Symbolic Verification of Cryptographic Protocols

The Bana-Comon Approach for Computationally Sound Symbolic Verification

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Intro

What would it take to make symbolic analyses of equivalences relevant to the computational model ?

- Interpret terms as computations of probabilistic Turing machine.
- Interpret equality as the almost-always coincidence of two computations, which may be larger than $=_{\mathsf{E}}$.
- Make precise assumptions about primitives e.g. IND-CCA1.
- Give up non-determinism: determinate processes only.
- Do not reason explicitly about these machines and interpretations.

Semantics: terms

We want to interpret t as probabilistic Turing computation $[\![t]\!]^{\sigma}_{\eta,\rho}$

- with security parameter $\eta \in \mathbb{N}$;
- with read-only infinite binary tapes $\rho = \rho_1, \rho_2$ of protocol and attacker randomness;
- where σ interprets free variables as Turing machines.

Definition

- $[\![x]\!]^{\sigma}_{\eta,\rho} = \sigma(x)(1^{\eta}; \rho)$
- names interpreted as non-overlapping portions $[\![n]\!]^{\sigma}_{\eta,\rho}$ of ρ_1
- any function symbol f of arity n interpreted as deterministic polynomial-time n-ary computation

Semantics: LTS

We modify the LTS for this computational setting:

$$(\mathbf{out}(c,u).P \mid Q, \Phi, \sigma) \xrightarrow{\mathbf{out}(c,w)} (P \mid Q, \Phi + \{w \mapsto u\}, \sigma)$$

$$(\mathbf{in}(c,x).P \mid Q, \Phi, \sigma) \xrightarrow{\mathbf{in}(c,x)} (P \mid Q, \Phi, \sigma + \{x \mapsto \mathcal{A}_x([\![\Phi]\!]^\sigma_{\eta,\rho}, \eta, \rho)\})$$
where $\mathcal{A}_x$ is a PPT Turing machine

$$(\mathbf{if}\ u = v\ \mathbf{then}\ P\ \mathbf{else}\ Q \mid R, \Phi, \sigma) \xrightarrow{\tau} (P \mid R, \Phi, \sigma) \quad \text{if } [\![u]\!]^\sigma_{\eta,\rho} = [\![v]\!]^\sigma_{\eta,\rho}$$

$$(\mathbf{if}\ u = v\ \mathbf{then}\ P\ \mathbf{else}\ Q \mid R, \Phi, \sigma) \xrightarrow{\tau} (Q \mid R, \Phi, \sigma) \quad \text{if } [\![u]\!]^\sigma_{\eta,\rho} \neq [\![v]\!]^\sigma_{\eta,\rho}$$

$$(\mathbf{new}\ n.P \mid Q, \Phi, \sigma) \xrightarrow{\tau} (P \mid Q, n.\Phi, \sigma) \quad \text{if } n \notin \mathrm{bn}(\Phi)$$

Define $K \stackrel{\alpha}{\Rightarrow} K'$ to hold iff $K \xrightarrow{\alpha} K_\alpha \xrightarrow{\tau}{}^* K'$ and $K' \not\xrightarrow{\tau}$.

Semantics: indistinguishability

We say that $P \sim_{\mathcal{M}} Q$ when, for any PPT adversary $\mathcal{A}$, the following advantage is negligible in η:

$$|\mathbf{Pr}\{\rho \ : \ \mathcal{A}^{P}_{\eta,\rho} = 1\} - \mathbf{Pr}\{\rho \ : \ \mathcal{A}^{Q}_{\eta,\rho} = 1\}|$$

Here $\mathcal{A}^{P}_{\eta,\rho}$ may extract randomness from ρ_2, interact with P by choosing observable actions and input values, and receiving output values.

Equivalently: for any tr, for any PPT machines $(\mathcal{A}_x)_{\mathbf{in}(c,x)\in \mathsf{tr}}$ and $\mathcal{A}$, the following is well-defined and negligible in η:

$$|\mathbf{Pr}\{\rho \ : \ \mathcal{A}([\![\Phi(P^{\mathsf{tr}})]\!]^{\sigma}_{\eta,\rho}) = 1\} - \mathbf{Pr}\{\rho \ : \ \mathcal{A}([\![\Phi(Q^{\mathsf{tr}})]\!]^{\sigma}_{\eta,\rho}) = 1\}|$$

where P^{tr} is the result of executing tr with inputs given by $(\mathcal{A}_x)_x$.

First-order logic

Turing machines are just one particular model for our terms.
Introduce attacker terms for a given tr:

- for each $\mathbf{in}(c, x) \in \mathrm{tr}$,
  take symbol g_x of arity the number of preceding outputs;
- conceptually replace $\mathcal{A}_x(\Phi, \eta, \rho)$ by $[\![g_x]\!]([\![\Phi]\!]^{\sigma}_{\eta,\rho}, \rho_2) \stackrel{def}{=} [\![g_x(\Phi)]\!]^{\sigma}_{\eta,\rho}$.

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Theorem

P and Q are computationally indistinguishable if:

- P and Q must perform the same traces tr;
- for each such tr, $\mathrm{fold}(P, \mathrm{tr}) \sim \mathrm{fold}(P, \mathrm{tr})$ is valid.

Example

$P = \mathbf{new}\ m.\ \mathbf{in}(c, x).\mathbf{out}(c, m)$
$Q = \mathbf{new}\ n.\ \mathbf{in}(c, x).\mathbf{if}\ x = n\ \mathbf{then}\ \mathbf{out}(c, \mathbf{0})\ \mathbf{else}\ \mathbf{out}(c, n)$
$\mathrm{tr} = \mathbf{in}(c, x).\mathbf{out}(c, w)$ induces $m \sim \mathbf{if}\ \mathrm{EQ}(g_x(), n)\ \mathbf{then}\ \mathbf{0}\ \mathbf{else}\ n$.

Axioms: basic examples

Force enough things that are valid in computational models, using recursive axiom schemes. For example:

$\forall \vec{x}, \vec{y}, \vec{z}, \vec{w}.$ $\quad \vec{x}, \vec{z} \sim \vec{y}, \vec{w} \Rightarrow f(\vec{x}), \vec{z} \sim f(\vec{y}), \vec{w}$ $\qquad |\vec{x}| = |\vec{y}| = \mathrm{ar}(f)$

$\forall \vec{x}, \vec{y}, z, z'.$ $\quad \vec{x}, z \sim \vec{y}, z' \Rightarrow \vec{x}, z, z \sim \vec{y}, z', z'$

$\forall \vec{x}, \vec{y}.$ $\quad x_1, \ldots, x_n \sim y_1, \ldots, y_n \Rightarrow x_{\pi(1)}, \ldots, x_{\pi(n)} \sim y_{\pi(1)}, \ldots, y_{\pi(n)}$
where π is any permutation

$\vec{u} \sim \vec{v} \Rightarrow \vec{u}, n \sim \vec{v}, m$
for any terms such that $n \notin \mathrm{fn}(\vec{u})$, $m \notin \mathrm{fn}(\vec{v})$

$\forall x, y, z, \vec{u}, \vec{v}.$ $\quad$ **if** $\mathrm{EQ}(x, y)$ **then** $t[x]$ **else** $z, \vec{u} \sim \vec{v} \Rightarrow$
if $\mathrm{EQ}(x, y)$ **then** $t[y]$ **else** $z, \vec{u} \sim \vec{v}$

$\forall b, \vec{w}, x, z.$ $\quad b, \vec{w}, x \sim b, \vec{w}, z \wedge b, \vec{w}, y \sim b, \vec{w}, z$
$\Rightarrow \vec{w},$ **if** b **then** x **else** $y \sim \vec{w}, z$

Axioms: equality

A computational model is a model where:

- $[\![\mathsf{EQ}(u,v)]\!]^{\sigma}_{\eta,\rho} = 1$ iff $[\![u]\!]^{\sigma}_{\eta,\rho} = [\![v]\!]^{\sigma}_{\eta,\rho}$;
- $\mathsf{EQ}(u,v) \sim \mathsf{true}$: $[\![u]\!]^{\sigma}_{\eta,\rho} = [\![v]\!]^{\sigma}_{\eta,\rho}$ with overwhelming probability;
- $u = v$: $[\![u]\!] = [\![v]\!]$, i.e. same computations, equal on all η, ρ.

Example axioms on equality:

$$
\begin{array}{rll}
\forall x, y. & x = y \wedge \Phi[x] \Rightarrow \Phi[y] & \\
\forall x, y. & \mathsf{EQ}(x,y) \sim \mathsf{true} \wedge \vec{u}[x] \sim \vec{v} \Rightarrow \vec{u}[y] \sim \vec{v} & \\
\forall x. & \mathsf{EQ}(x,x) = \mathsf{true} & \\
 & \mathsf{EQ}(m,\mathsf{n}) \sim \mathsf{false} & \text{if } n \notin \mathsf{fn}(m) \\
\forall x, y. & \textbf{if } \mathsf{true} \textbf{ then } x \textbf{ else } y = x & \\
\forall x, y. & \textbf{if } \mathsf{false} \textbf{ then } x \textbf{ else } y = y & \\
\forall b, x, y, z, . & \textbf{if } b \textbf{ then if } b \textbf{ then } x \textbf{ else } y \textbf{ else } z = \textbf{if } b \textbf{ then } x \textbf{ else } z & \\
\forall x, y. & \textbf{proj}_1(\textbf{pair}(x,y)) = x &
\end{array}
$$

Axioms: encryption

The following axiom scheme is satisfied when $[\![\mathbf{aenc}]\!]$ is IND-CCA1:

$$\vec{v}, \mathbf{if}\ \mathrm{EQL}(u, u')\ \mathbf{then}\ \{u\}^{r}_{\mathrm{pk}_a}\ \mathbf{else}\ u'' \sim \vec{v}, \mathbf{if}\ \mathrm{EQL}(u, u')\ \mathbf{then}\ \{u'\}^{r'}_{\mathrm{pk}_a}\ \mathbf{else}\ u''$$

provided:

- r and r' do not appear in other terms;
- sk_a only occurs in decryption position in $\vec{v}, u, u', u''$.

If $[\![\mathbf{aenc}]\!]$ ensures key privacy, then

$$\vec{v}, \{u\}^{r}_{\mathrm{pk}_a} \sim \vec{v}, \{u\}^{r'}_{\mathrm{pk}_{a'}}$$

holds whenever r and r' are fresh and secret keys for a and a' do not occur, in particular there are no decryptions using these keys.

Private authentication: attacks / counter-models

$I(sk_a, pk_b)$	$R(sk_b, pk_a)$
new r, n_a.	**new** r, n_b.
let $pk_a = \mathbf{pub}(sk_a)$ **in**	**let** $pk_b = \mathbf{pub}(sk_b)$ **in**
$\mathbf{out}(c, \{\mathbf{pair}(n_a, pk_a)\}^r_{pk_b}))$.	$\mathbf{in}(c, x).\mathbf{let}\ y = \mathbf{adec}(x, sk_b)$ **in**
...	**if** $\mathbf{proj}_2(y) = pk_a$
	then $\mathbf{out}(c, \{\langle \mathbf{proj}_1(y), n_b \rangle\}^r_{pk_a})$
	else $\mathbf{out}(c, \{n_b\}^r_{pk_a}))$

Anonymity

$R(sk_b, \mathbf{pub}(sk_a)) \approx^? R(sk_b, \mathbf{pub}(sk_c))$

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Anonymity

$\Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_a)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b \rangle\}^r_{pk_a}\ \mathbf{else}\ \{n_b\}^r_{pk_a} \sim^?$
$\Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_c)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b \rangle\}^r_{pk_c}\ \mathbf{else}\ \{n_b\}^r_{pk_c}$
where $h = \mathbf{adec}(g_x(\Phi_0), sk_b)$.

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...	**if** $\mathbf{proj}_2(y) = pk_a \wedge \mathrm{EQL}(\mathbf{proj}_1(y), n_b)$
	then $\mathbf{out}(c, \{\langle \mathbf{proj}_1(y), n_b \rangle\}^r_{pk_a})$
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$\Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_a)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b \rangle\}^r_{pk_a}\ \mathbf{else}\ \{\langle n_b, n_b \rangle\}^r_{pk_a} \sim^?$
$\Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_c)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b \rangle\}^r_{pk_c}\ \mathbf{else}\ \{\langle n_b, n_b \rangle\}^r_{pk_c}$
where $h = \mathbf{adec}(g_x(\Phi_0), sk_b)$.

Private authentication: proof / validity

$$\Phi_a = \Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_a) \wedge \mathrm{EQL}(\mathbf{proj}_1(h), n_b)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b\rangle\}^r_{pk_a}\ \mathbf{else}\ \{\langle n_b, n_b\rangle\}^r_{pk_a}$$

$$\Phi_c = \Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_c) \wedge \mathrm{EQL}(\mathbf{proj}_1(h), n_b)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b\rangle\}^r_{pk_c}\ \mathbf{else}\ \{\langle n_b, n_b\rangle\}^r_{pk_c}$$

$\Phi_a \sim \Phi_c$ in any model of the axioms, in particular in any computational model whose encryption scheme satisfies IND-CCA1 and key privacy.

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Private authentication: proof / validity

$$\Phi_a = \Phi_0, \mathbf{if}\ \mathrm{EQ}(\mathbf{proj}_2(h), pk_a) \wedge \mathrm{EQL}(\mathbf{proj}_1(h), n_b)\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b\rangle\}^r_{pk_a}\ \mathbf{else}\ \{\langle n_b, n_b\rangle\}^r_{pk_a}$$

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$\Phi_a \sim \Phi_c$ in any model of the axioms, in particular in any computational model whose encryption scheme satisfies IND-CCA1 and key privacy.

- $\mathrm{EQ}_a, \mathbf{if}\ \mathrm{EQL}\ \mathbf{then}\ \{\langle n_b, n_b\rangle\}^r_{pk_a}\ \mathbf{else}\ \{\langle n_b, n_b\rangle\}^r_{pk_a} \sim \mathrm{EQ}_a, \{\langle n_b, n_b\rangle\}^r_{pk_a}$;
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- $\mathrm{EQ}_a, \mathbf{if}\ \mathrm{EQL}\ \mathbf{then}\ \{\langle \mathbf{proj}_1(h), n_b\rangle\}^r_{pk_a}\ \mathbf{else}\ \{\langle n_b, n_b\rangle\}^r_{pk_a} \sim \mathrm{EQ}_a, \{\langle n_b, n_b\rangle\}^r_{pk_a}$ by transitivity;
- $\Phi_a \sim \Phi_0, \{\langle n_b, n_b\rangle\}^r_{pk_a}$ by axioms on conditional;
- $\Phi_a \sim \Phi_b$ by the previous item for a and a' and key privacy.

Conclusion

In a nutshell

We have reduced protocol equivalences

. . . to symbolic indistinguishabilities $\vec{u} \sim \vec{v}$

. . . to proving these in FOL under some recursive axiom schemes.

This allows to prove security against computational attackers, under clearly identified assumptions on crypto primitives.

Ongoing research

- Proving security against stronger attackers, e.g. side-channel attacks
- Avoiding the explicit enumeration of symbolic traces
- Unbounded sessions