rules:

1. Player 1 points a card
2. Player 2 flips one remaining black card
3. Player 1 may change his mind, wins if pointed card is red

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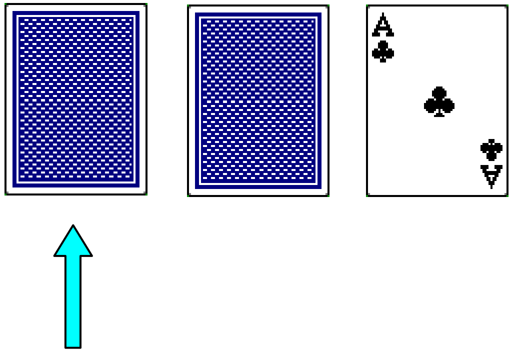

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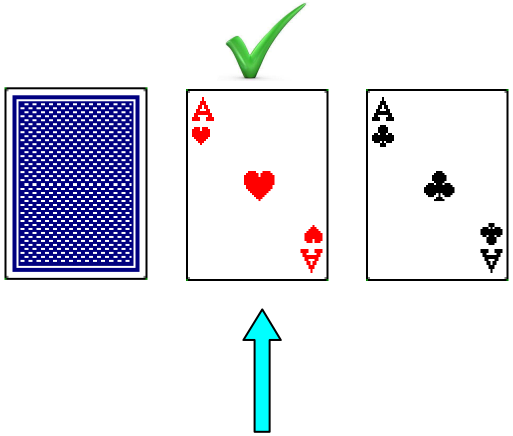

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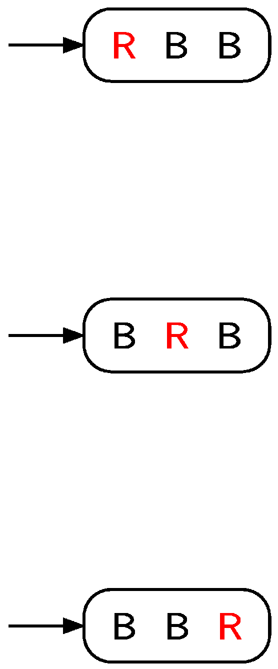
Bonneteau: Game Model



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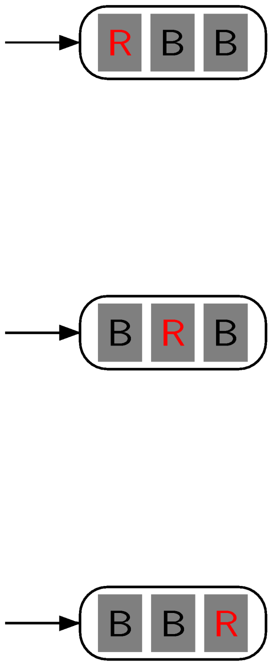
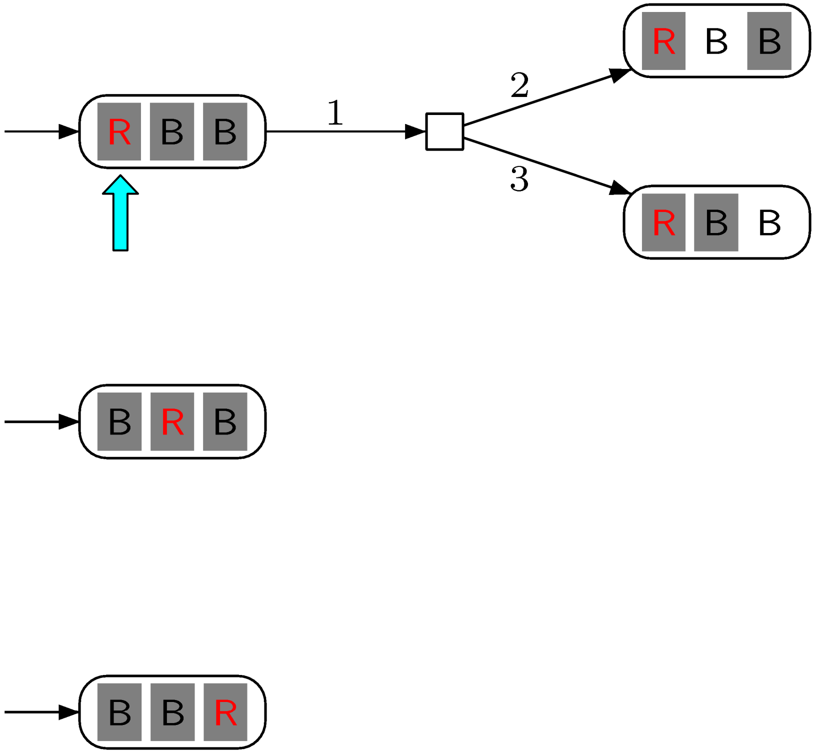


Game Model



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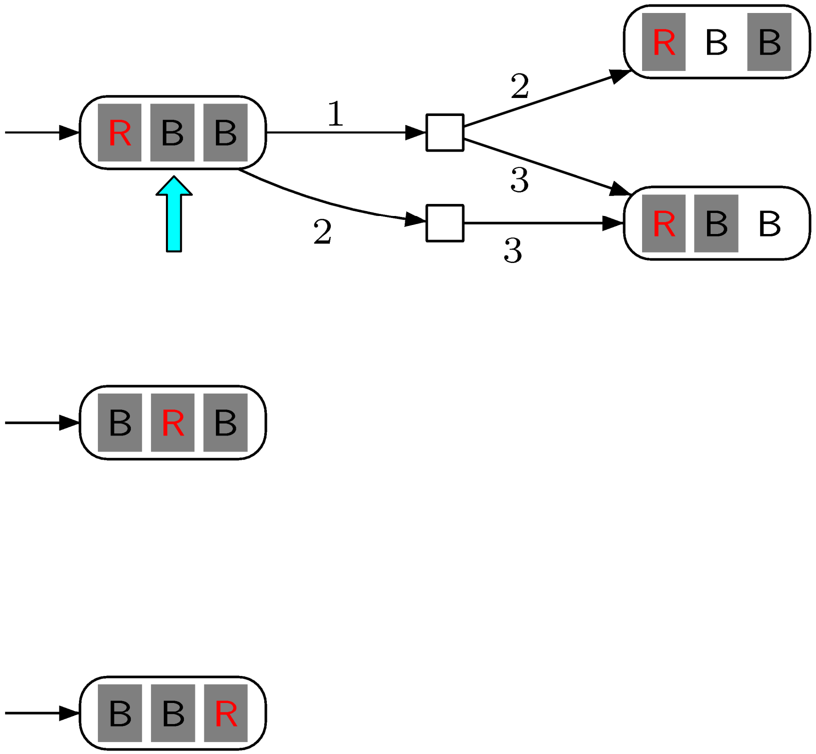

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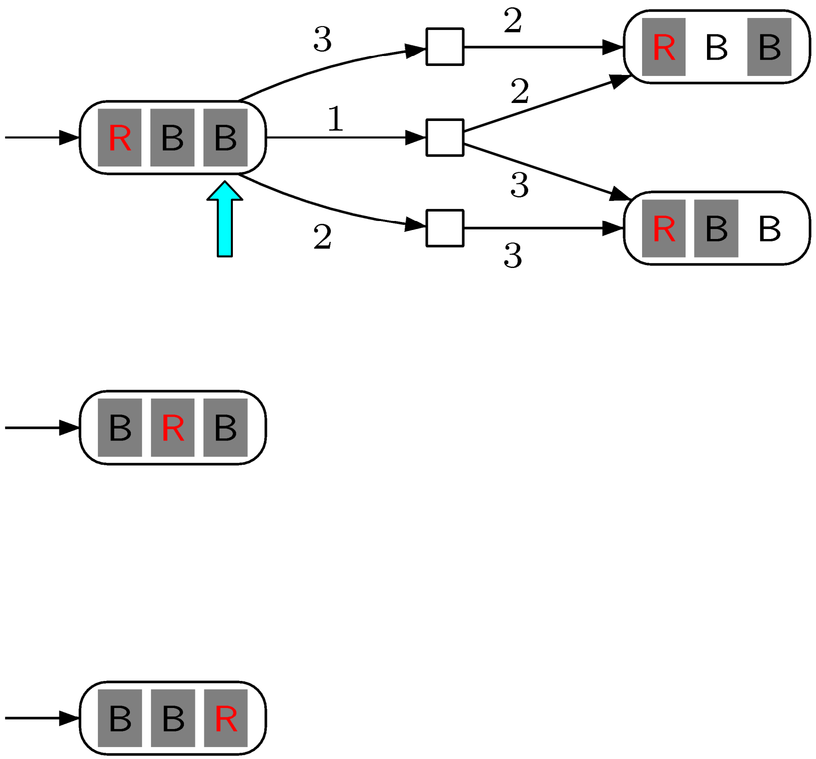

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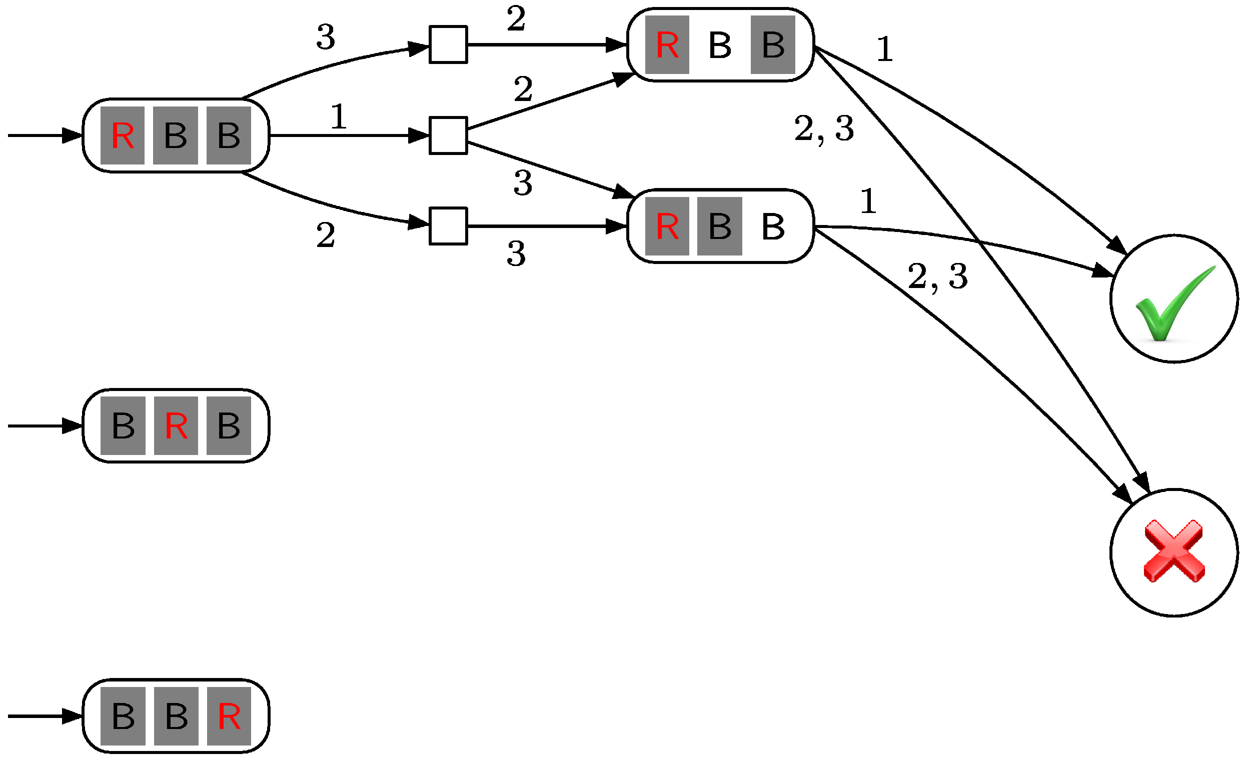

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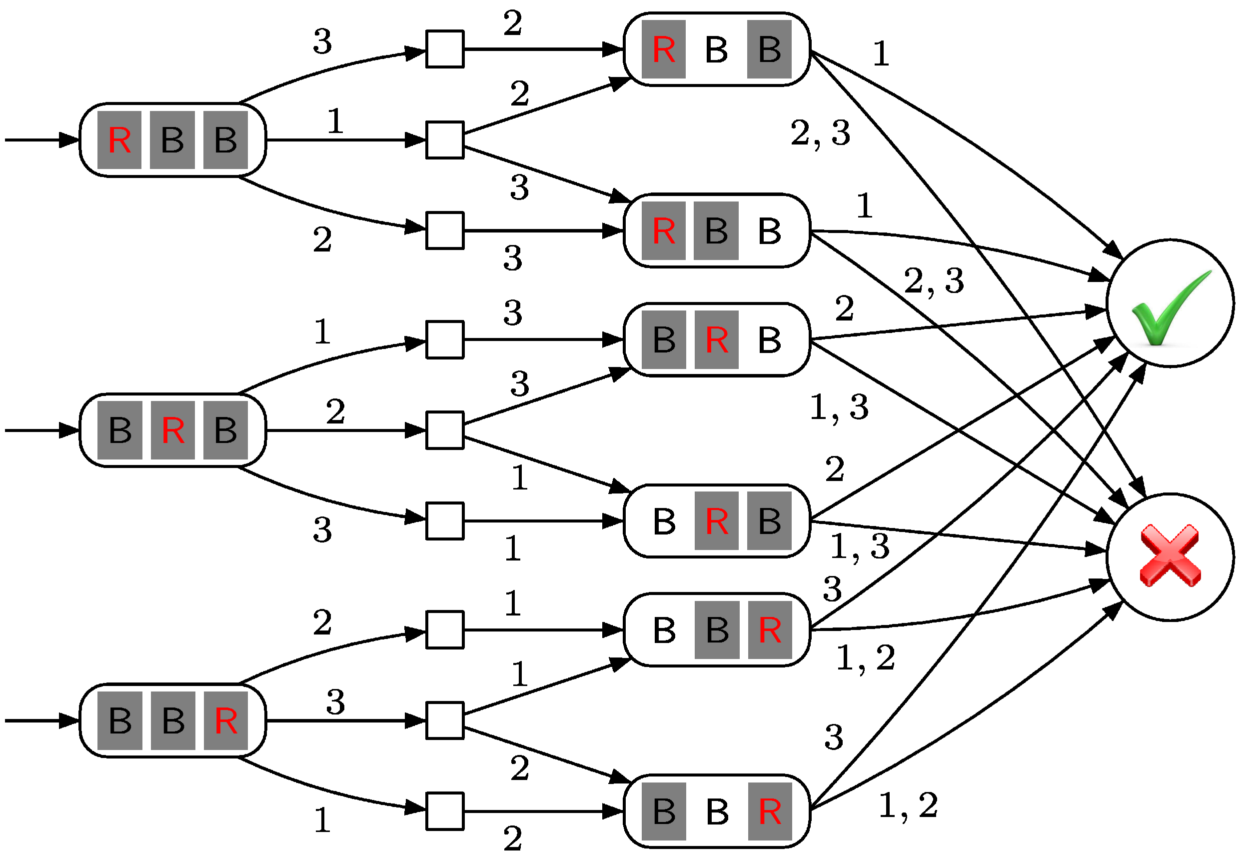

3
2
R B B
1
2
2, 3
R B B
1
3
1
2
R B B
3
2, 3
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B R B
2
B R B
2
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1, 3
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B R B
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1, 3
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B B R
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B B R
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B B R
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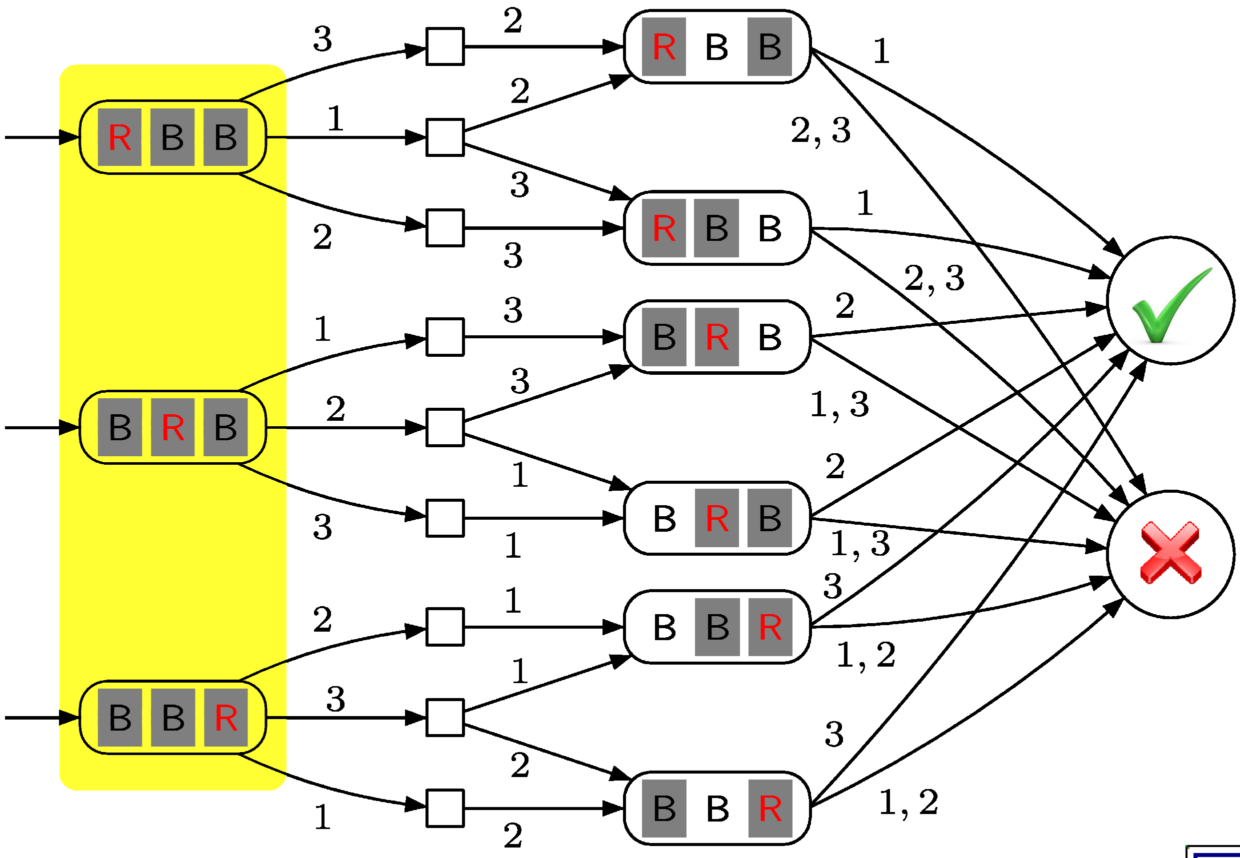
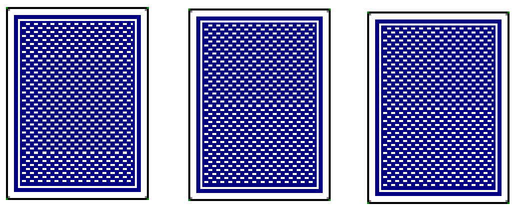
1, 2

Observations (for player 1)





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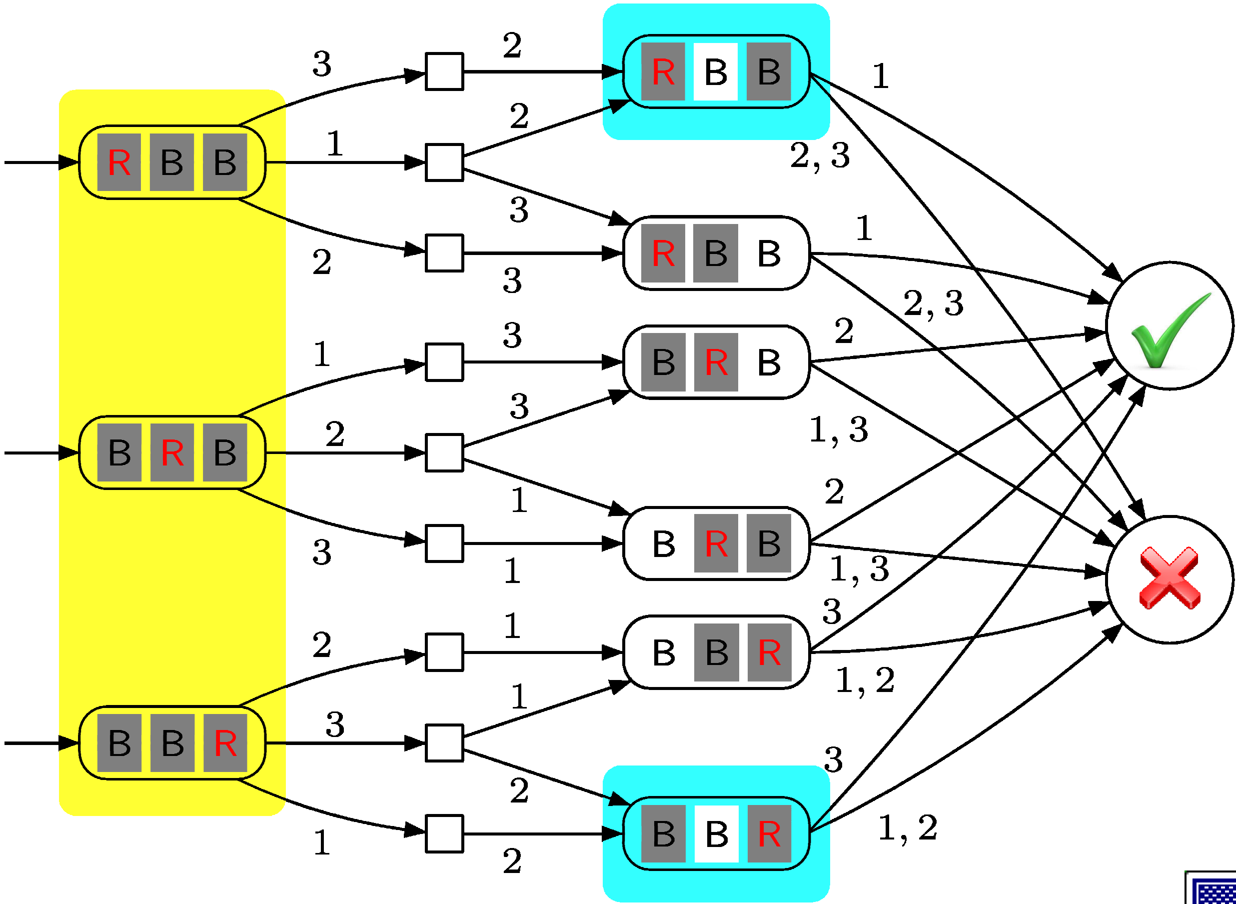

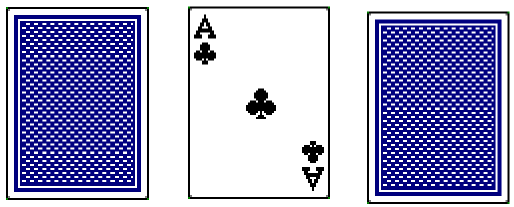

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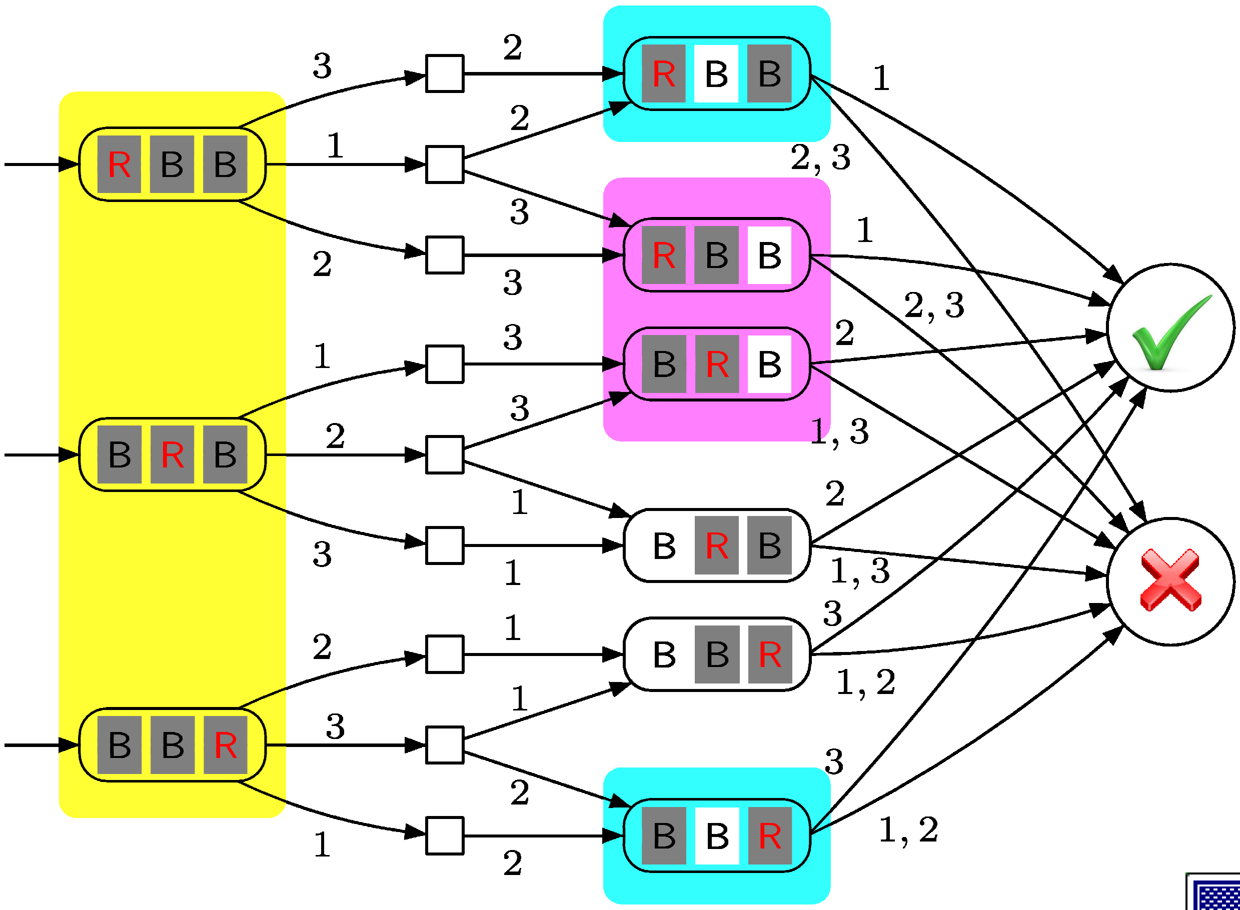

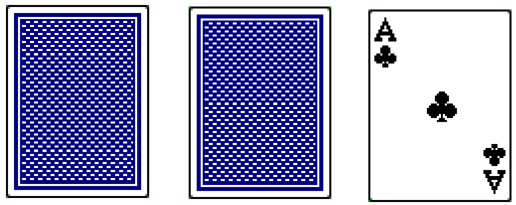

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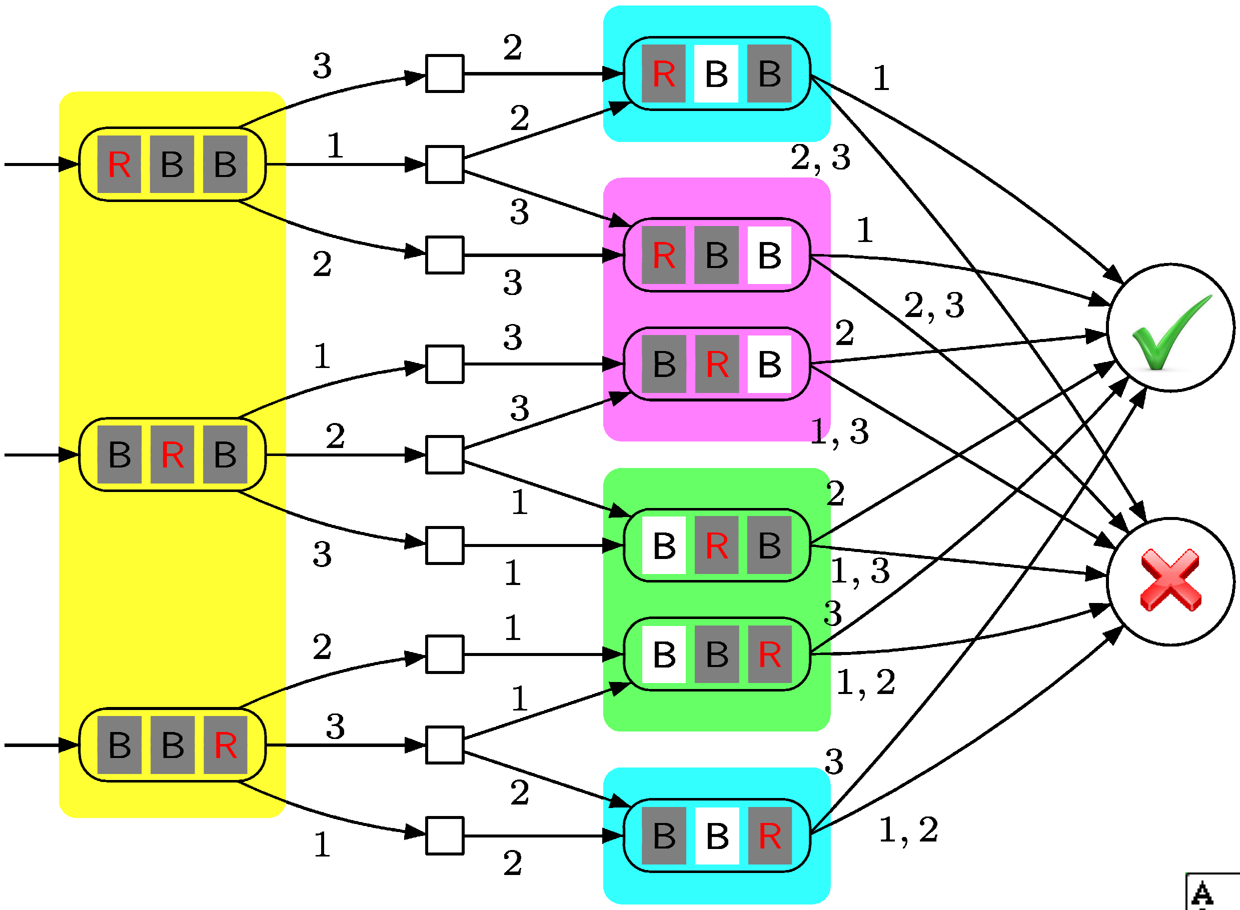

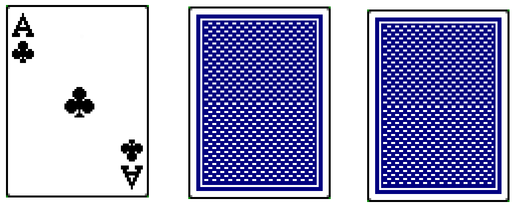

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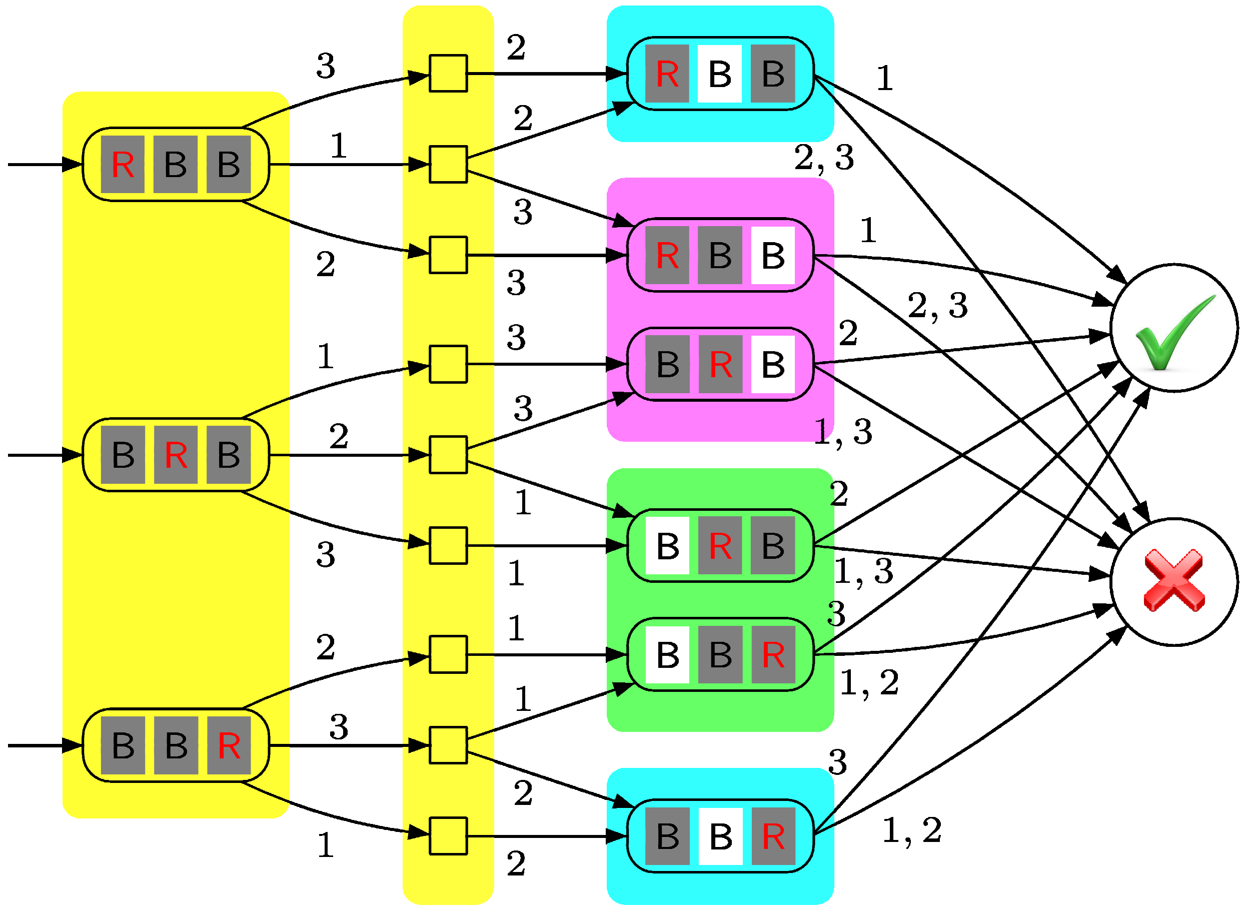
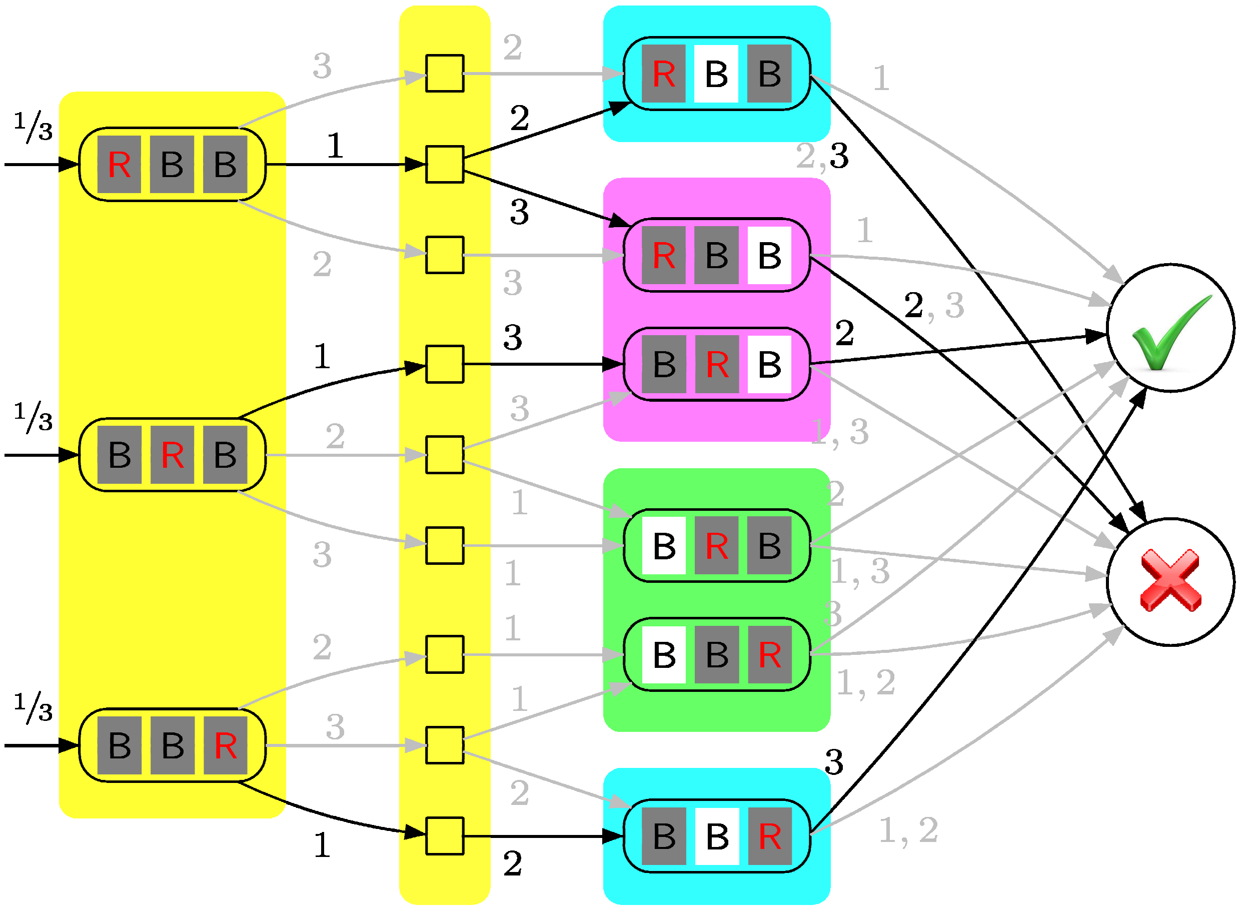
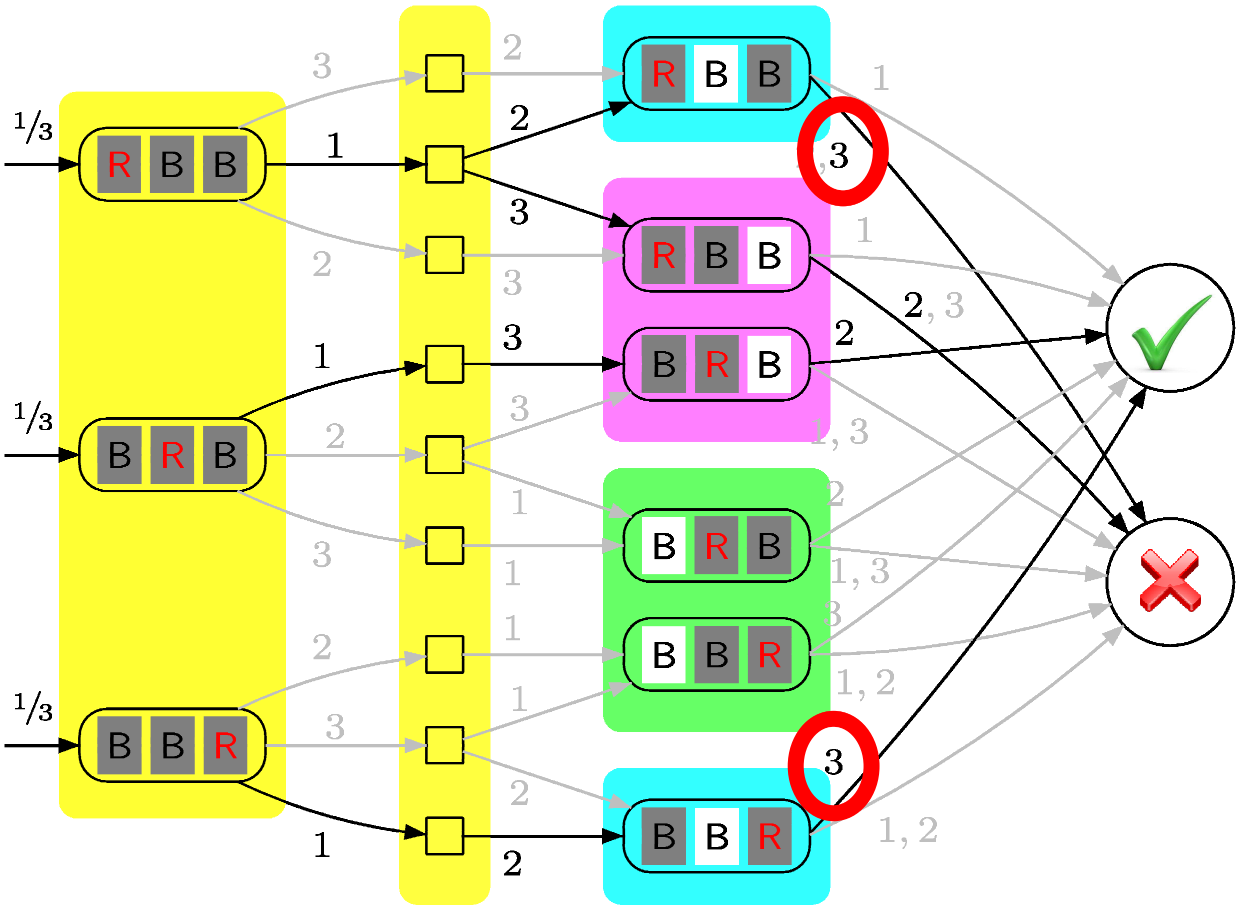


Observation-based strategy



This strategy is observation-based,
e.g. after , , it plays 3

Observation-based strategy



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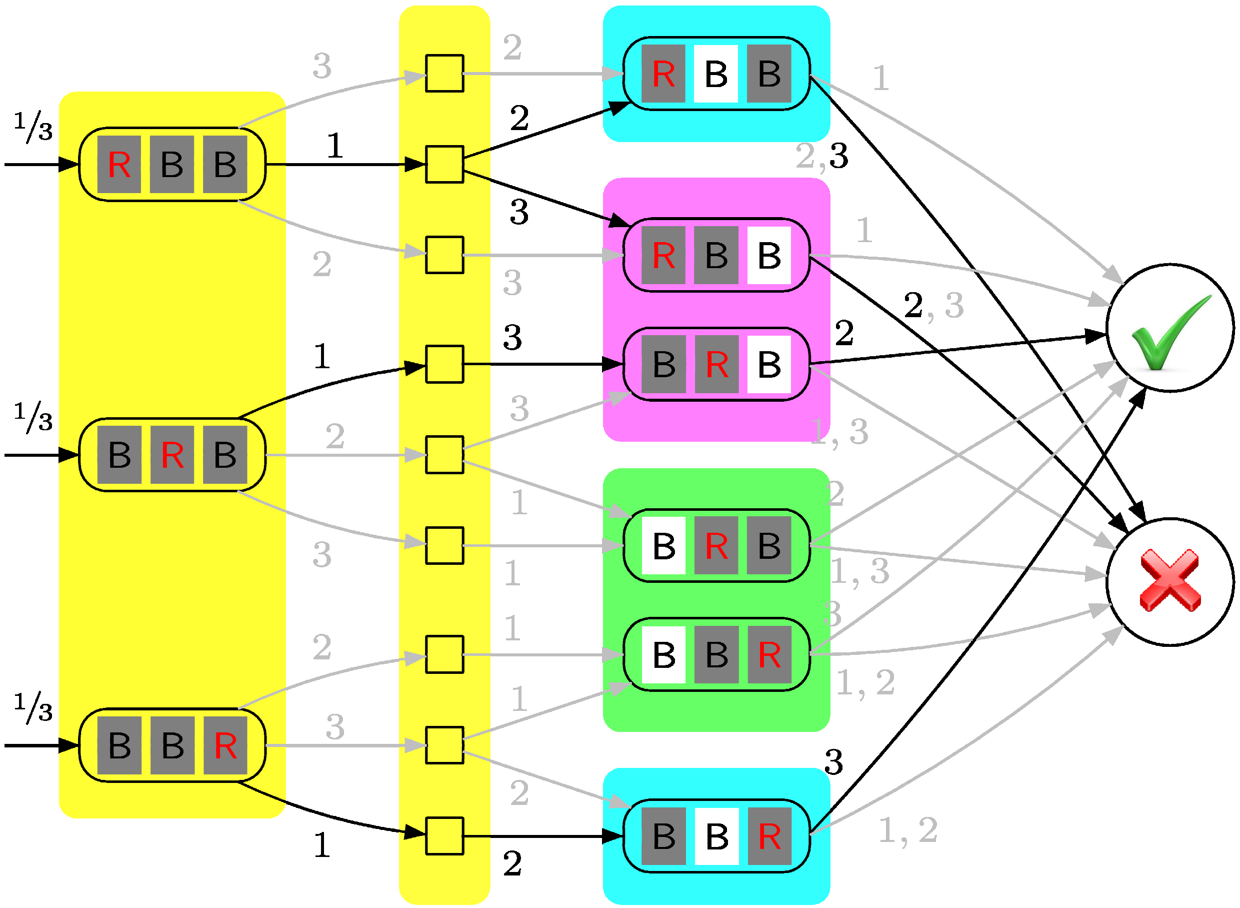
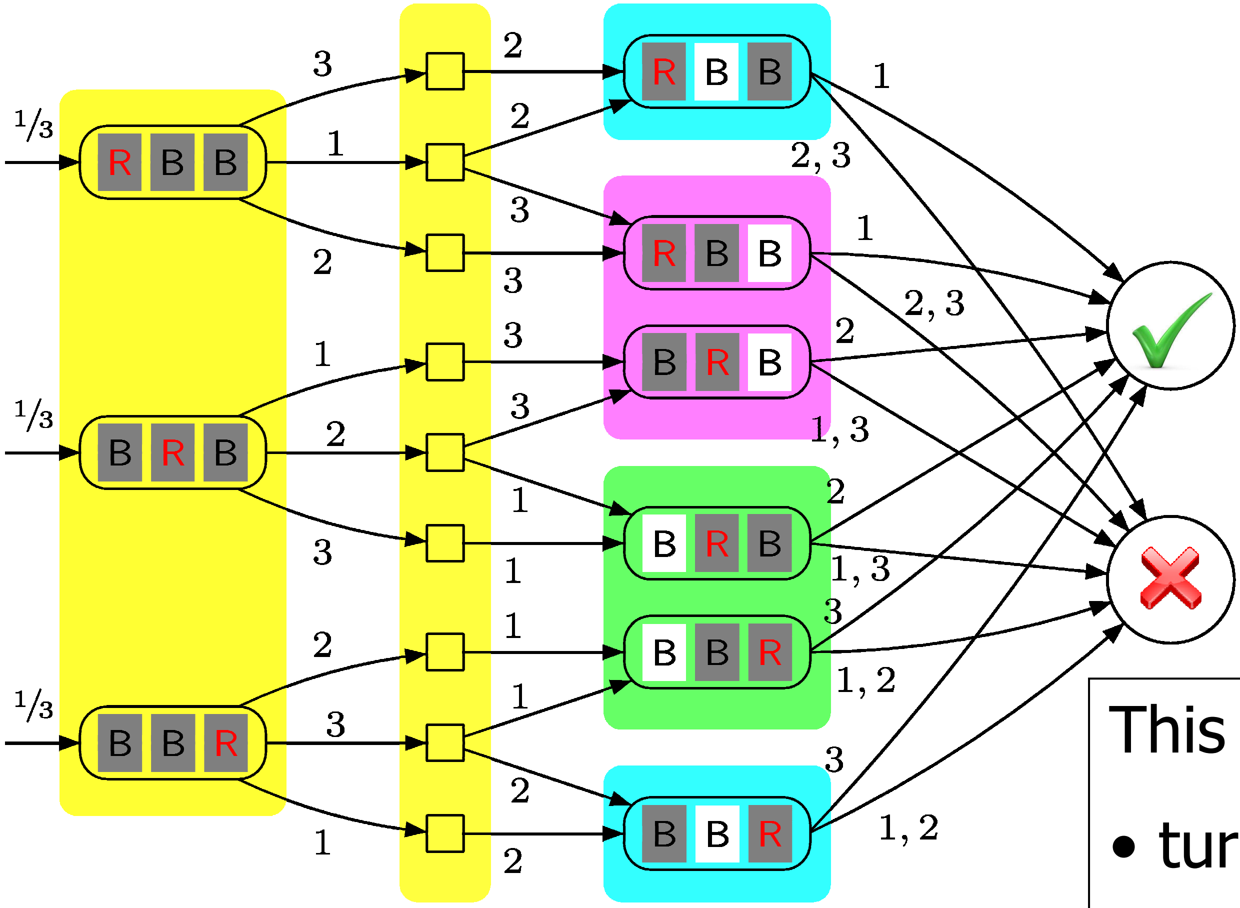
Optimal ?



This strategy is winning with probability 2/3

Example



This game is:

- turn-based
- (almost) non-stochastic
- player 2 has perfect observation

Interaction

General case: concurrent & stochastic

$$\delta : Q \times A_1 \times A_2 \rightarrow \mathcal{D}(Q)$$

Player 1's move

Player 2's move

Players choose their moves simultaneously and independently

Interaction

General case: concurrent & stochastic

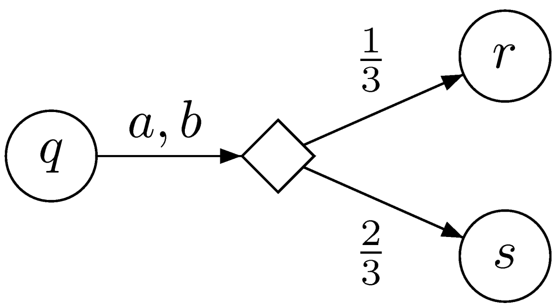
$$\delta : Q \times A_1 \times A_2 \to \mathcal{D}(Q)$$

$$\mathcal{D}(Q) = \{f : Q \to [0,1] \mid \sum_q f(q) = 1\}$$

Player 1's move

Player 2's move

Probability distribution on successor state



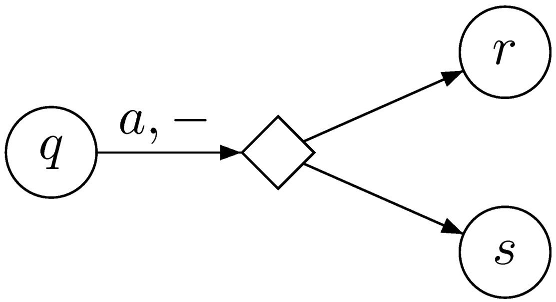
$$\delta(q, a, b)(r) = \frac{1}{3}$$

$$\delta(q, a, b)(s) = \frac{2}{3}$$

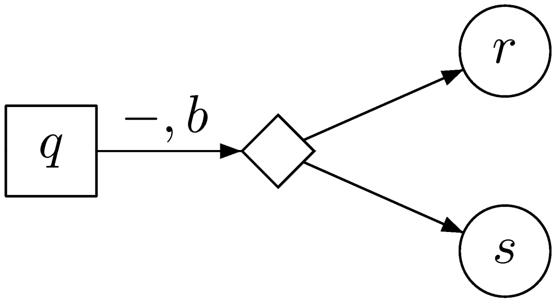
Interaction

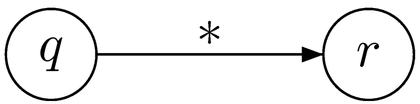
Special cases: **Turn-based games**

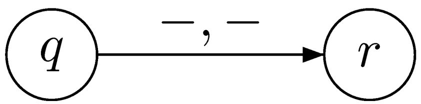
- player-1 state



- player-2 state



Note: 

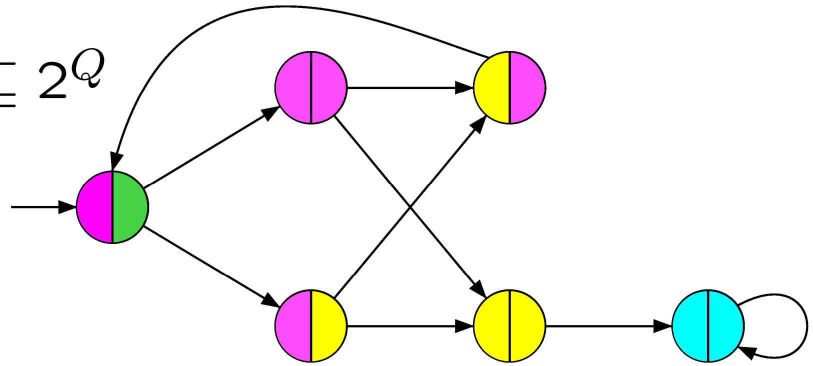
means 

Partial-observation

Observations: partitions induced by coloring

General case: 2-sided partial observation

Two partitions $\texttt{Obs}_1 \subseteq 2^Q$ and $\texttt{Obs}_2 \subseteq 2^Q$



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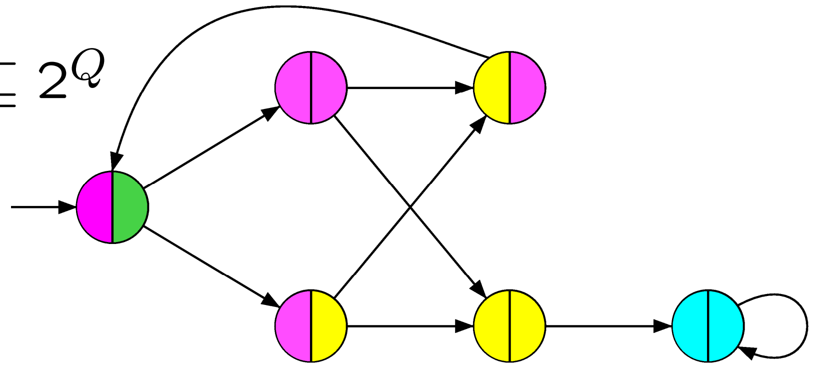
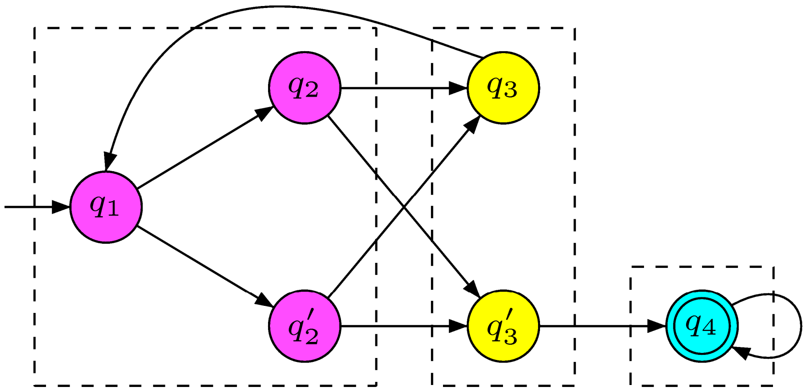
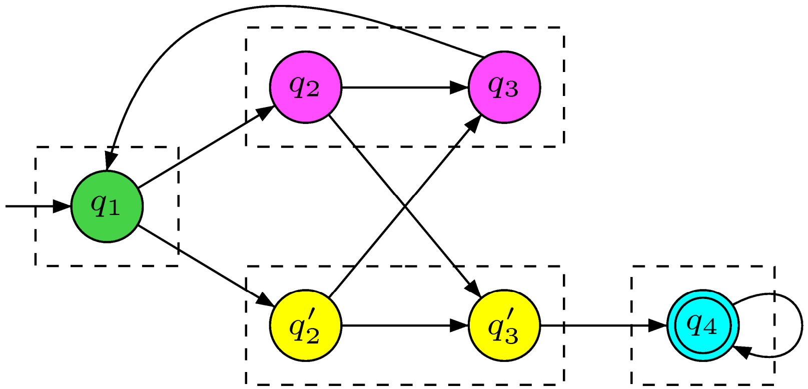




Player 1's view



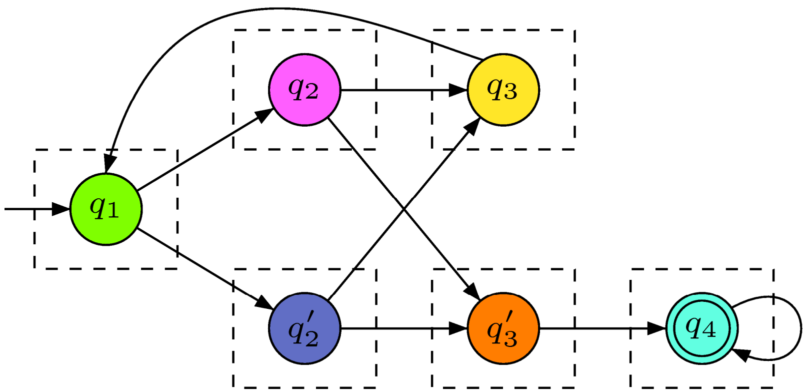
Player 2's view

Partial-observation

Observations: partitions induced by coloring

Special case: 1-sided partial observation

$\mathsf{Obs}_1 = \{\{q\} \mid q \in Q\}$ or $\mathsf{Obs}_2 = \{\{q\} \mid q \in Q\}$



View of perfect-observation player

Strategies & objective

A strategy for Player i is a function $\sigma_i : \mathsf{Obs}_i^+ \to \mathcal{D}(A_i)$ that maps histories (sequences of observations) to probability distribution over actions.

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History-depedent randomized

Strategies & objective

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Reachability objective: $\mathcal{T} \subseteq Q$

Winning probability of σ_1: $\inf_{\sigma_2} Pr_{q_0}^{\sigma_1,\sigma_2}(\exists i \geq 0 : q_i \in \mathcal{T})$

Qualitative analysis

The following problem is undecidable:
(already for probabilistic automata [Paz71])

Decide if there exists a strategy for player 1 that is winning with probability at least ½.

[Paz71] Paz. *Introduction to Probabilistic Automata*. Academic Press 1971.

Qualitative analysis

The following problem is undecidable:
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Decide if there exists a strategy for player 1 that is winning with probability at least ½.

Qualitative analysis:

- Almost-sure: … winning with probability 1
- Positive: … winning with probability > 0

$$\exists \sigma_1 \cdot \forall \sigma_2 : Pr_{q_0}^{\sigma_1, \sigma_2}(\exists i \geq 0 : q_i \in \mathcal{T}) \begin{cases} = 1 \\ > 0 \end{cases}$$

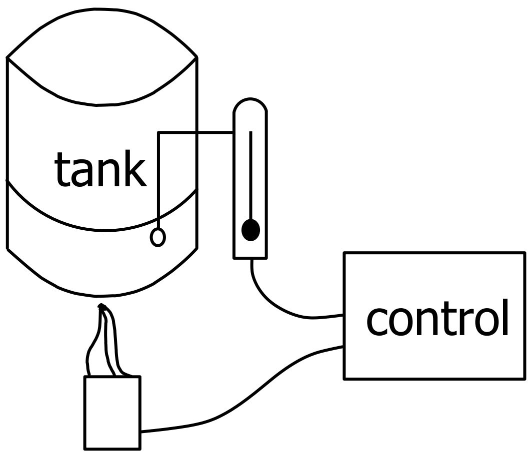
Applications in verification

- Control with inaccurate digital sensors
- multi-process control with private variables
- multi-agent protocols
- planning with uncertainty/unknown

```
void main () {
    int got_lock = 0;
    do {
1:          if (*) {
2:              lock ();
3:              got_lock++;
            }
4:          if (got_lock != 0) {
5:              unlock ();
            }
6:          got_lock--;
    } while (*);
}
```

```
void lock () {
    assert(L == 0);
    L = 1;
}
```

```
void unlock () {
    assert(L == 1);
    L = 0;
}
```



Outline

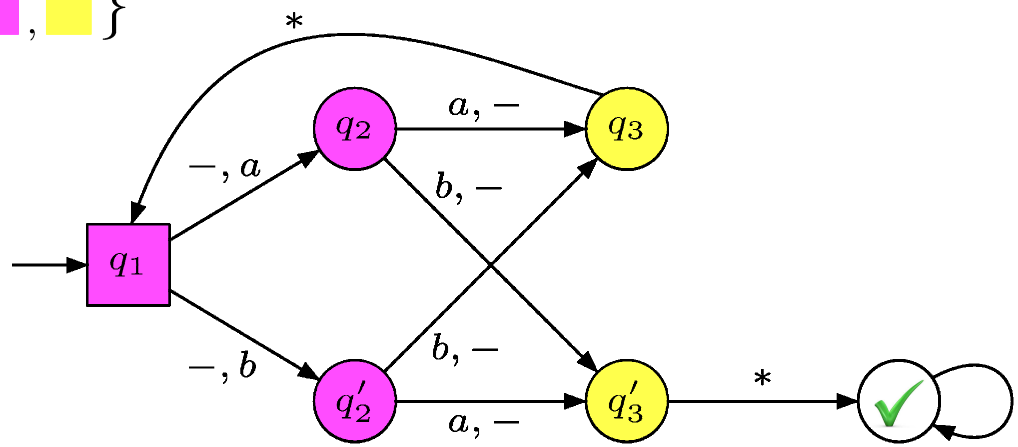
- Game Model: example
- Challenges & Results: examples
- Solution insights: examples

Randomization is necessary

Player 1 partial, player 2 perfect

$\sigma_i : \mathsf{Obs}_i^+ \to \mathcal{D}(A_i)$

$\mathsf{Obs}_1 = \{$ ■, ■ $\}$



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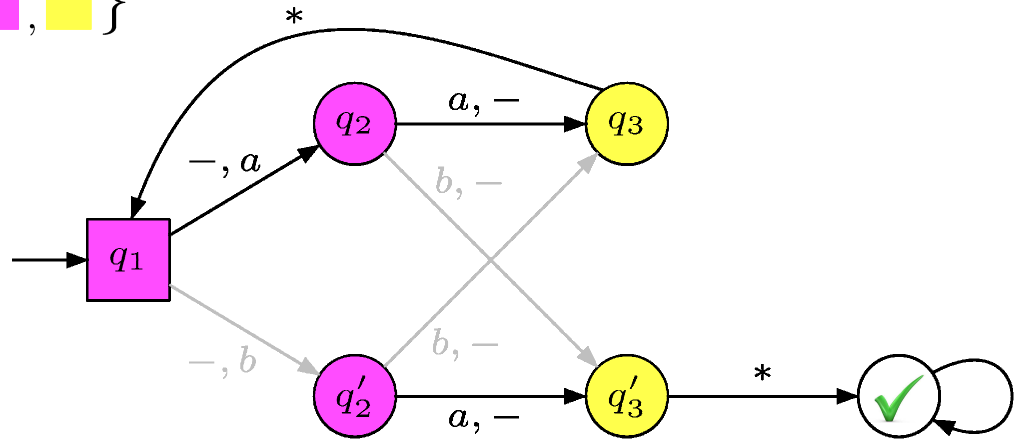


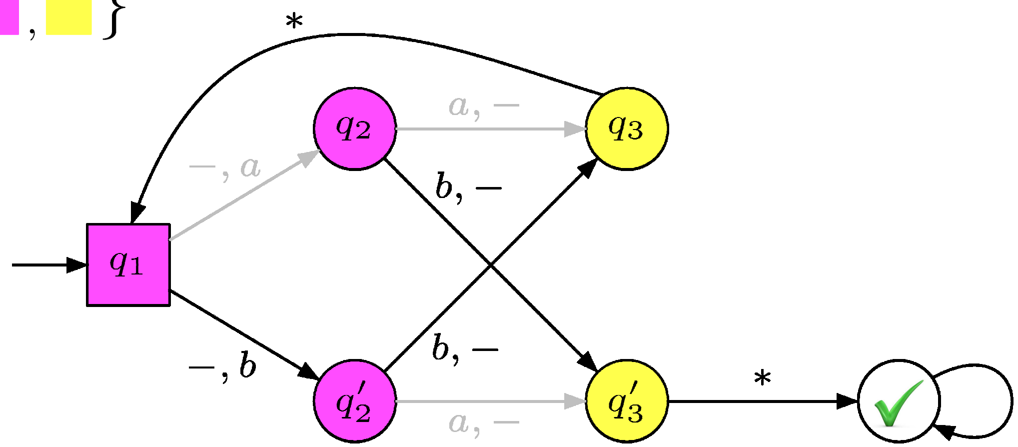
No pure strategy of Player 1 is winning with probability 1 (example from [CDHR06]).

Randomization is necessary

Player 1 partial, player 2 perfect

$\sigma_i : \text{Obs}_i^+ \to \mathcal{D}(A_i)$

$\text{Obs}_1 = \{\blacksquare, \blacksquare\}$



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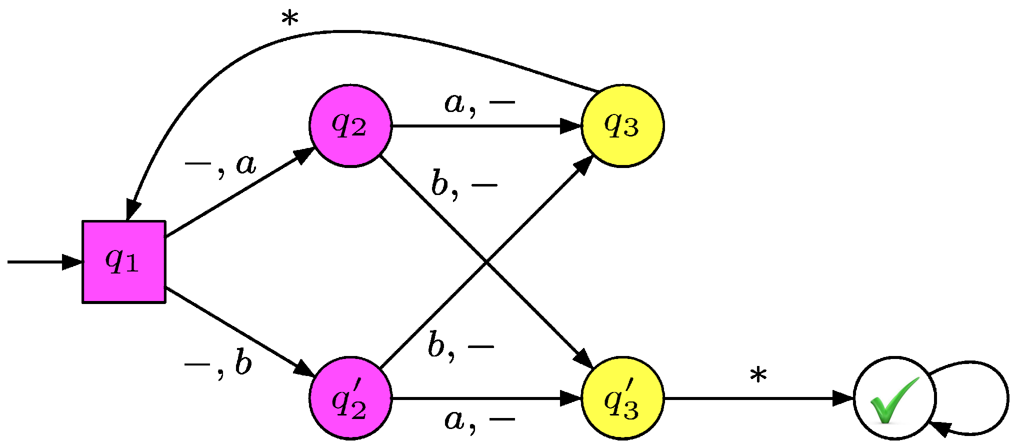
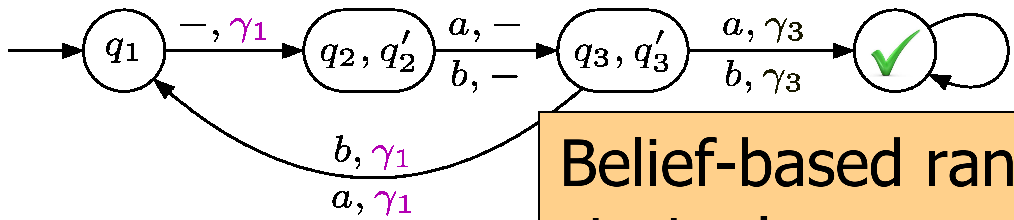
Memory and Randomization

Player 1 partial, player 2 perfect

$\sigma_i : \mathsf{Obs}_i^+ \to \mathcal{D}(A_i)$



Player 1 wins with probability 1, and needs randomization

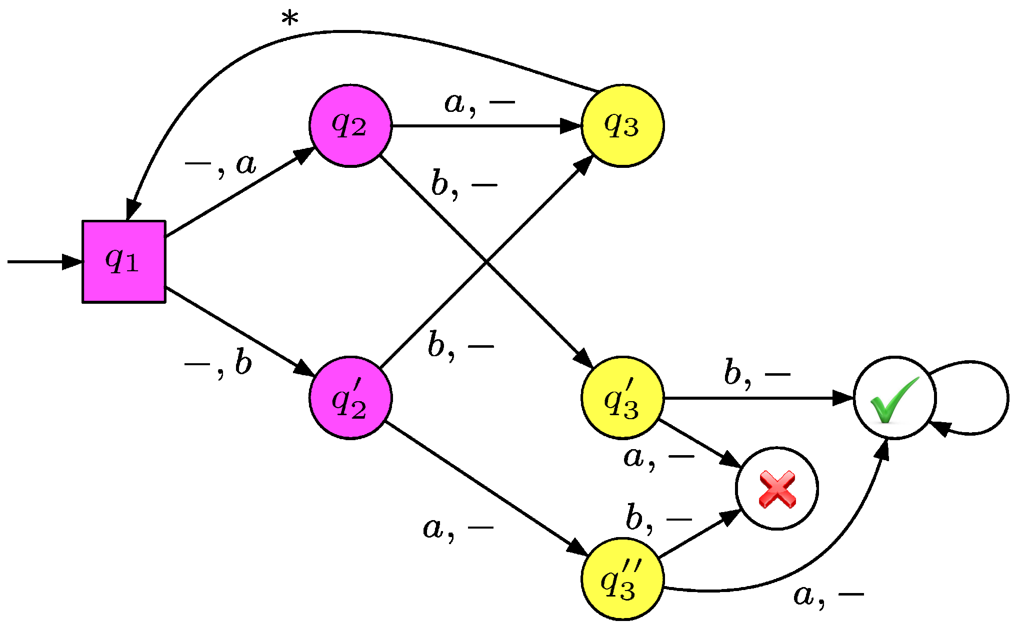


Belief-based randomized strategies are sufficient

Example 2

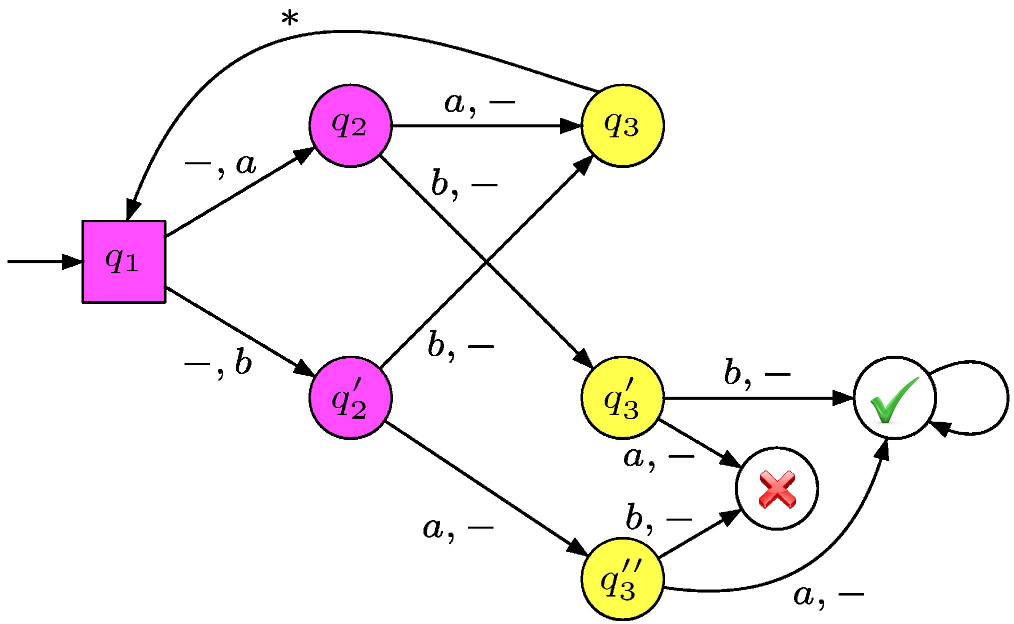
Player 1 partial, player 2 perfect

$\sigma_i : \mathsf{Obs}_i^+ \to \mathcal{D}(A_i)$



Example 2

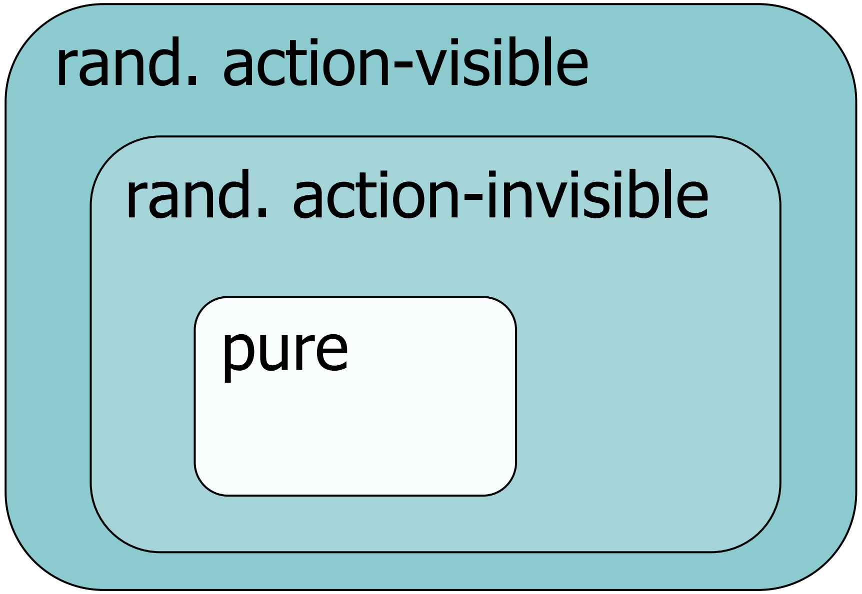
Player 1 partial, player 2 perfect



To win with probability 1, player 1 needs to observe his own actions. (example from [CDH10]).

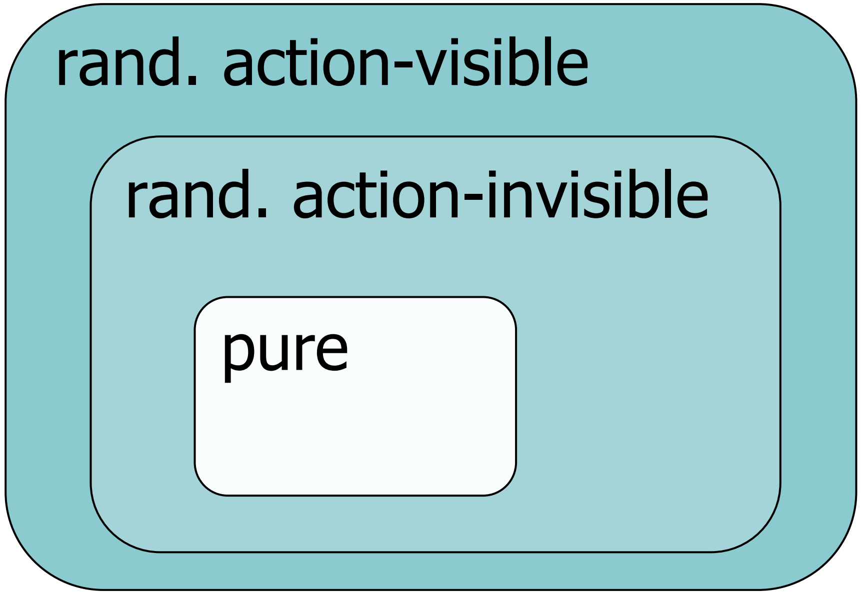
Randomized action-visible strategies: $\sigma_i : (\mathsf{Obs}_i A_i)^* \mathsf{Obs}_i \to \mathcal{D}(A_i)$

Classes of strategies



Classification according to the power of strategies

Classes of strategies

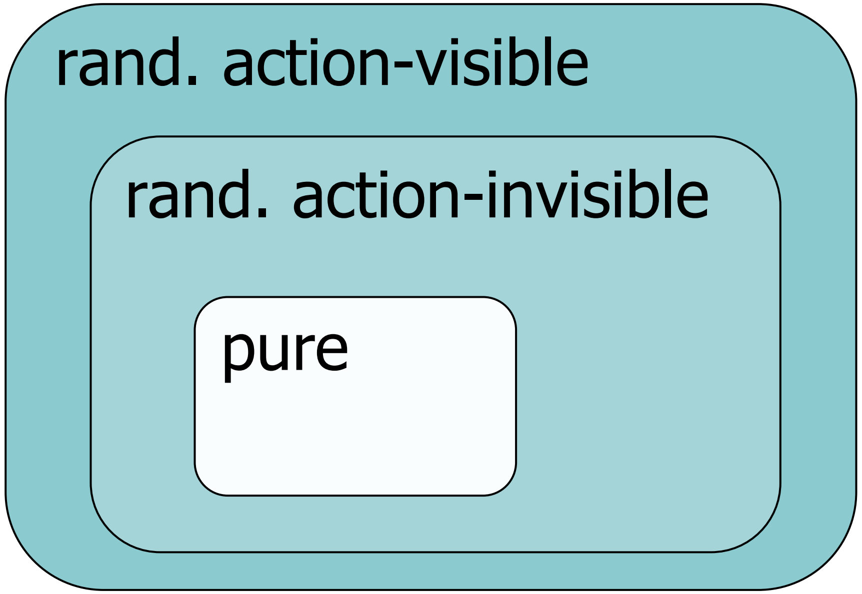


Classification according to the power of strategies

Poly-time reduction from decision problem of rand. act.-vis. to rand. act.-inv.

The model of rand. act.-inv. is more general

Classes of strategies



Classification according to the power of strategies

Computational complexity (algorithms)

Strategy complexity (memory)

Known results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.			
rand. act.-inv.			
pure			

Known results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (belief) **[CDHR'06(remark), GS'09]**		exponential (belief) **[GS'09]**
pure	?	?	?

[BGG09] Bertrand, Genest, Gimbert. ***Qualitative Determinacy and Decidability of Stochastic Games with Signals*. LICS'09.**
[CDHR06] Chatterjee, Doyen, Henzinger, Raskin. ***Algorithms for ω-Regular games with Incomplete Information*. CSL'06.**
[GS09] Gripon, Serre. ***Qualitative Concurrent Stochastic Games with Imperfect Information*. ICALP'09.**

Known results

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rand. act.-inv.	exponential (belief) **[CDHR'06(remark), GS'09]**		exponential (belief) **[GS'09]**
pure	?	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	?	?	?

About beliefs

Three prevalent beliefs:

- Belief is sufficient.

- Randomized action invisible or visible almost same.

- The general case memory is similar (or in some cases exponential blow up) as compared to the one-sided case.

Pure Strategies

Belief

- Belief is sufficient.

Proofs

- Doubts.

Pure Strategies

Belief

- Belief is sufficient.

Proofs

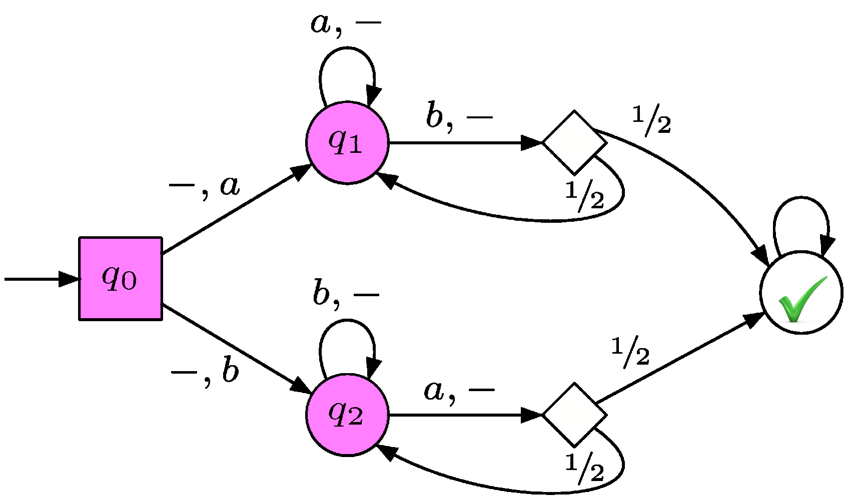
- Doubts.

Lesson:

Doubt your belief and believe in your doubts !! See the unexpected.

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning

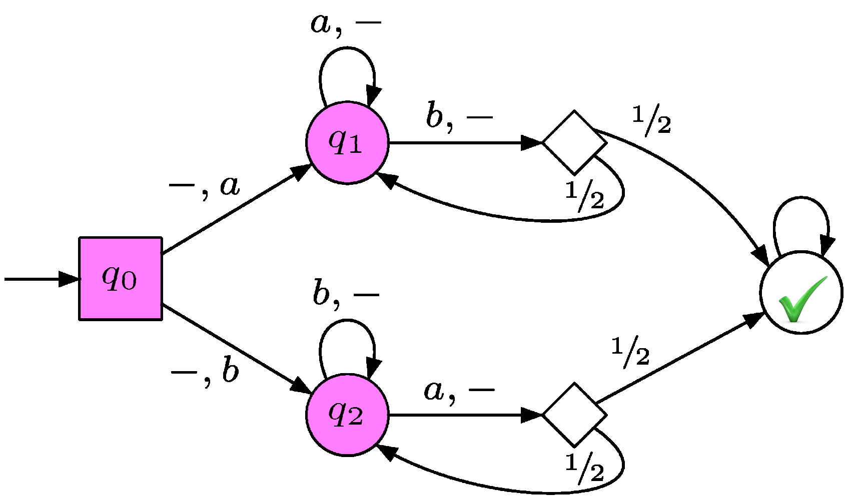


player 1 partial

player 2 perfect

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

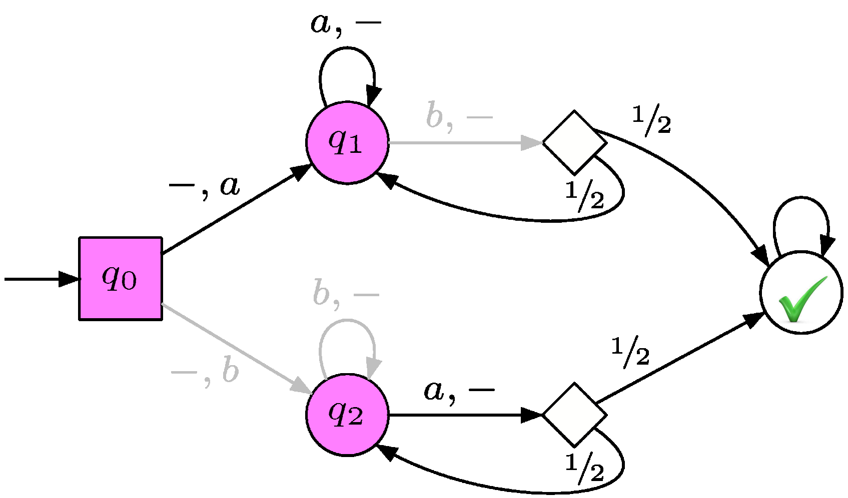
player 2 perfect

There are two belief-based-only pure strategies:

1. When belief is $\{q_1, q_2\}$, play a
2. When belief is $\{q_1, q_2\}$, play b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

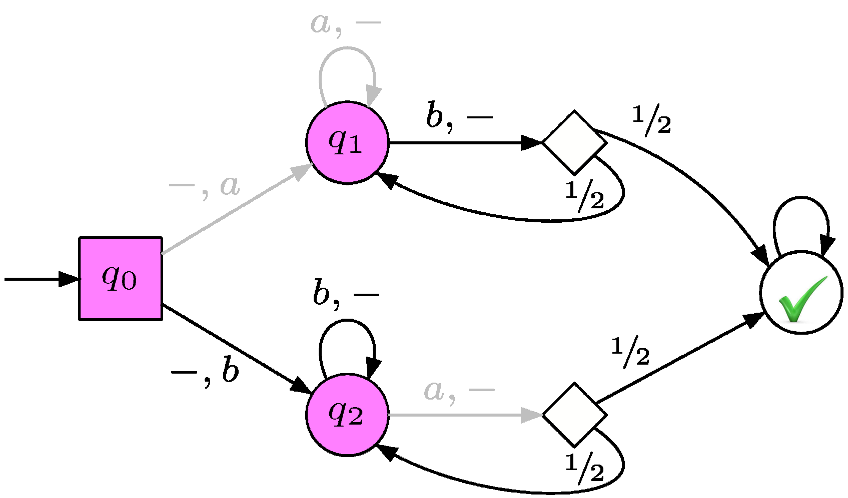
player 2 perfect

There are two belief-based-only pure strategies:

1. When belief is $\{q_1, q_2\}$, play a ⇐ not winning
2. When belief is $\{q_1, q_2\}$, play b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

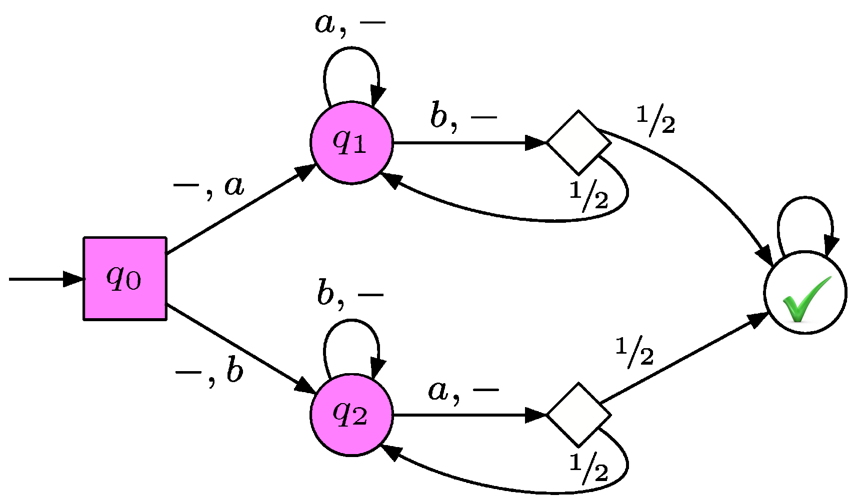
player 2 perfect

There are two belief-based-only pure strategies:

1. When belief is $\{q_1, q_2\}$, play a
2. When belief is $\{q_1, q_2\}$, play b ⟵ not winning

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

player 2 perfect

There are two belief-based-only pure strategies:

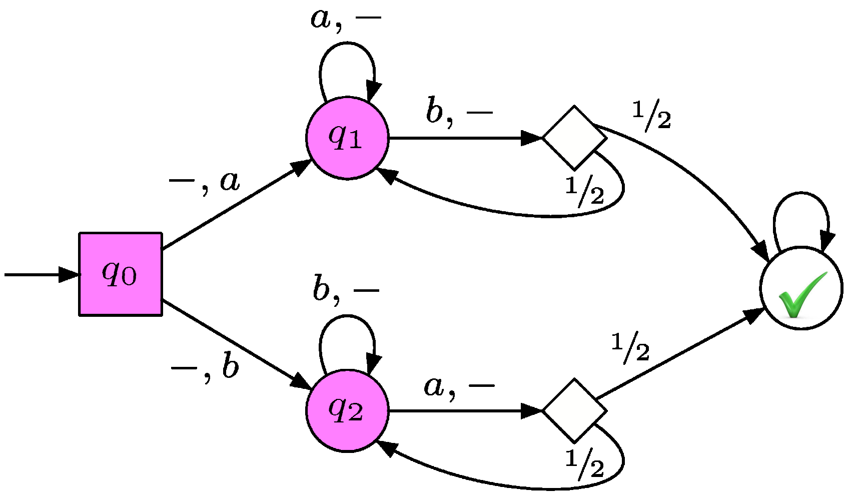
1. W

Neither is winning !

2. When belief is $\{q_1, q_2\}$, play b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



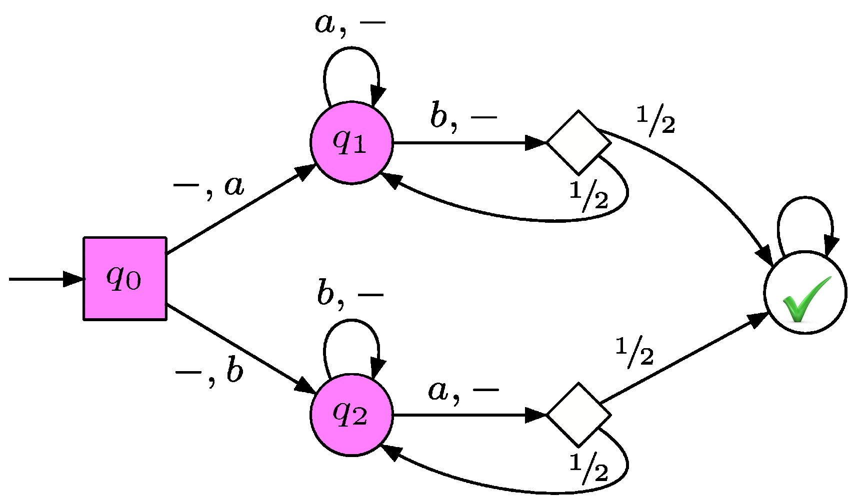
player 1 partial

player 2 perfect

When belief is $\{q_1, q_2\}$, alternate a and b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

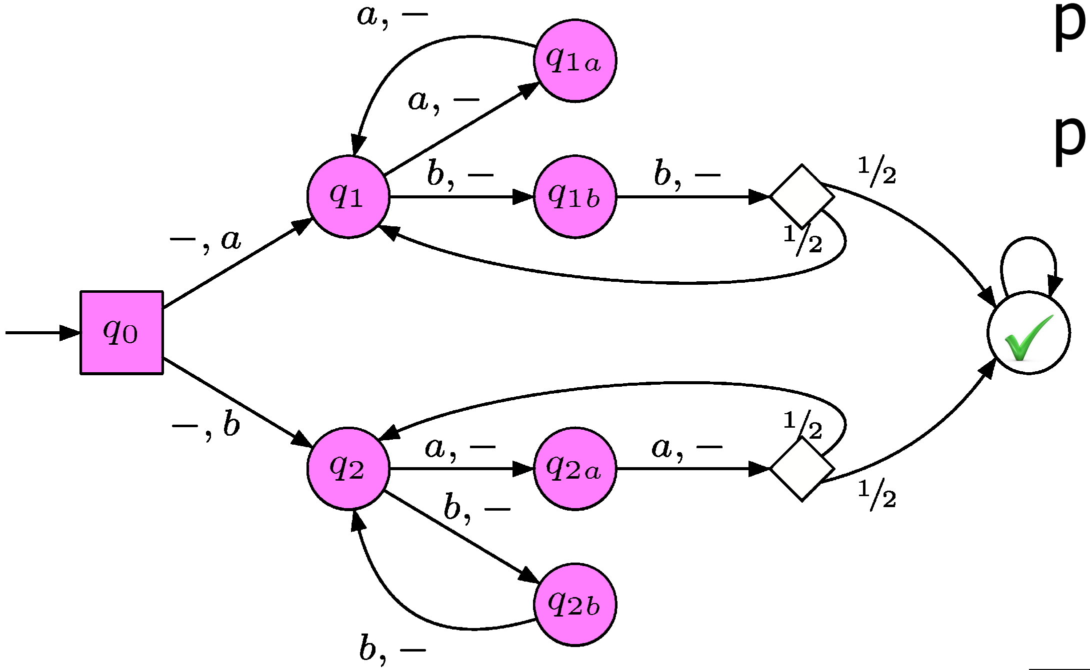
player 2 perfect

When belief is $\{q_1, q_2\}$, alternate a and b

This strategy is almost-sure winning !

When belief fails (2/2)

Using the trick of "repeated actions" we construct an example where belief-only randomized action-invisible strategies are not sufficient (for almost-sure winning)



player 1 partial

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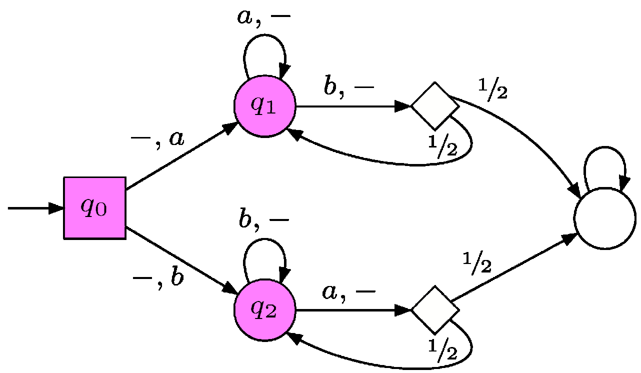

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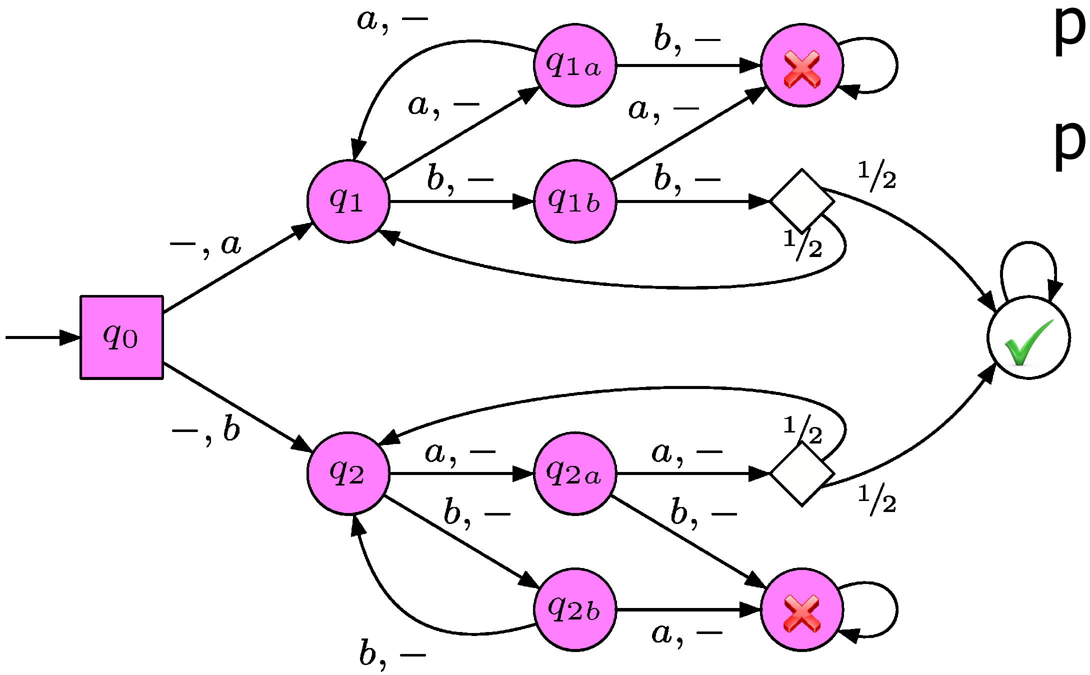

player 1 partial

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When belief fails (2/2)

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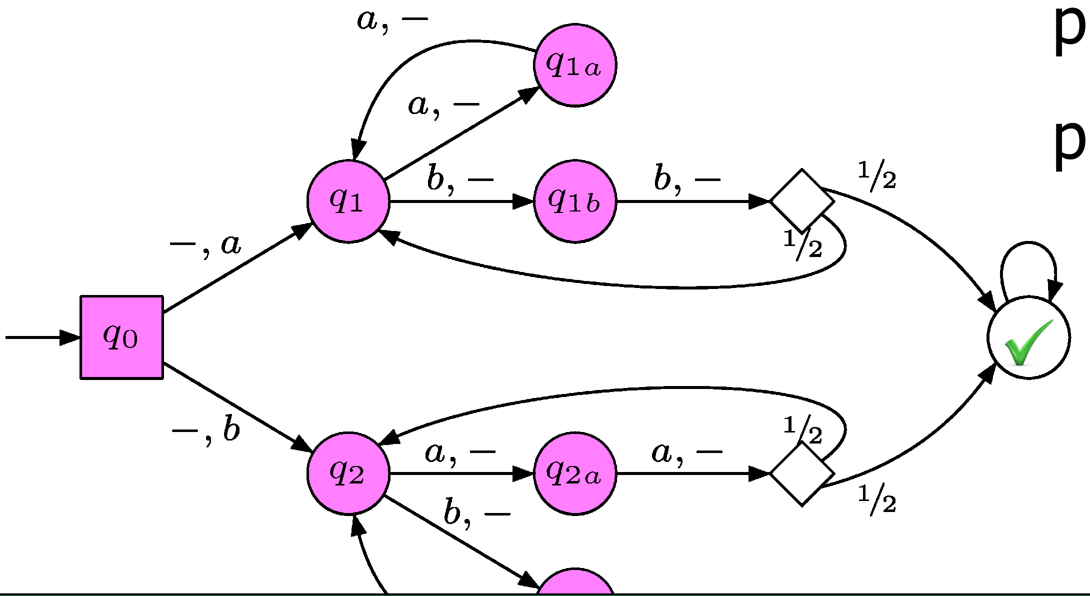


player 1 partial

player 2 perfect

Almost-sure winning requires to play pure strategy,
with more-than-belief memory !

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR’06]**	memoryless **[BGG’09]**	exponential (belief) **[BGG’09]**
rand. act.-inv.	exponential (belief) **[CDHR’06(remark), GS’09]**		exponential (belief) **[GS’09]**
pure	?	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	?	?	?

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (belief not sufficient)		?
pure	exponential (belief not sufficient)	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (belief not sufficient)	?	?

Pure Strategies: Player 1 Perfect, Player 2 Partial

Pl1 Perfect, Pl 2 Partial :
Stochastic, Randomized.
Memoryless

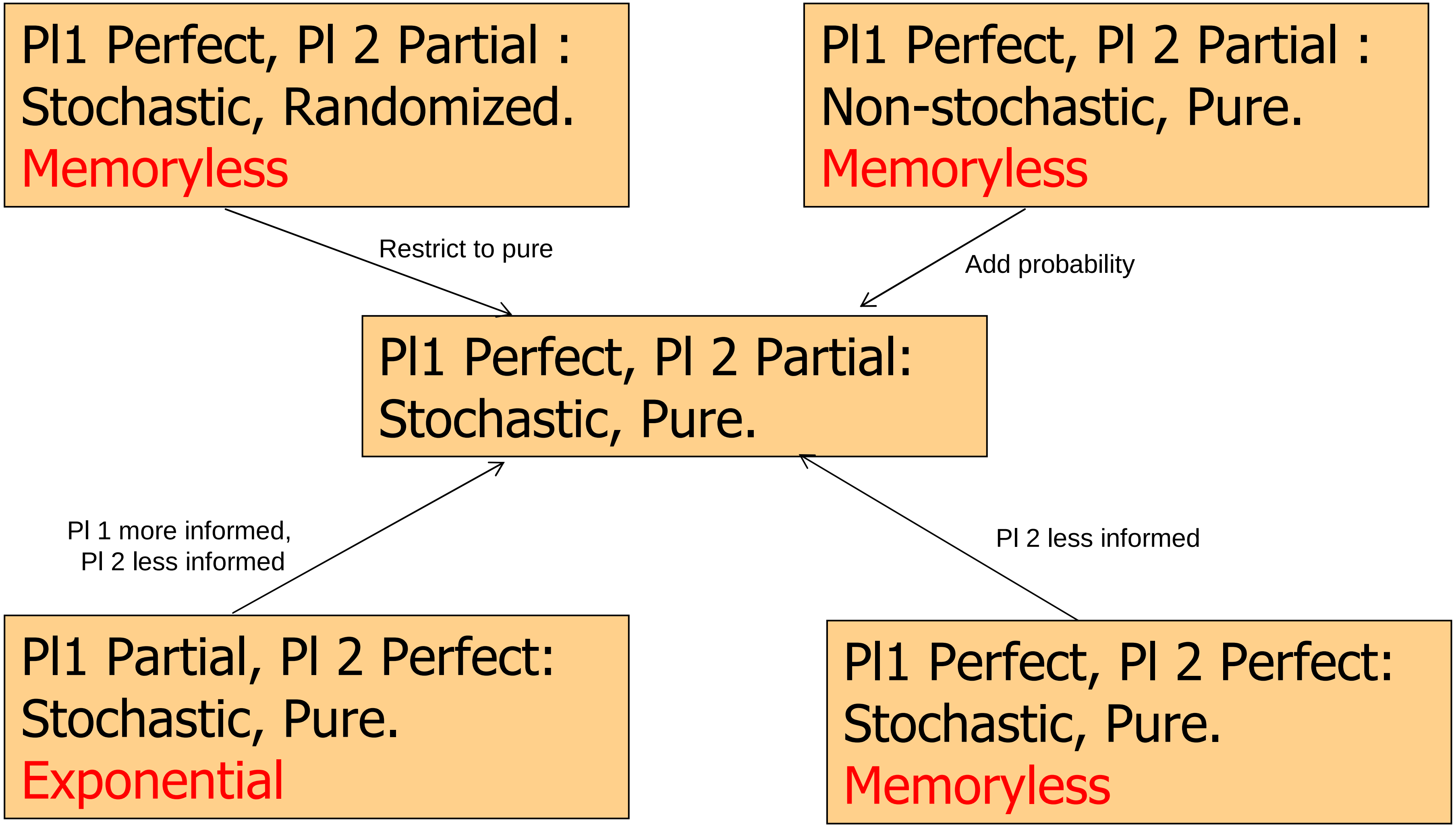
Pl1 Perfect, Pl 2 Partial :
Non-stochastic, Pure.
Memoryless

Pl1 Perfect, Pl 2 Partial:
Stochastic, Pure.

Pl1 Partial, Pl 2 Perfect:
Stochastic, Pure.
Exponential

Pl1 Perfect, Pl 2 Perfect:
Stochastic, Pure.
Memoryless

Pure Strategies: Player 1 Perfect, Player 2 Partial



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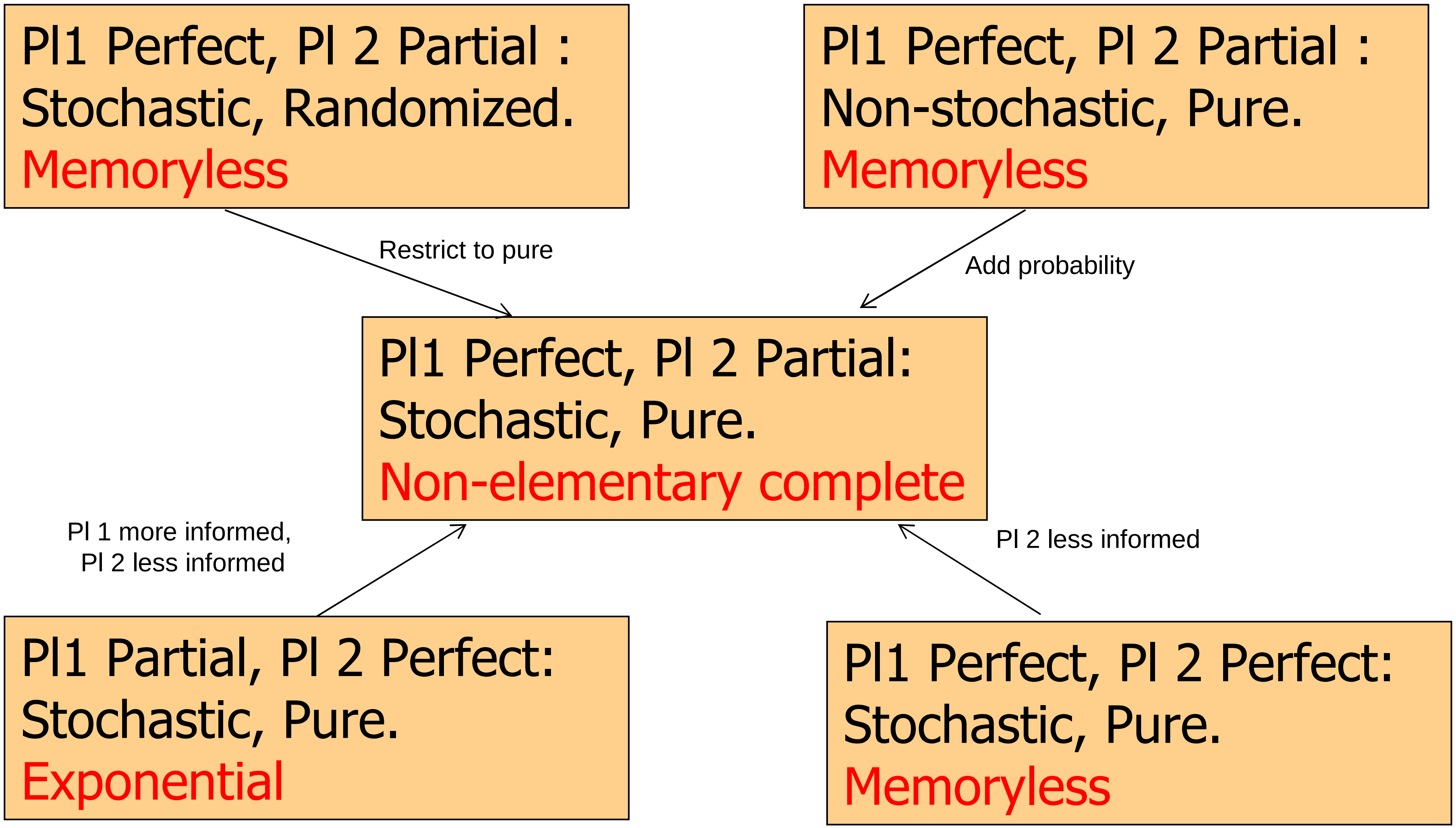

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rand. act.-inv.	exponential (belief not sufficient)		?
pure	exponential (belief not sufficient)	non-elementary complete	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
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rand. act.-inv.	exponential (belief not sufficient)		?
pure	exponential (belief not sufficient)	non-elementary complete	?

Positive	player 1 partial player 2 perfect	[illegible]	[illegible]
rand. act.-vis.	memoryless	[illegible]	[illegible]
rand. act.-inv.	memoryless		memoryless
pure	exponential (belief not sufficient)	non-elementary complete	?

Player 1 wins from more states, but needs more memory !

Player 1 perfect, player 2 partial

Memory of **non-elementary** size for pure strategies

- lower bound: simulation of counter systems with increment and division by 2
- upper bound:
  positive: non-elementary counters simulate randomized strategies
  almost-sure: reduction to iterated positive

Counter systems with {+1,÷2} require non-elementary counter value for reachability

$$2^{2^{\cdot^{\cdot^{2}}}} \Big\} \text{height } n$$

Player 1 perfect, player 2 partial

More information:

- Win from more places.
- Winning strategy is very hard to implement.

Information is useful, but ignorance is bliss ☺ !

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
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Pure ≡ randomized-invisible

Equivalence of the decision problems for almost-sure reach with **pure** strategies and **rand. act.-inv.** strategies

- Reduction of rand. act.-inv. to pure
  choice of a subset of actions (support of prob. dist.)
- Reduction of pure to rand. act.-inv.
  repeated-action trick (holds for almost-sure only)

It follows that the memory requirements for pure hold for rand. act.-inv. as well !

New results

Reachability - Memory requirement (for player 1)

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Beliefs

Three prevalent beliefs:

• Belief is sufficient.

• Randomized action invisible or visible almost same.

• The general case memory is similar (or in some cases exponential blow up) as compared to the one-sided case.

Belief fails !

Summary of our results

Pure strategies (for almost-sure and positive):

- player 1 partial: exponential memory, belief not sufficient
- player 1 perfect: non-elementary memory (complete)
- 2-sided: finite, at least non-elementary memory

  (as compared to previously claimed exponential upper bound)

Randomized action-invisible strategies (for almost-sure) :

- player 1 partial: exponential memory, belief not sufficient
- 2-sided: finite, at least non-elementary memory

More results & open questions

Computational complexity for 1-sided:

- Player 1 partial: reduction to Büchi game, EXPTIME-complete
- Player 2 partial: non-elementary complexity

  (note: almost-sure Büchi is poly-time equivalent to almost-sure reachability, positive Büchi is undecidable [BBG08])

Open questions:

- Whether non-elementary size memory is sufficient in 2-sided
- Exact computational complexity

Details

Details can be found in:

[CD11] **Chatterjee, Doyen.** ***Partial-Observation Stochastic Games: How to Win when Belief Fails*****. CoRR abs/1107.2141, July 2011.**

Extended abstract @ LICS'12:

[CD12] **Chatterjee, Doyen.** ***Partial-Observation Stochastic Games: How to Win when Belief Fails*****. LICS'12, pp. 175-184, IEEE Press.**

References

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Other references:

[BBG08] Baier, Bertrand, Grösser. ***On Decision Problems for Probabilistic Büchi automata*****. FoSSaCS'08.**

[BGG09] Bertrand, Genest, Gimbert. ***Qualitative Determinacy and Decidability of Stochastic Games with Signals*****. LICS'09.**

[CDHR06] Chatterjee, Doyen, Henzinger, Raskin. ***Algorithms for ω-Regular games with Incomplete Information*****. CSL'06.**

[CDH10] Cristau, David, Horn. ***How do we remember the past in randomised strategies?*****. GANDALF'10.**

[GS09] Gripon, Serre. ***Qualitative Concurrent Stochastic Games with Imperfect Information*****. ICALP'09.**

[Paz71] Paz. ***Introduction to Probabilistic Automata*****. Academic Press 1971.**

Outline

- Game Model: example
- Challenges & Results: examples
- Solution insights: examples

Some proof ideas

► Skip

New results

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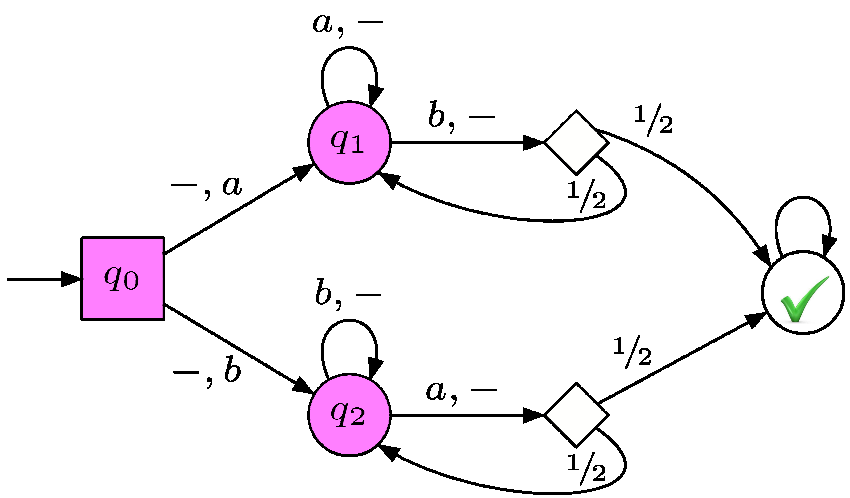
When belief fails

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



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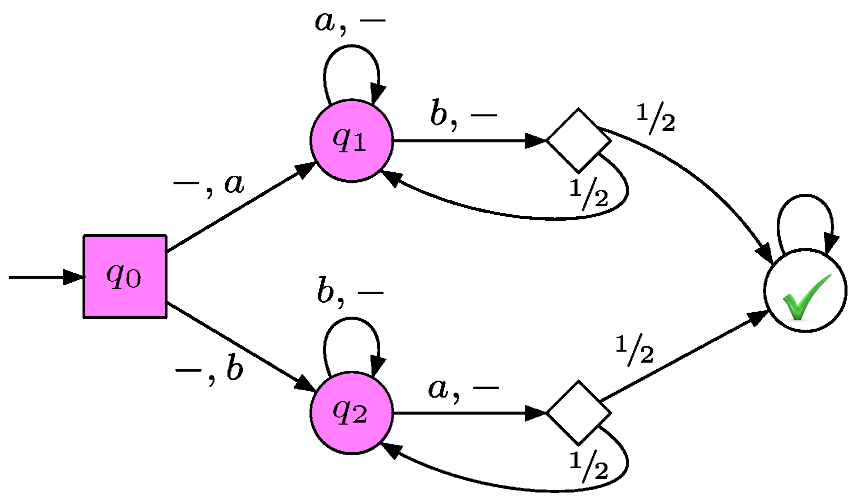

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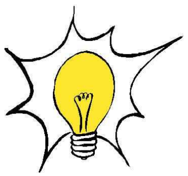
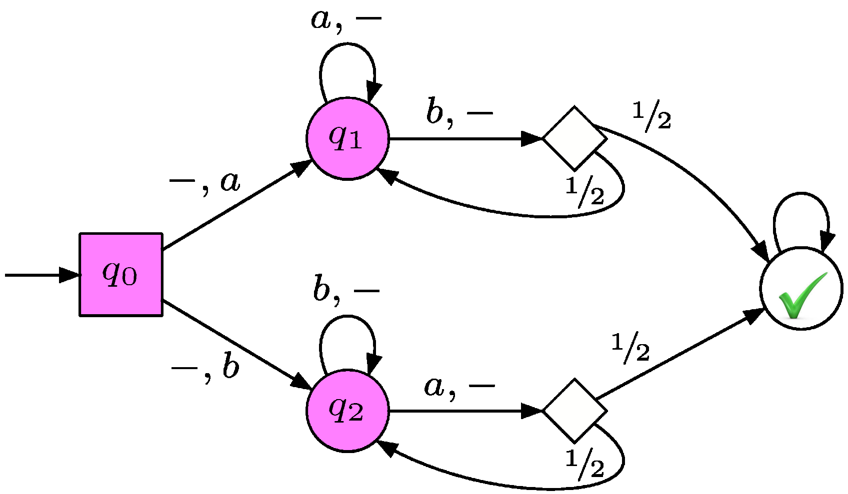
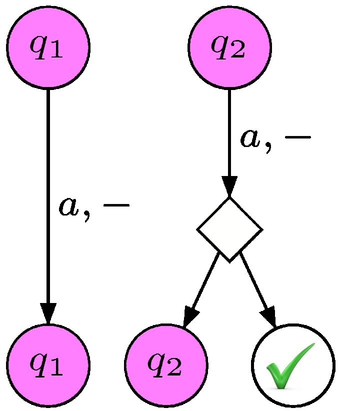


When belief is $\{q_1,q_2\}$, then ensure that the target is reached from **both** q_1 and q_2.

When belief fails





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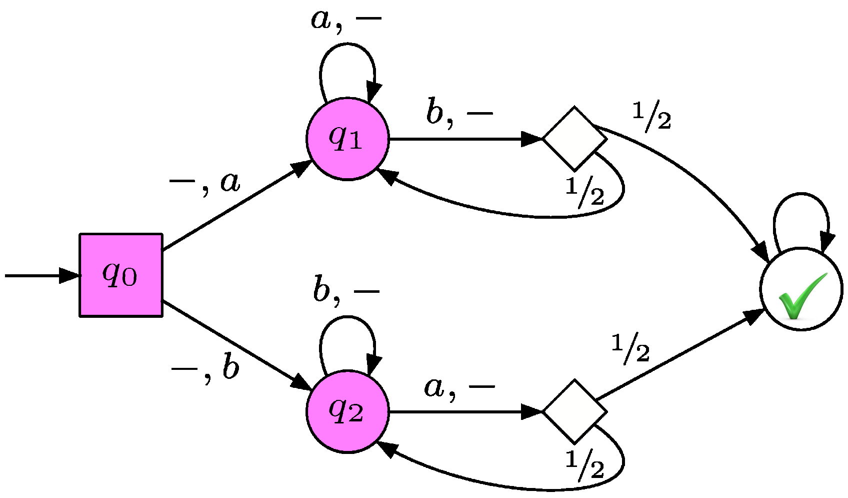

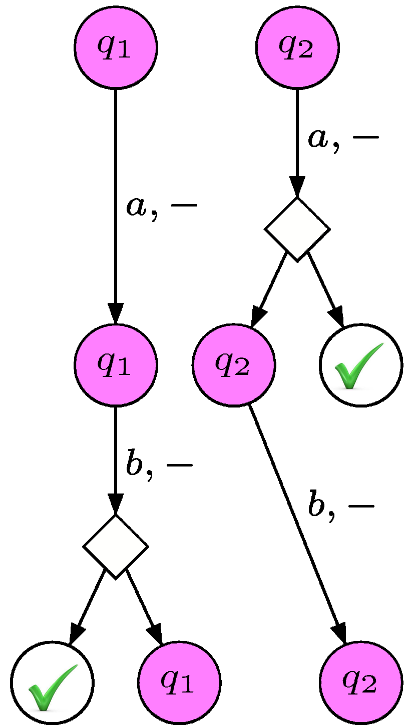

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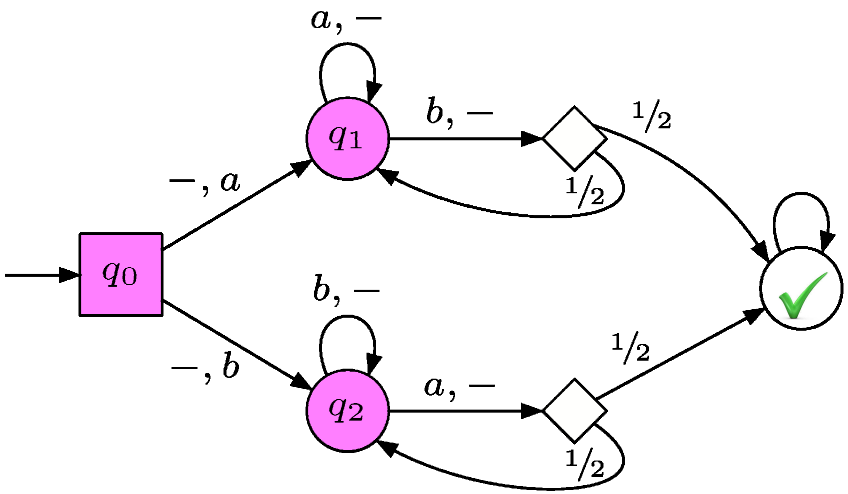

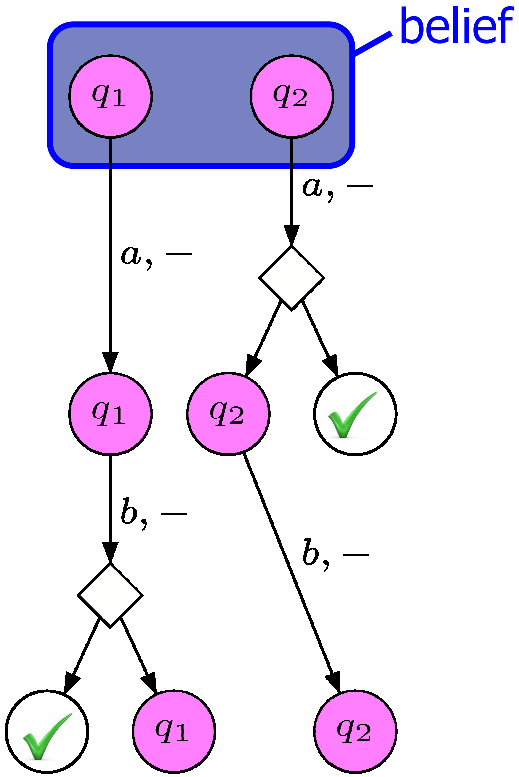

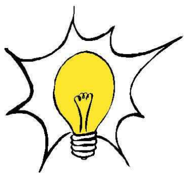

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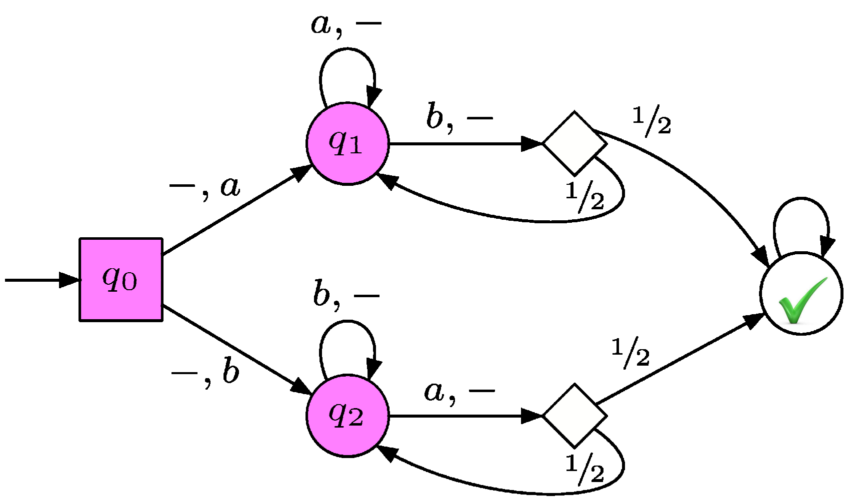
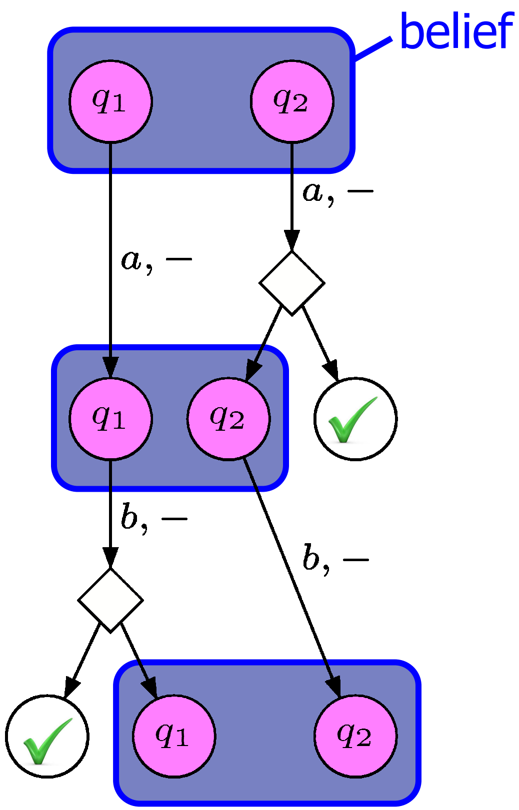
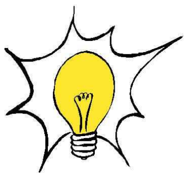


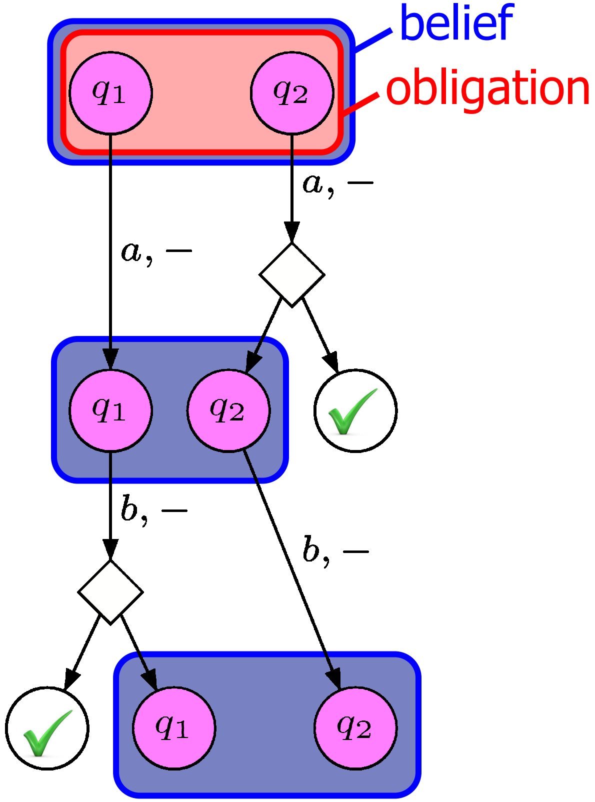


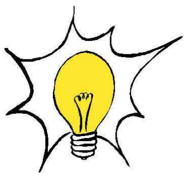


When belief is $\{q_1,q_2\}$, then ensure that the target is reached from **both** q_1 and q_2.

When belief fails

Obligation = from every state in belief, reach target with positive probability





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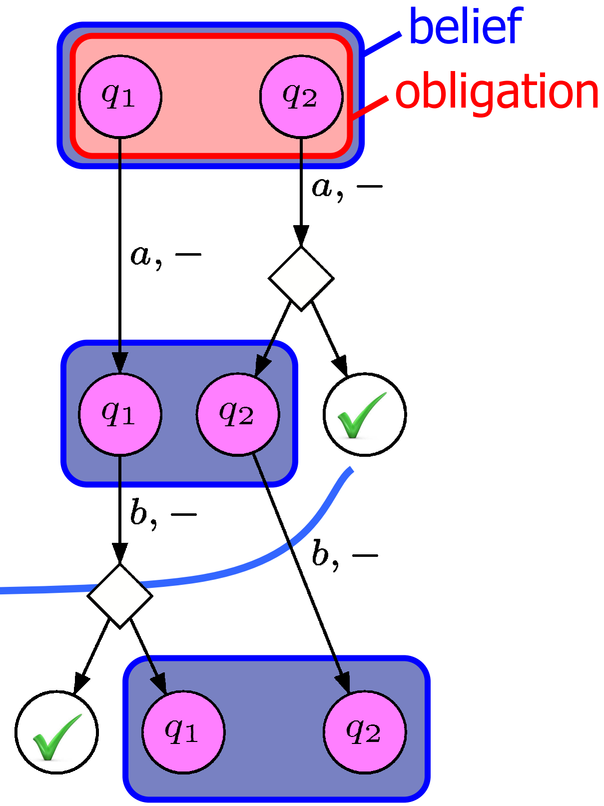
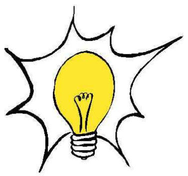


At this point, obligation of q_2 is satisfied



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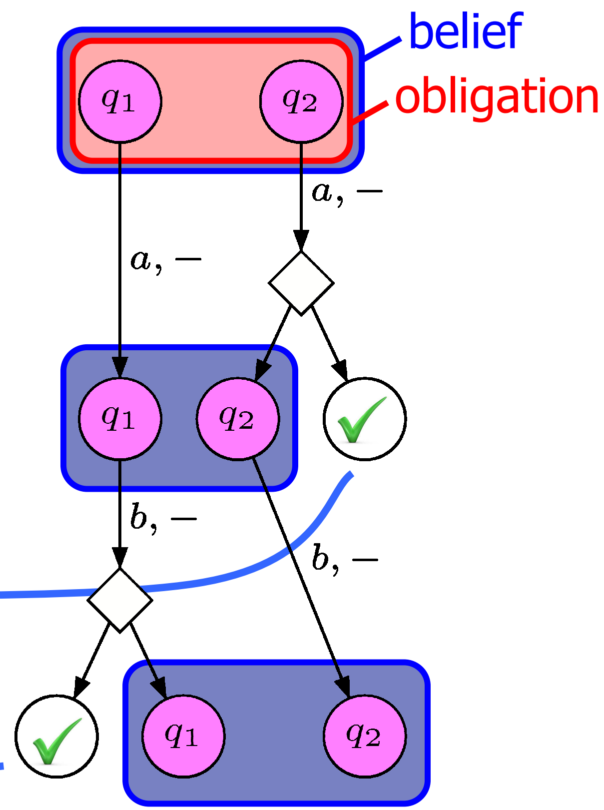

At this point, obligation of q_2 is satisfied

Here, obligation of q_1 is satisfied

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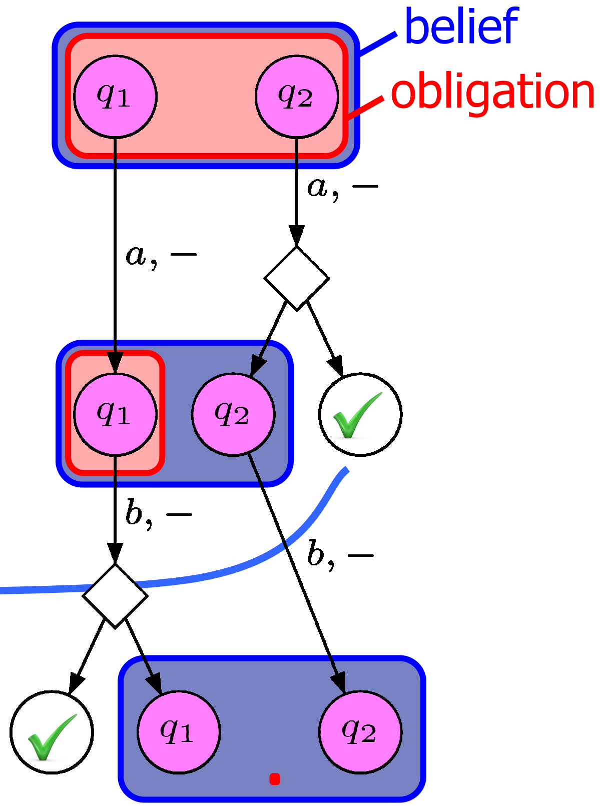

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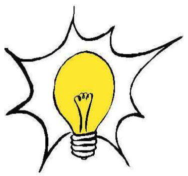

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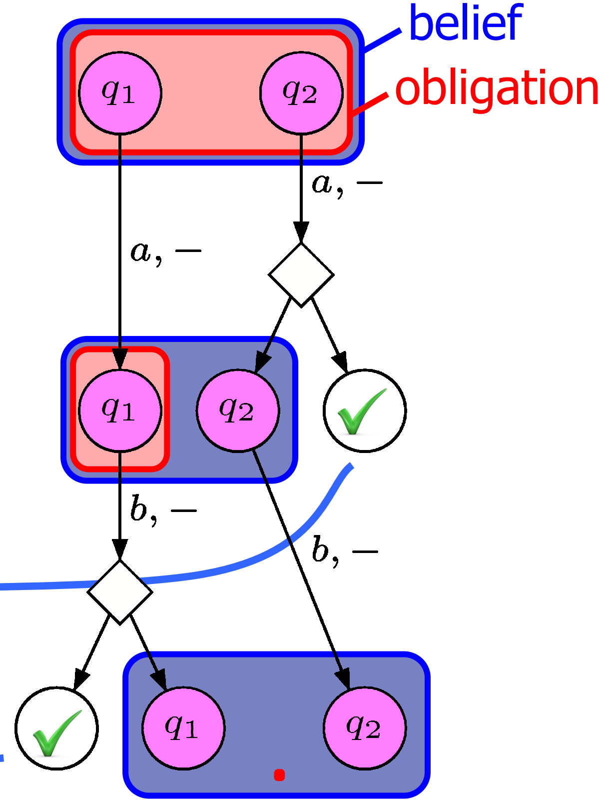


At this point, obligation of q_2 is satisfied

Here, obligation of q_1 is satisfied

Empty obligation set

⇔

All obligations fulfilled

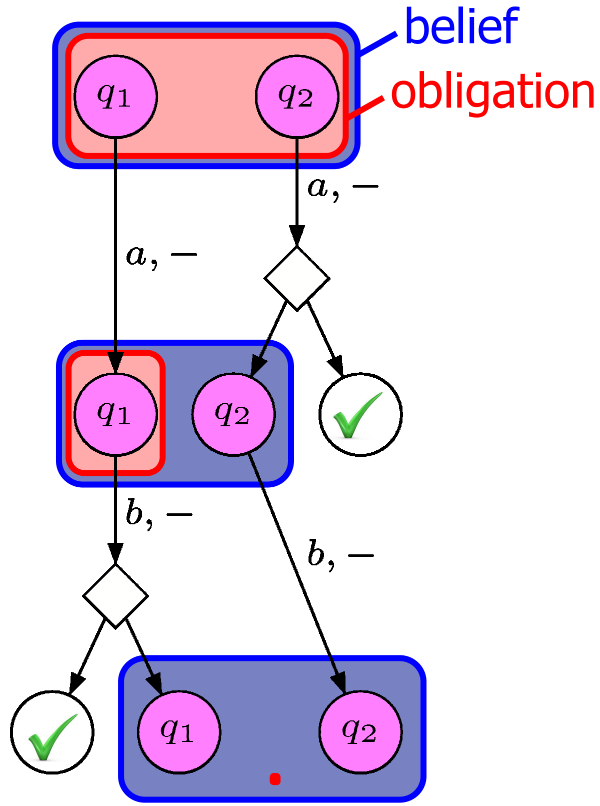
When belief fails

Positive reachability: ensure empty obligation once

Reachability condition

Almost-sure reachability: ensure empty obligation infinitely often (and recharge when empty)

Büchi condition



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$\Longleftrightarrow$

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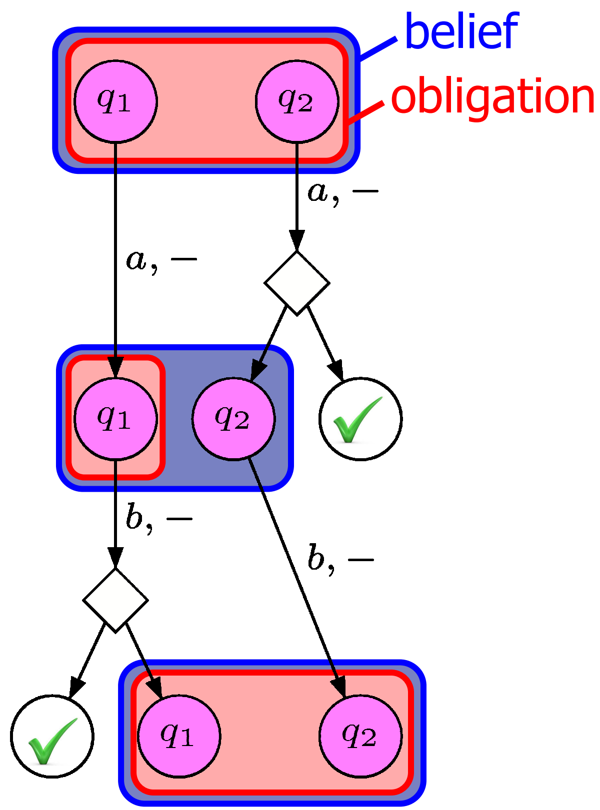

Empty obligation set

$\Leftrightarrow$

All obligations fulfilled

New results

Reachability - Memory requirement (for player 1)

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rand. act.-inv.	exponential (belief not sufficient)		finite (at least non-elementary)
pure	exponential (belief not sufficient)	non-elementary complete	finite (at least non-elementary)

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rand. act.-vis.	memoryless	memoryless	memoryless
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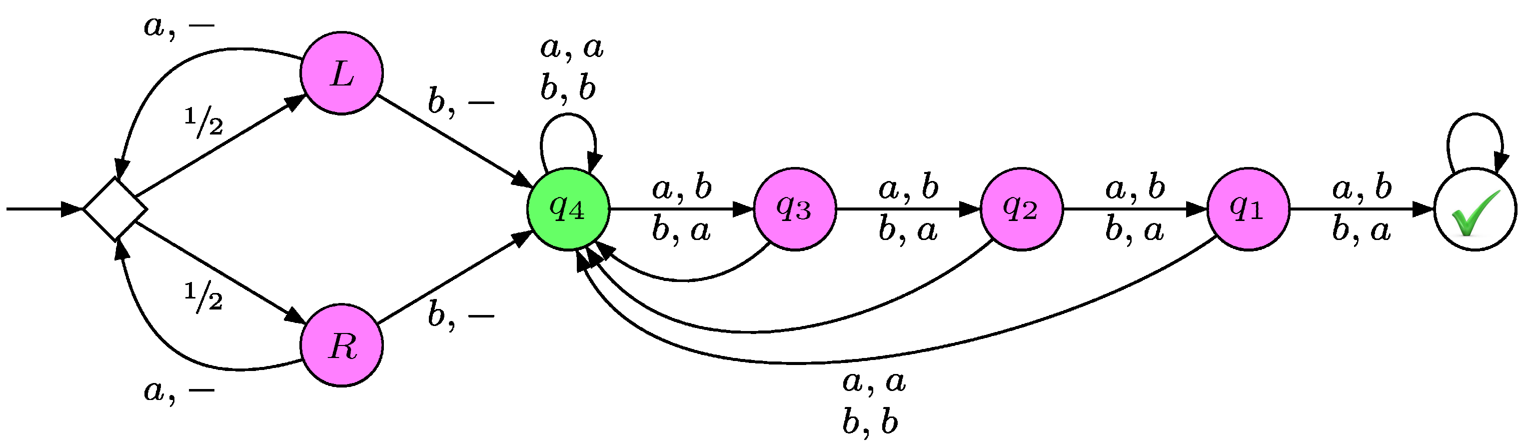
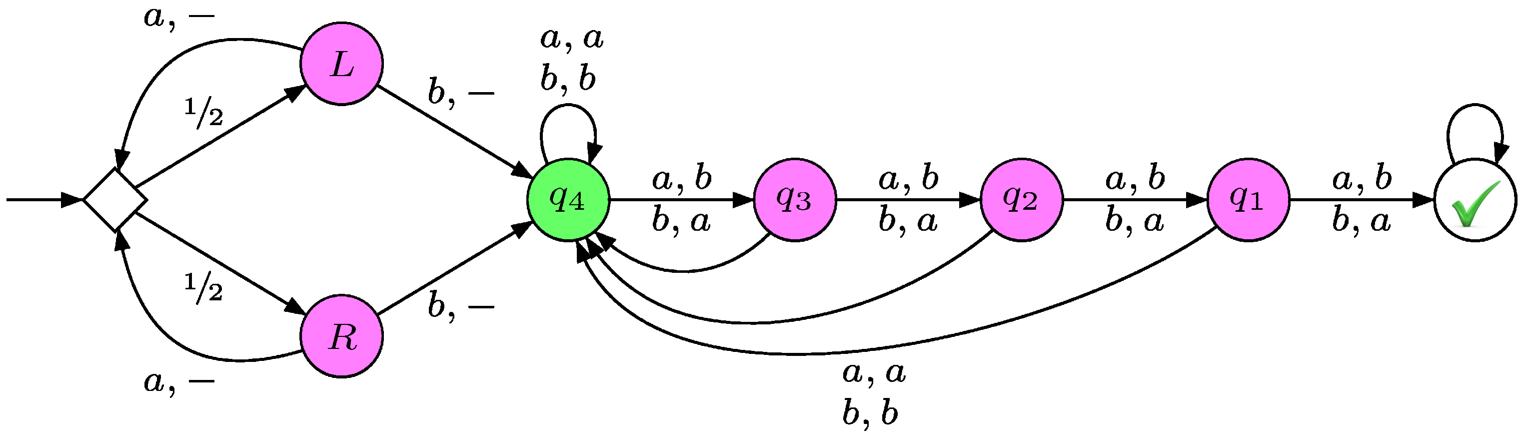
Player 1 perfect, player 2 partial



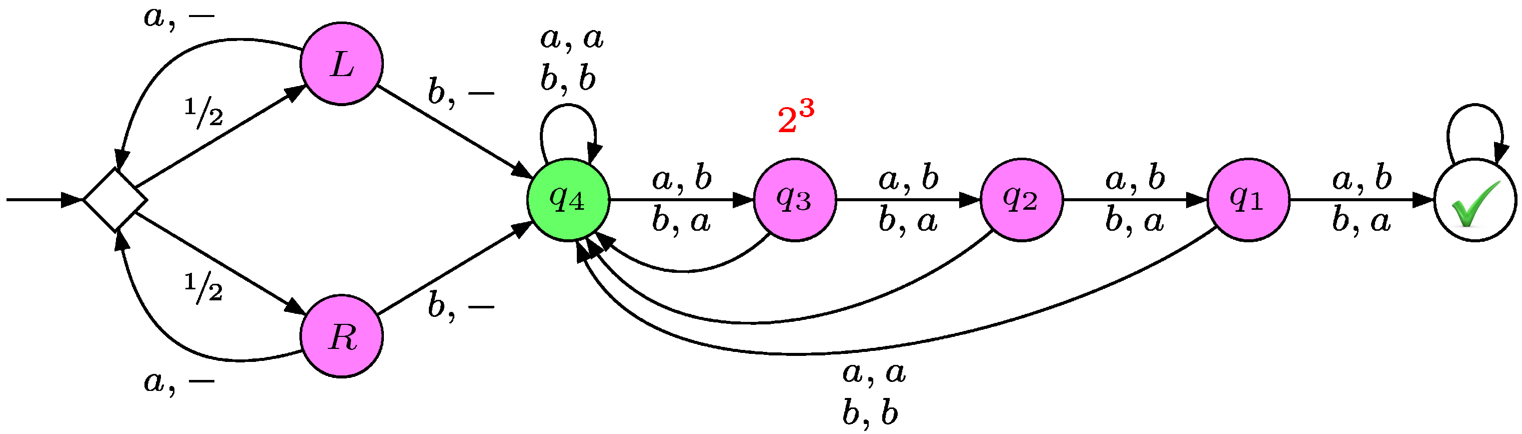
Player 1 perfect, player 2 partial



1. Play 3 times a to generate 2^4 indistinguishable paths (with observation )
2. Play b, then in q_4 play a over half of the paths ... er the rest
3. In q_i, play analogously, and ensure 2^{i-1} paths ... 1
4. Reach q_0 with positive probability

$LLLL$
$LLLR$
$LLRL$
$LLRR$
$LRLL$
...

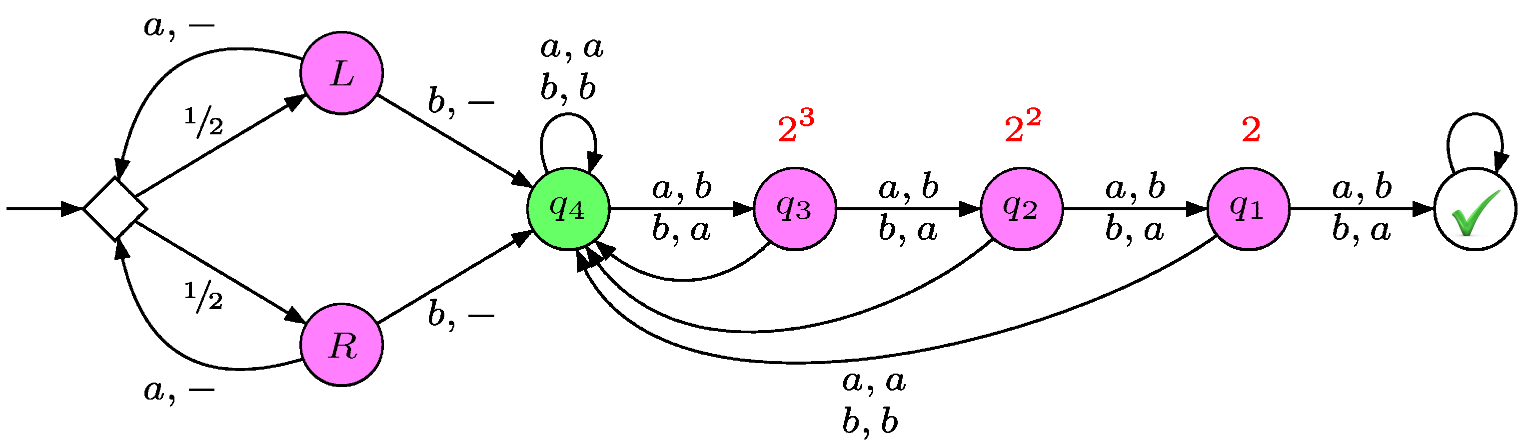
Player 1 perfect, player 2 partial



1. Play 3 times a to generate 2^4 indistinguishable paths
2. Play b, then in q_4 play a over half of the paths, b over the rest
3. In q_i, play analogously, and ensure 2^{i-1} paths
4. Reach q_0 with positive probability

$LLLLq_4$
$LLLRq_4$
$LLRLq_4$
...

Player 1 perfect, player 2 partial

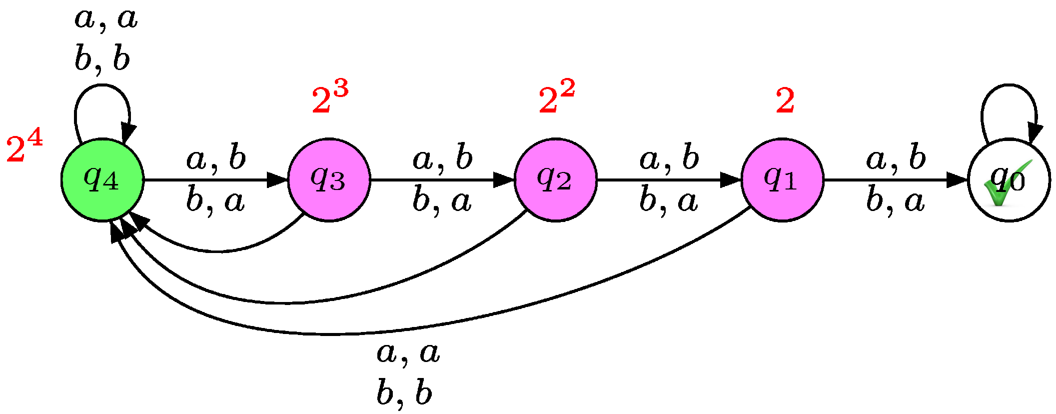


1. Play 3 times a to generate 2^4 indistinguishable paths
2. Play b, then in q_4 play a over half of the paths, b over the rest
3. In q_i, play analogously, and ensure 2^{i-1} paths in q_{i-1}
4. Reach q_0 with positive probability

Player 1 perfect, player 2 partial

$$\left.\begin{array}{l} LLLLq_4 \\ LLLRq_4 \\ LLRLq_4 \\ LLRRq_4 \\ LRLLq_4 \\ LRLRq_4 \\ LRRLq_4 \\ LRRRq_4 \end{array}\right\} a$$

$$\left.\begin{array}{l} RLLLq_4 \\ RLLRq_4 \\ RLRLq_4 \\ RLRRq_4 \\ RRLLq_4 \\ RRLRq_4 \\ RRRLq_4 \\ RRRRq_4 \end{array}\right\} b$$



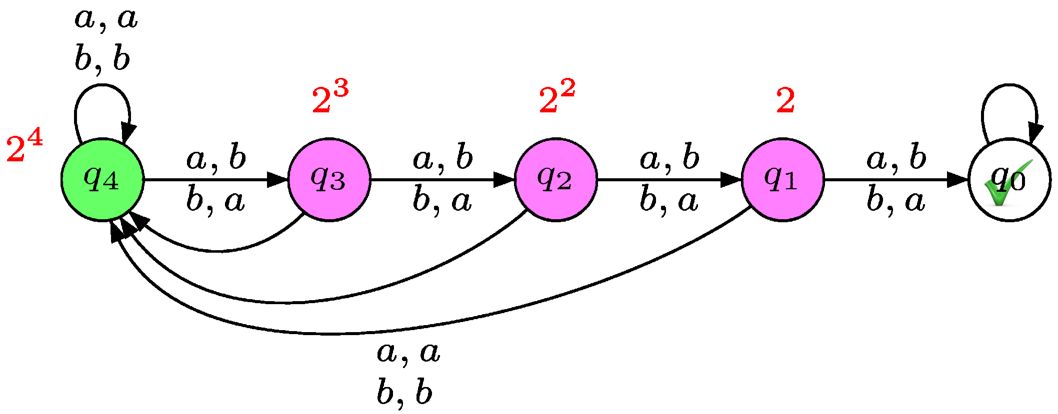
Player 1 perfect, player 2 partial

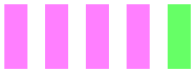
$LLLLq_4$
$LLLRq_4$
$LLRLq_4$
$LLRRq_4$
$LRLLq_4$
$LRLRq_4$
$LRRLq_4$
$LRRRq_4$

} a, a ✗

$RLLLq_4$
$RLLRq_4$
$RLRLq_4$
$RLRRq_4$
$RRLLq_4$
$RRLRq_4$
$RRRLq_4$
$RRRRq_4$

} b, a





q_3

Player 1 perfect, player 2 partial

$LLLLq_4$
$LLLRq_4$
$LLRLq_4$
$LLRRq_4$
$LRLLq_4$
$LRLRq_4$
$LRRLq_4$
$LRRRq_4$

a, a ✗

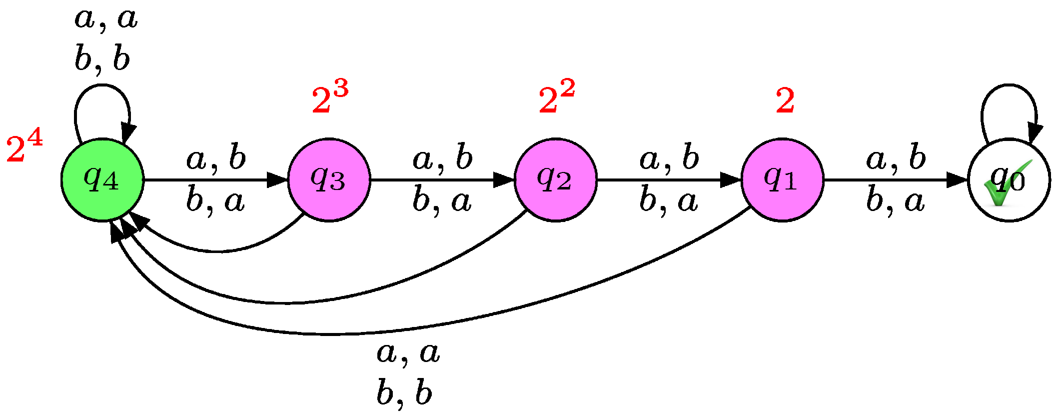
$RLLLq_4$
$RLLRq_4$
$RLRLq_4$
$RLRRq_4$
$RRLLq_4$
$RRLRq_4$
$RRRLq_4$
$RRRRq_4$

b, a

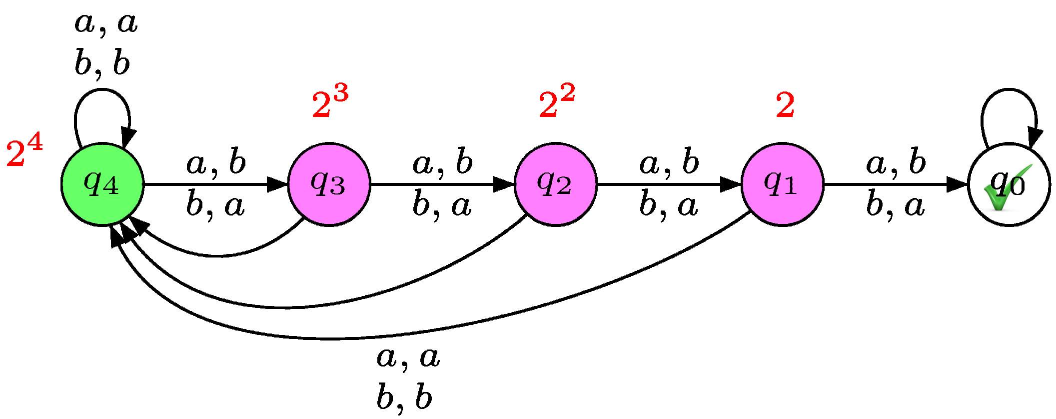
a, a ✗

b, a

q_3 q_2



Player 1 perfect, player 2 partial



$LLLLq_4$
$LLLRq_4$
$LLRLq_4$
$LLRRq_4$
$LRLLq_4$
$LRLRq_4$
$LRRLq_4$
$LRRRq_4$
} a, a ✗

$RLLLq_4$
$RLLRq_4$
$RLRLq_4$
$RLRRq_4$
} b, a } a, a ✗

$RRLLq_4$
$RRLRq_4$
$RRRLq_4$
$RRRRq_4$
} b, a } b, a } a, a ✗

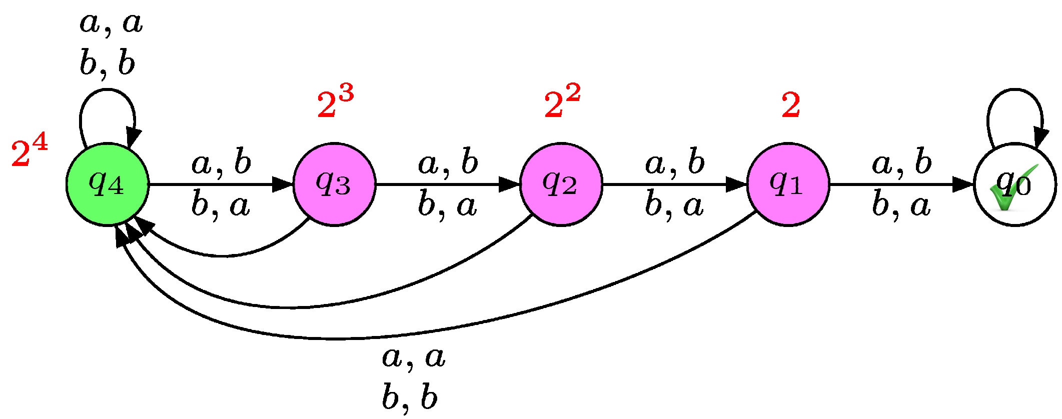
} b, a } b, a } b, a } a, a ✗ } b, a ✓

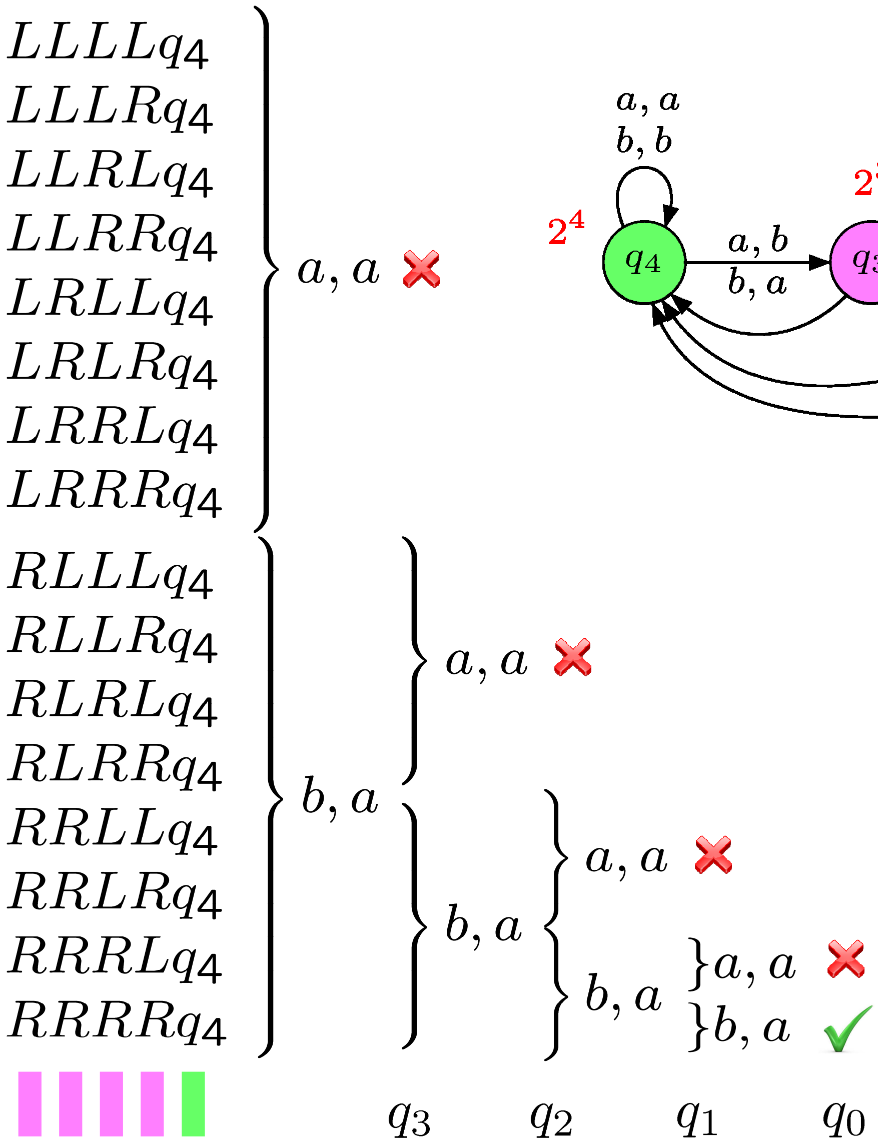
q_3 q_2 q_1 q_0

1 out of 16 paths reaches q_0

Winning probability = 1/16 !

Player 1 perfect, player 2 partial

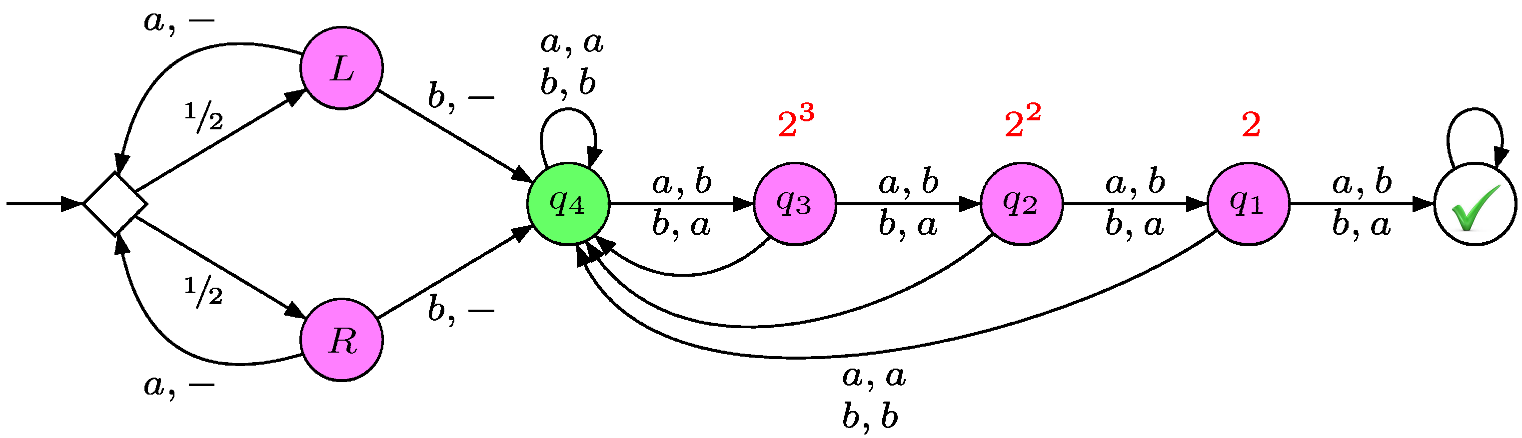




1 out of 16 paths reaches q_0

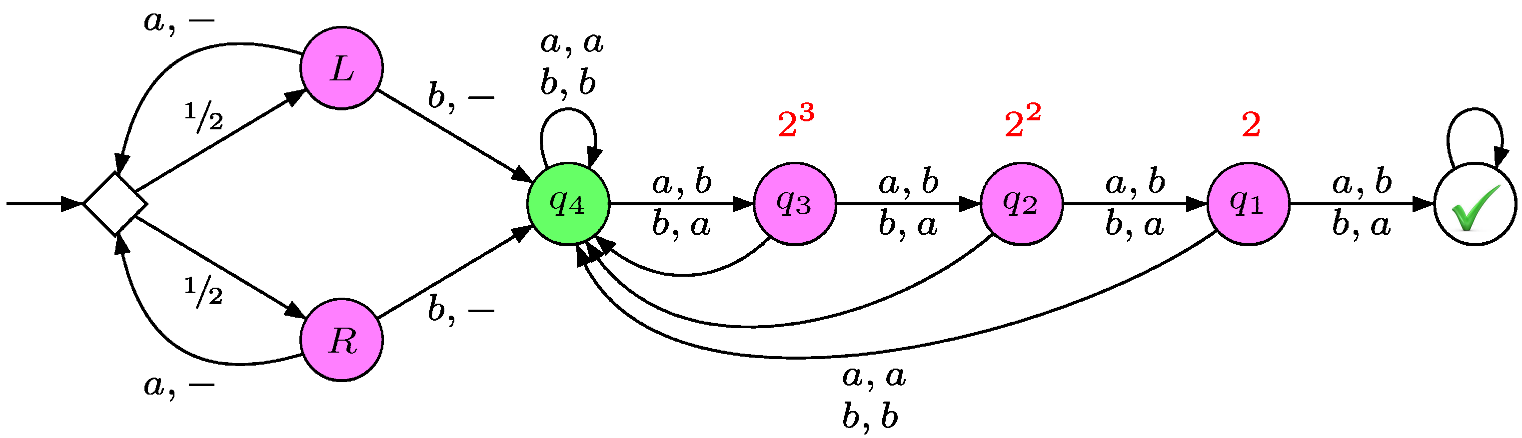
Winning probability = 1/16 !

Player 1 perfect, player 2 partial



1. Play 3 times a to generate 2^4 indistinguishable paths
2. Play b, then in q_4 play a over half of the paths, b over the rest
3. In q_i, play analogously, and ensure 2^{i-1} paths in q_{i-1}
4. Reach q_0 with positive probability ...

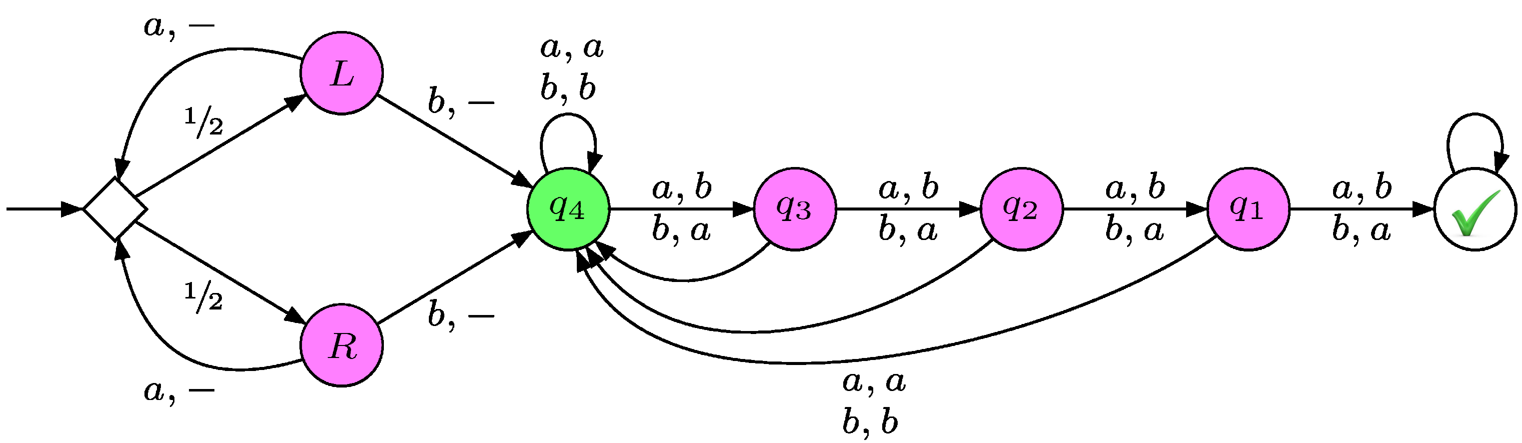
Player 1 perfect, player 2 partial



1. Play 3 times a to generate 2^4 indistinguishable paths
2. Play b, then in q_4 play a over half of the paths, b over the rest
3. In q_i, play analogously, and ensure 2^{i-1} paths in q_{i-1}
4. Reach q_0 with positive probability ...

   ... using exponential memory

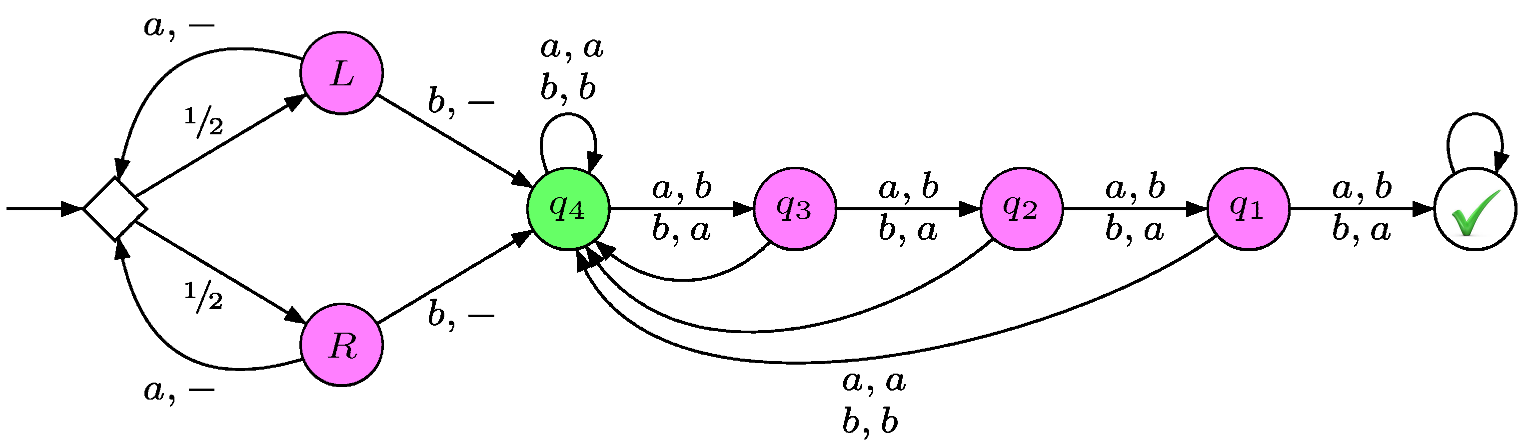
Player 1 perfect, player 2 partial



Flavor of a counter system with:

- increment,
- division by 2 (size of alphabet)

Player 1 perfect, player 2 partial

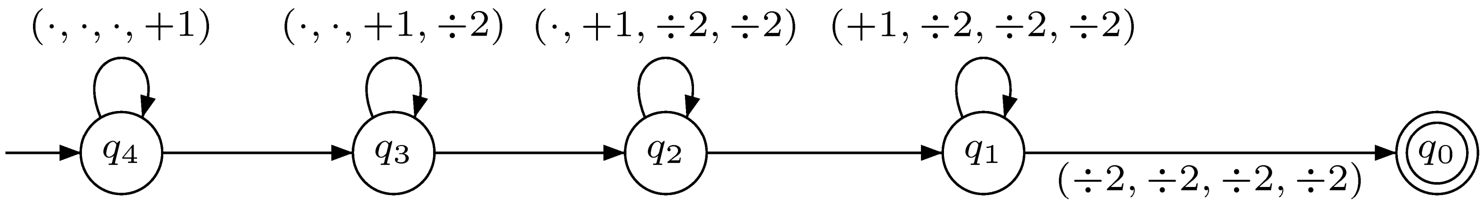


Show that:

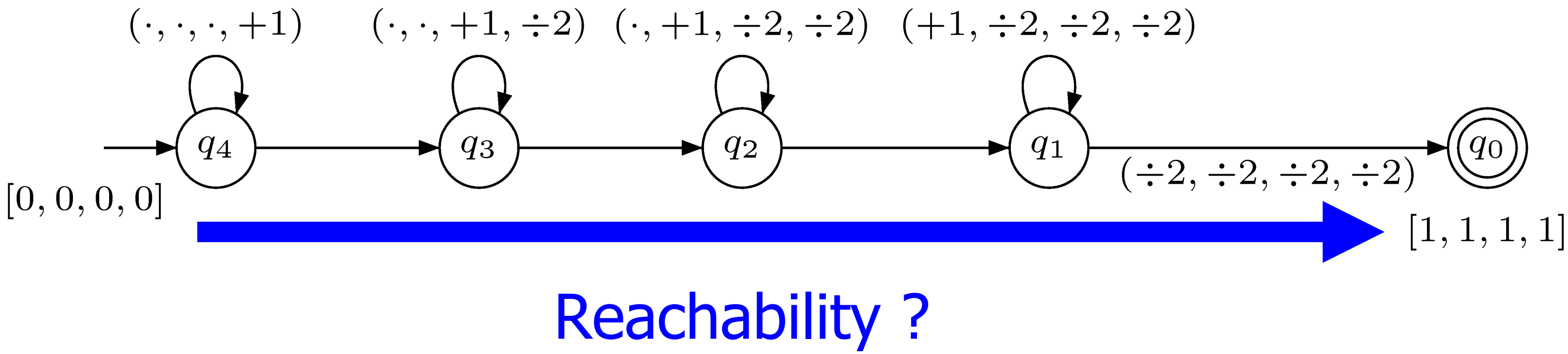
1. games can simulate increment and division by 2
2. Such counter systems require non-elementary counter value for reachability

$$2^{2^{\cdot^{\cdot^{2}}}} \Big\} \text{height } n$$

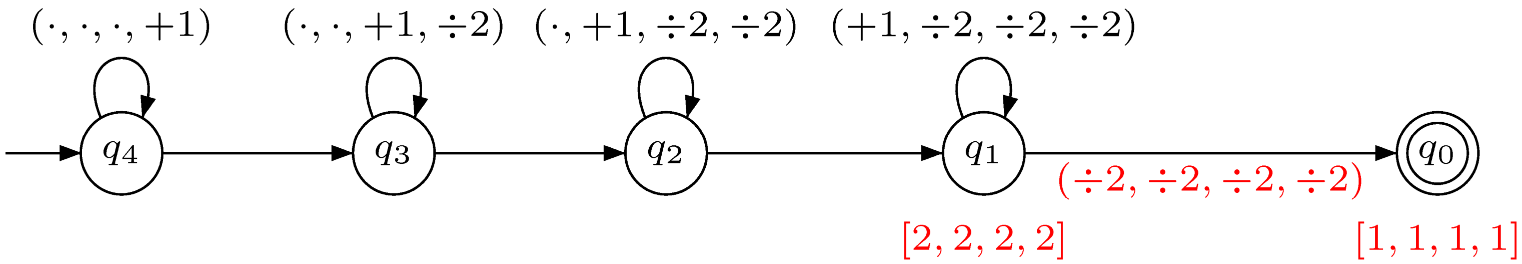
Counter system with {+1,÷2}



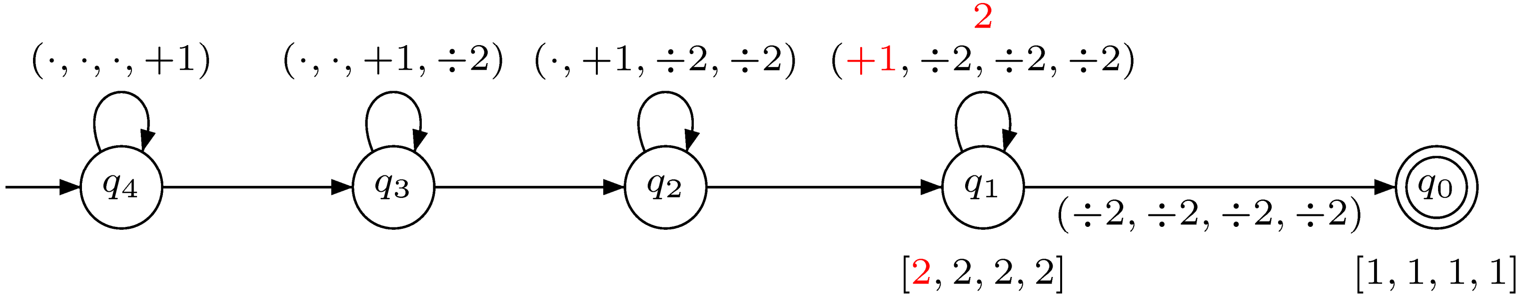
Counter system with {+1,÷2}



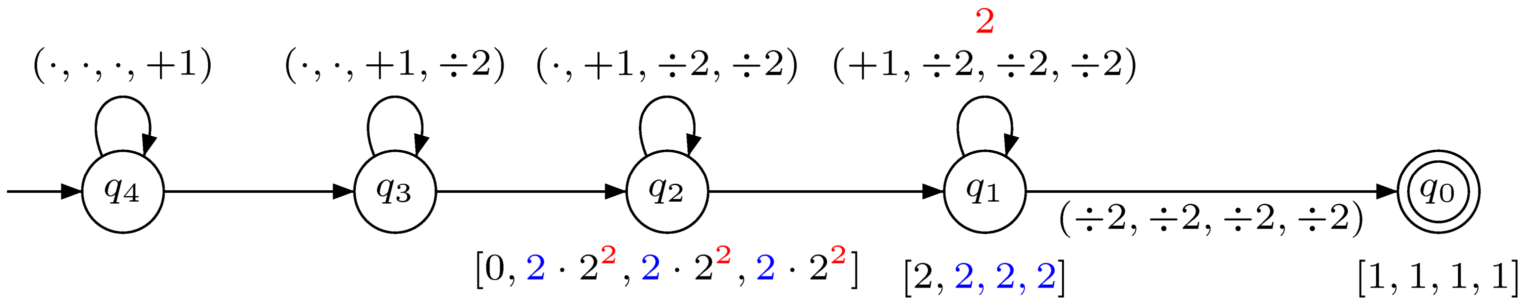
Counter system with {+1,÷2}



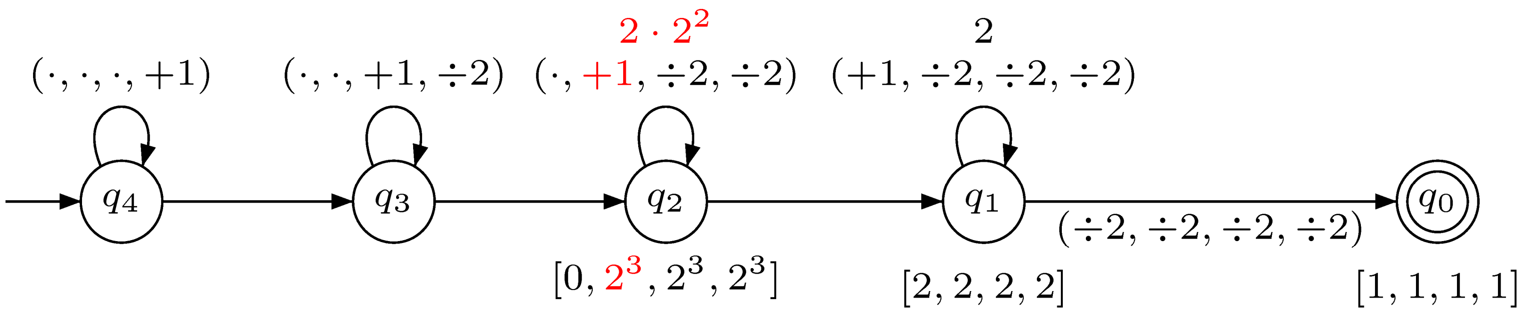
Counter system with {+1,÷2}



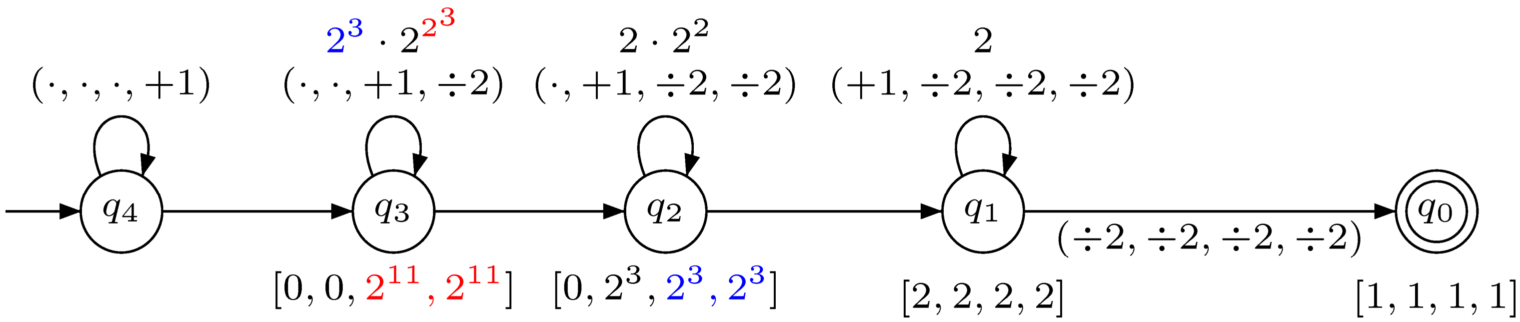
Counter system with {+1,÷2}



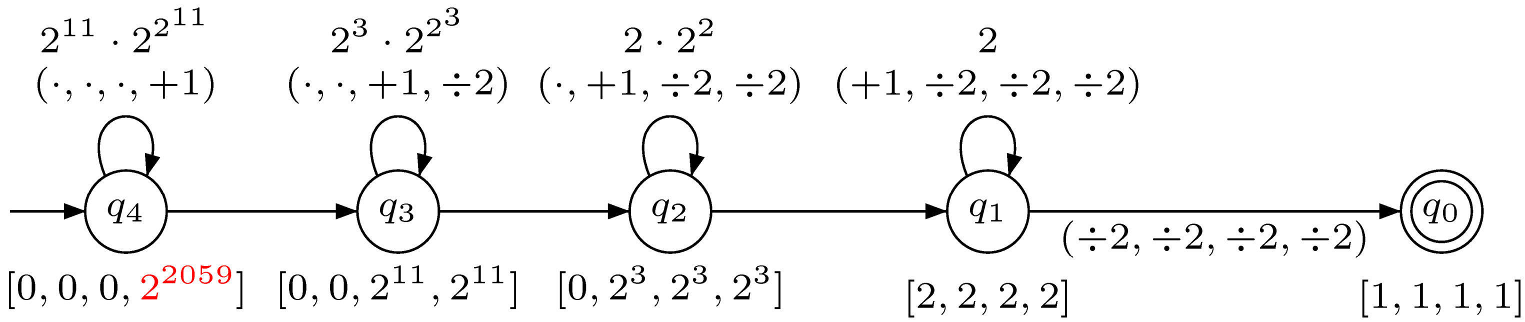
Counter system with {+1,÷2}



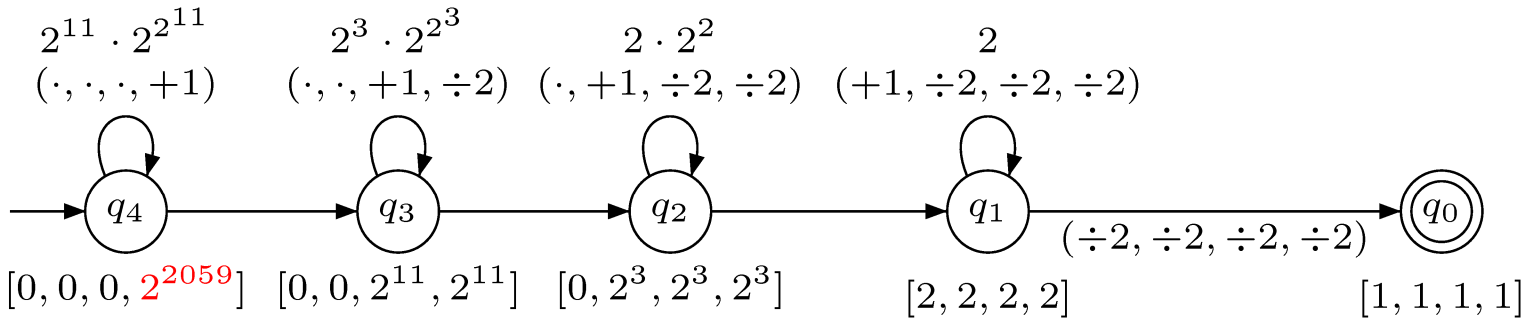
Counter system with {+1,÷2}



Counter system with {+1,÷2}

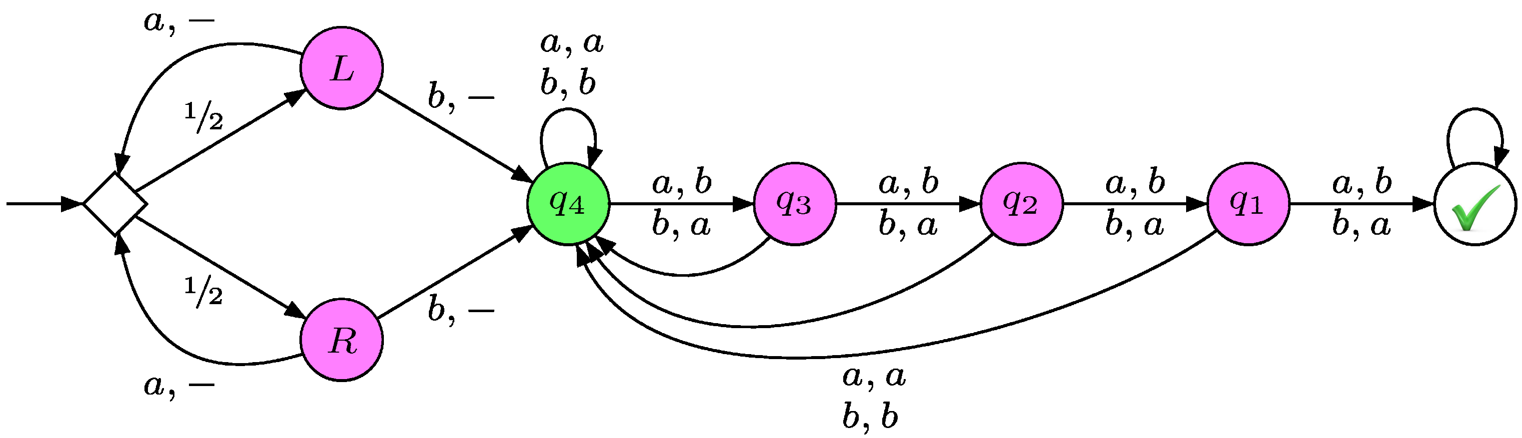


Counter system with {+1,÷2}



non-elementary growth !

Player 1 perfect, player 2 partial

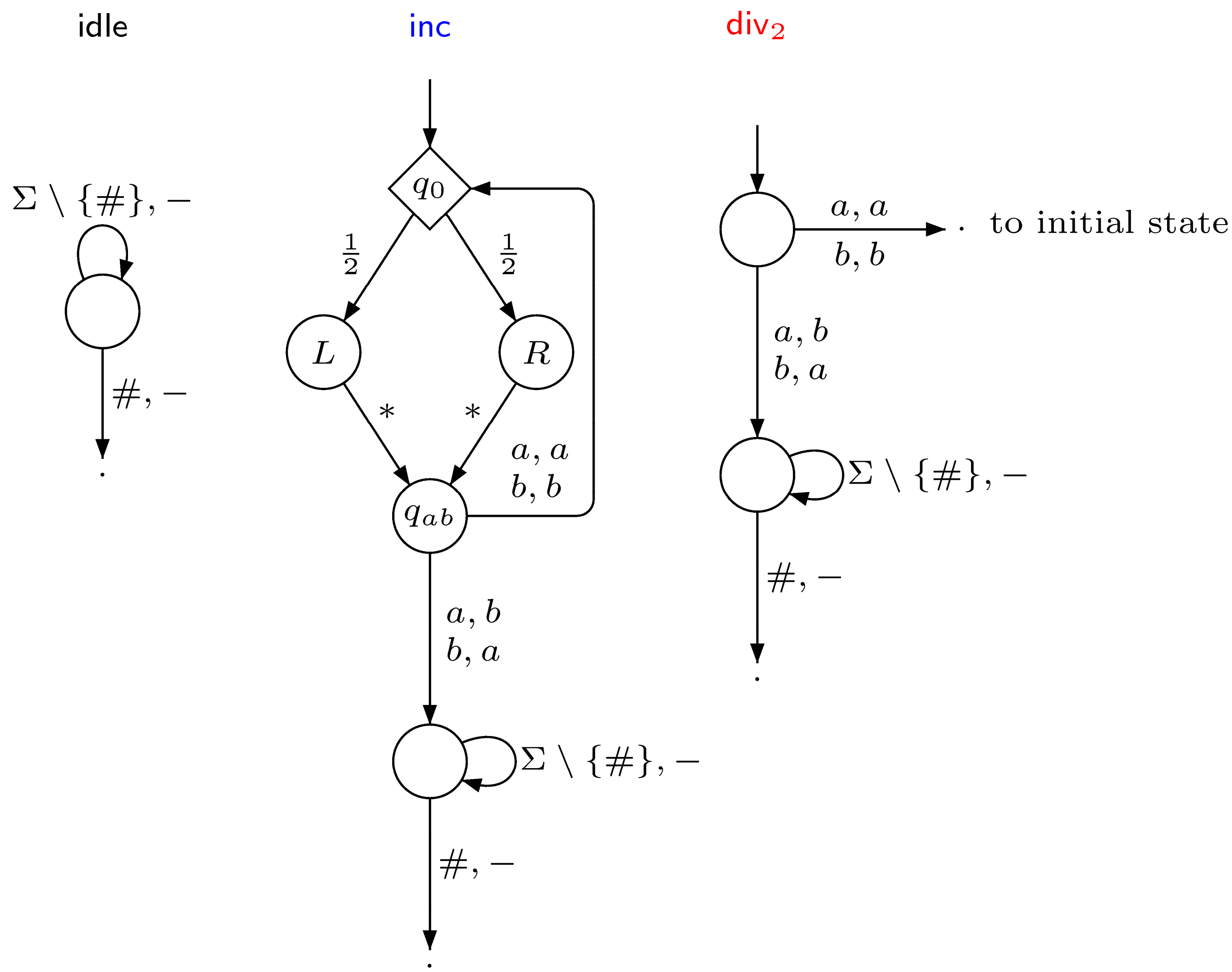


Show that:

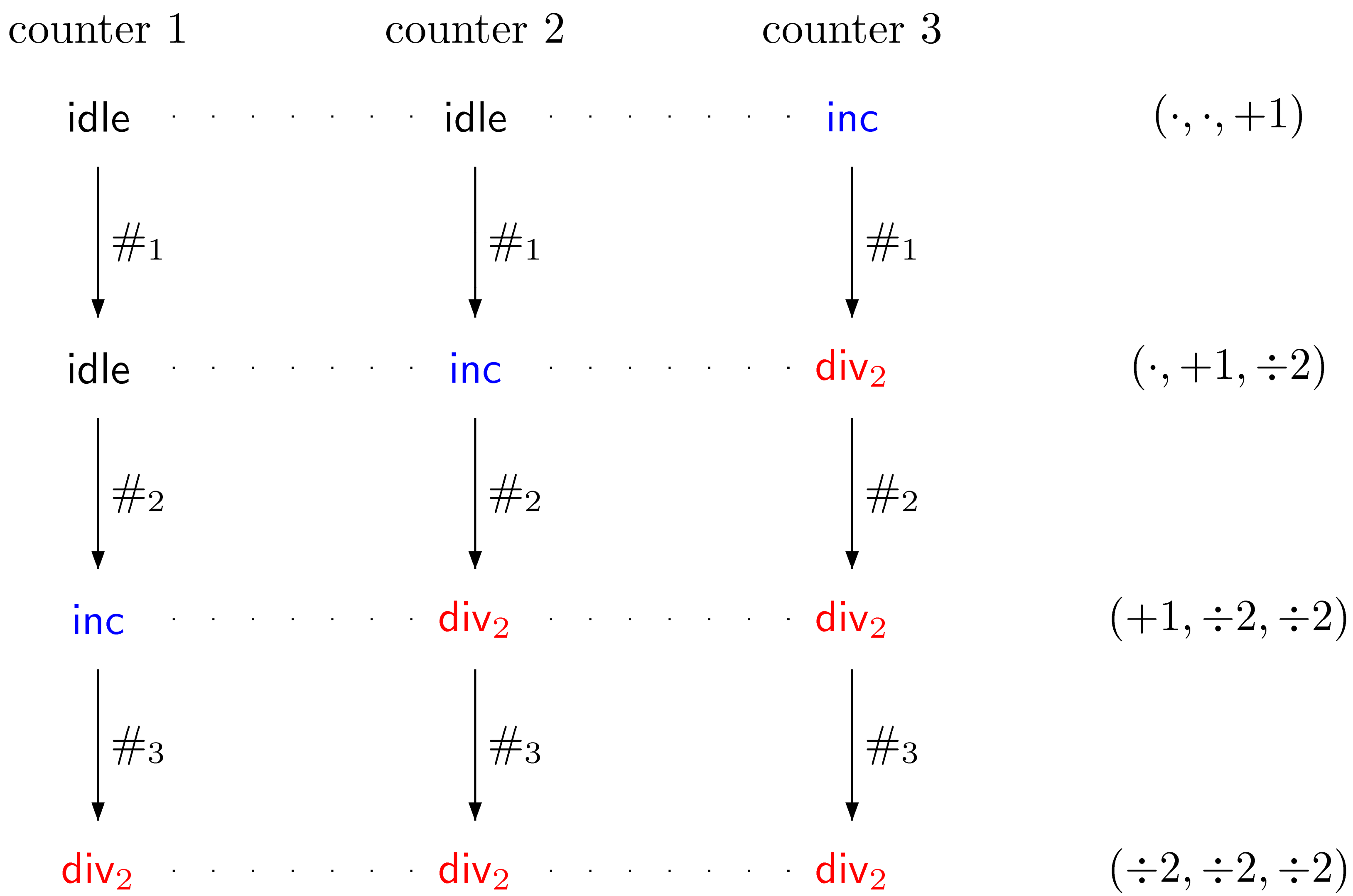
1. games can simulate increment and division by 2
2. Such counter systems require non-elementary counter value for reachability

$$2^{2^{\cdot^{\cdot^{2}}}} \Big\} \text{height } n$$

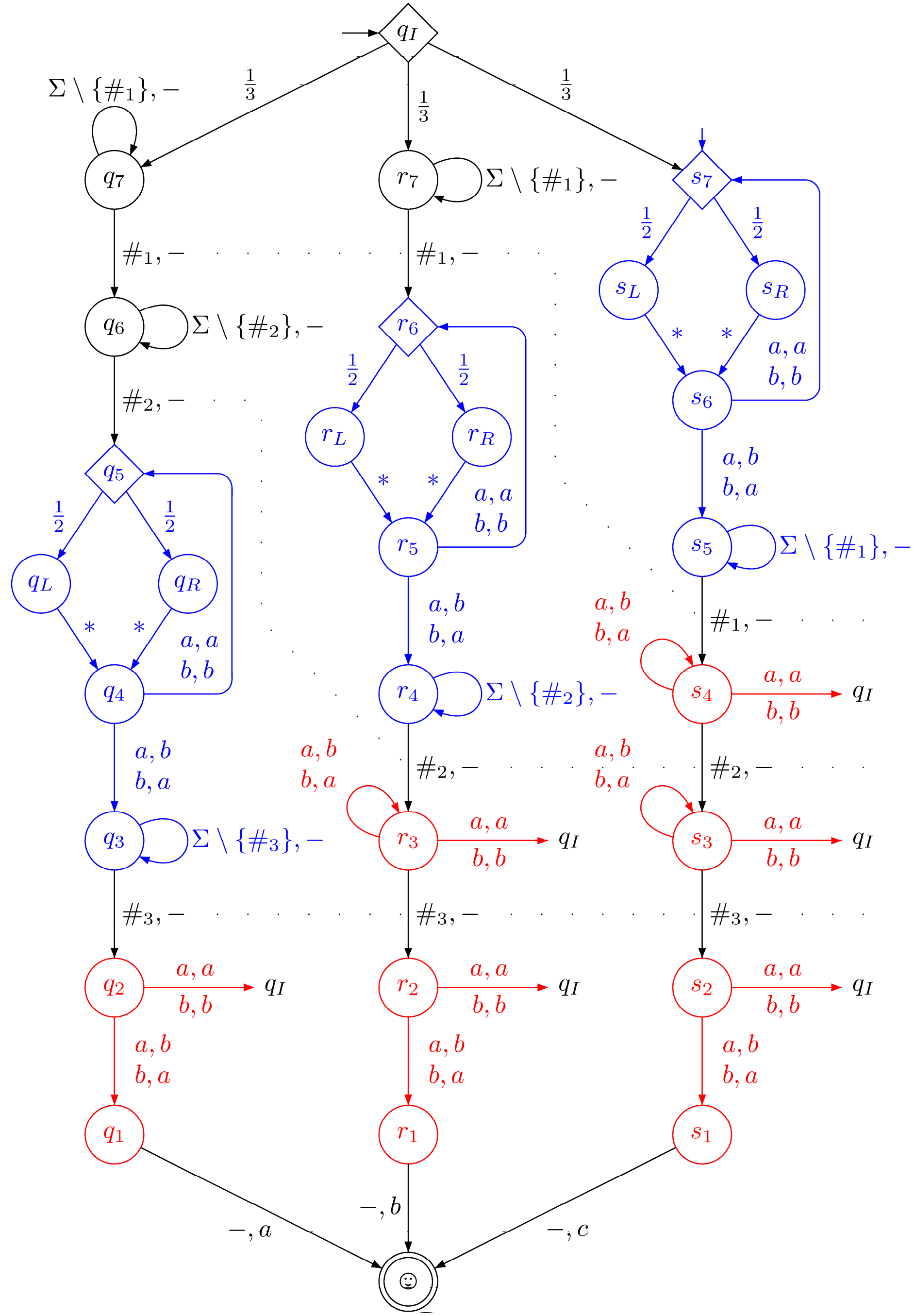
Game gadgets for {idle,+1,÷2}



Game gadgets for {idle,+1,÷2}



Game that simulates 3 counters...



Pure Strategies: Player 1 Perfect, Player 2 Partial

Pl1 Perfect, Pl 2 Partial:
Stochastic, Pure.
Non-elementary lower bound

Player 1 perfect, player 2 partial

Memory of **non-elementary** size for pure strategies

- lower bound: simulation of counter systems with increment and division by 2

- upper bound:
  positive: non-elementary counters simulate randomized strategies
  almost-sure: reduction to iterated positive

Counter systems with {+1,÷2} require non-elementary counter value for reachability

$$2^{2^{\cdot^{\cdot^{2}}}}\Big\}\text{height } n$$

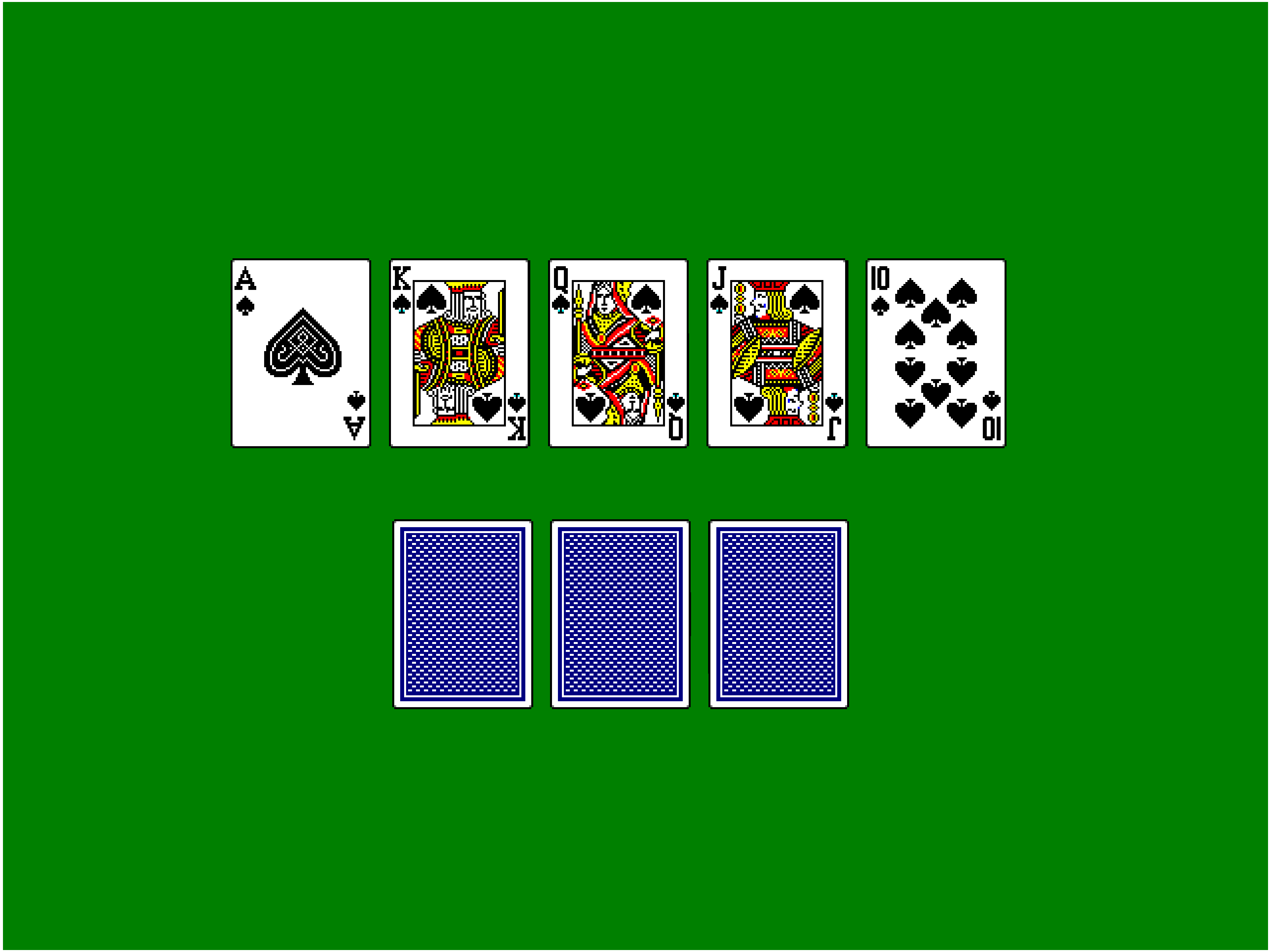
New results

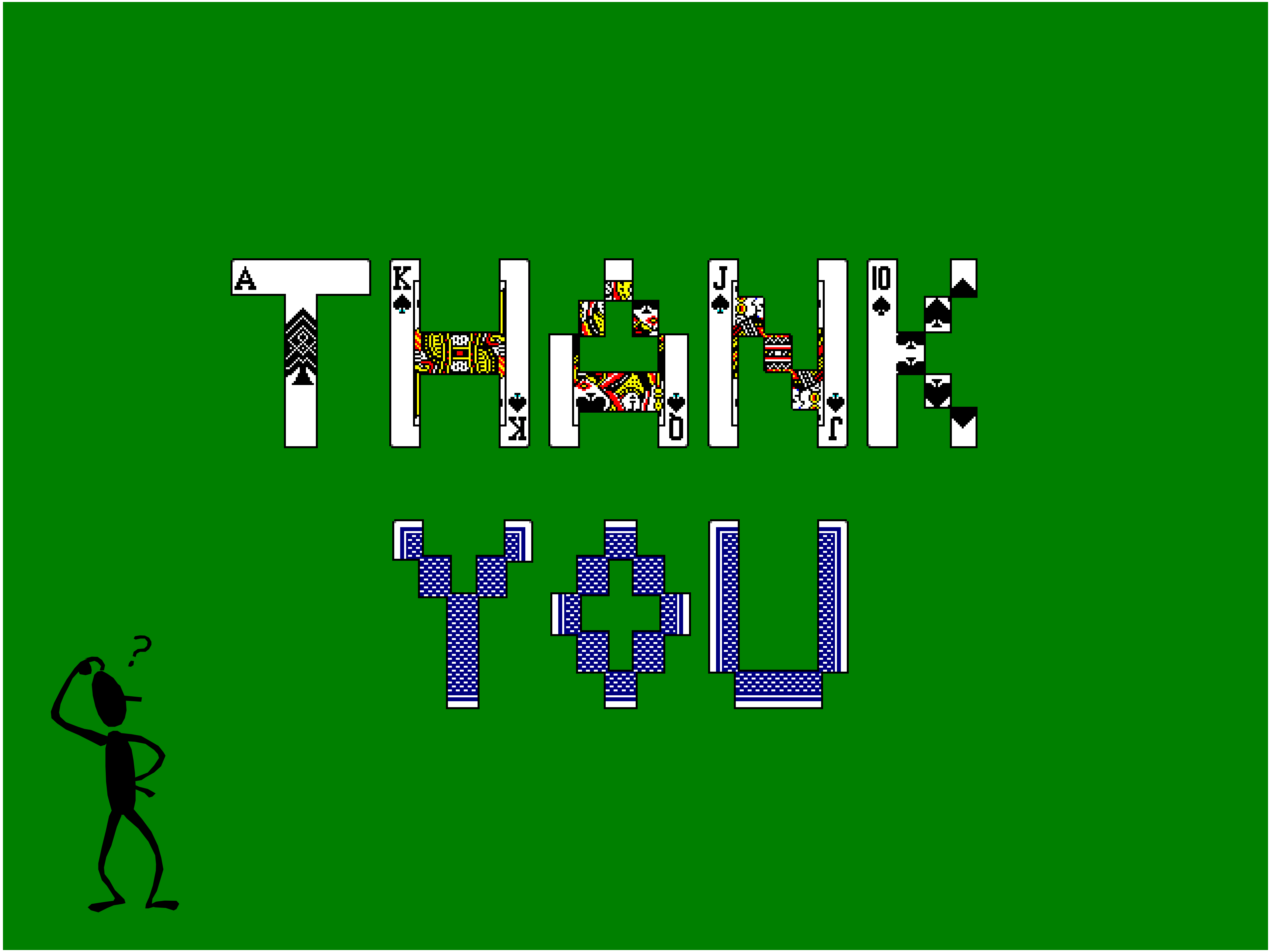
Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (belief not sufficient)		finite (at least non-elementary)
pure	exponential (belief not sufficient)	non-elementary complete	finite (at least non-elementary)

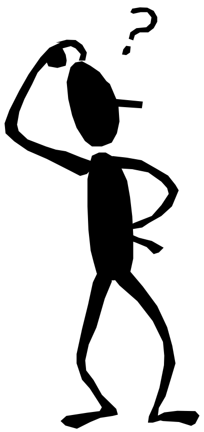
Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (belief not sufficient)	non-elementary complete	finite (at least non-elementary)

► Proof


A
K
Q
J
10


THANK YOU

Thank you !



Questions ?