Symbolic Verification of Cryptographic Protocols

Protocol Analysis in the Applied Pi-Calculus

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Intruder detection

Problem

Given Φ and u, does $S \vdash u$?

Theorem

For the standard primitives, the intruder detection problem is in PTIME.

Deducibility constraints

Definition

A deducibility constraint system is either $\perp$ or a (possibly empty) conjunction of deducibility constraints of the form

$$T_1 \vdash^? u_1 \wedge \ldots \wedge T_n \vdash^? u_n$$

such that

- $T_1 \subseteq T_2 \subseteq \ldots \subseteq T_n$ (monotonicity)
- for every i, $\mathrm{fv}(T_i) \subseteq \mathrm{fv}(u_1, \ldots, u_{i-1})$ (origination)

Definition

The substitution σ is a solution of $\mathcal{C} = T_1 \vdash^? u_1 \wedge \ldots \wedge T_n \vdash^? u_n$ when $T_i\sigma \vdash u_i\sigma$ for all i and $\mathrm{img}(\sigma) \subseteq T_c(\mathcal{N})$.

Example: Needham-Schroeder

- $S_1 := \langle sk_i, \mathrm{pub}(sk_a), \mathrm{pub}(sk_b)\rangle, \mathrm{aenc}(\langle \mathrm{pub}(sk_a), n_a\rangle, \mathrm{pub}(sk_i))$
  $S_1 \vdash^? x$

Example: Needham-Schroeder

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  $S_1 \vdash^? \mathrm{aenc}(\langle x_a, x_{na}\rangle, \mathrm{pub}(sk_b))$

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  $S_1 \vdash^? \mathrm{aenc}(\langle x_a, x_{na} \rangle, \mathrm{pub}(sk_b))$
- $S_2 := S_1, \mathrm{aenc}(\langle x_{na}, n_b \rangle, x_a)$
  $S_2 \vdash^? \mathrm{aenc}(\langle n_a, x_{nb} \rangle, \mathrm{pub}(sk_a))$

Example: Needham-Schroeder

- $S_1 := \langle sk_i, \mathsf{pub}(sk_a), \mathsf{pub}(sk_b)\rangle, \mathsf{aenc}(\langle \mathsf{pub}(sk_a), n_a\rangle, \mathsf{pub}(sk_i))$
  $S_1 \vdash^? \mathsf{aenc}(\langle x_a, x_{na}\rangle, \mathsf{pub}(sk_b))$
- $S_2 := S_1, \mathsf{aenc}(\langle x_{na}, n_b\rangle, x_a)$
  $S_2 \vdash^? \mathsf{aenc}(\langle n_a, x_{nb}\rangle, \mathsf{pub}(sk_a))$
- $S_3 := S_2, \mathsf{aenc}(x_{nb}, \mathsf{pub}(sk_i))$
  $S_3 \vdash^? \mathsf{aenc}(n_b, \mathsf{pub}(sk_b))$

Example: Needham-Schroeder

- $S_1 := \langle sk_i, \mathrm{pub}(sk_a), \mathrm{pub}(sk_b)\rangle, \mathrm{aenc}(\langle \mathrm{pub}(sk_a), n_a\rangle, \mathrm{pub}(sk_i))$
  $S_1 \vdash^? \mathrm{aenc}(\langle x_a, x_{na}\rangle, \mathrm{pub}(sk_b))$
- $S_2 := S_1, \mathrm{aenc}(\langle x_{na}, n_b\rangle, x_a)$
  $S_2 \vdash^? \mathrm{aenc}(\langle n_a, x_{nb}\rangle, \mathrm{pub}(sk_a))$
- $S_3 := S_2, \mathrm{aenc}(x_{nb}, \mathrm{pub}(sk_i))$
  $S_3 \vdash^? \mathrm{aenc}(n_b, \mathrm{pub}(sk_b))$
- $S_4 := S_3, \mathrm{senc}(\mathrm{secret}, n_b)$ and $x_a = \mathrm{pub}(sk_a)$
  $S_4 \vdash^? \mathrm{secret}$

Constraint resolution

Solved form

A system is solved if it is of the form

$$T_1 \vdash^? x_1 \wedge \ldots \wedge T_n \vdash^? x_n$$

Proposition

If $\mathcal{C}$ is solved, then it admits a solution.

Constraint resolution

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Proposition

If $\mathcal{C}$ is solved, then it admits a solution.

Theorem

There exists a terminating relation $\rightsquigarrow$ such that for any $\mathcal{C}$ and θ, $\theta \in \mathrm{Sol}(\mathcal{C})$ iff there is $\mathcal{C} \rightsquigarrow^*_\sigma \mathcal{C}'$ solved and $\theta = \sigma\theta'$ for some $\theta' \in \mathrm{Sol}(\mathcal{C}')$.

Simplification of constraint systems

Here systems are considered modulo AC of $\wedge$.

$$(R_1) \quad \mathcal{C} \wedge T \vdash^? u \rightsquigarrow \mathcal{C} \qquad \text{if } T \cup \{x \mid (T' \vdash^? x) \in \mathcal{C}, T' \subsetneq T\} \vdash u$$

$$(R_2) \quad \mathcal{C} \wedge T \vdash^? u \rightsquigarrow_\sigma \mathcal{C}\sigma \wedge T\sigma \vdash^? u\sigma$$
$$\text{if } \sigma = \text{mgu}(t, u), t \in \text{st}(T), t \neq u, \text{ and } t, u \notin \mathcal{X}$$

$$(R_3) \quad \mathcal{C} \wedge T \vdash^? u \rightsquigarrow_\sigma \mathcal{C}\sigma \wedge T\sigma \vdash^? u\sigma$$
$$\text{if } \sigma = \text{mgu}(t_1, t_2), t_1, t_2 \in \text{st}(T), t_1 \neq t_2$$

$$(R_4) \quad \mathcal{C} \wedge T \vdash^? u \rightsquigarrow \bot \qquad \text{if } \text{fv}(T \cup \{u\}) = \emptyset, T \nvdash u$$

$$(R_f) \quad \mathcal{C} \wedge T \vdash^? f(u_1, \ldots, u_n) \rightsquigarrow \mathcal{C} \wedge \textstyle\bigwedge_i T \vdash^? u_i \qquad \text{for } f \in \Sigma_c$$

$$(R_{\text{pub}}) \quad \mathcal{C} \rightsquigarrow \mathcal{C}[x := \text{pub}(x)] \quad \text{if aenc}(t, x) \in T \text{ for some } (T \vdash^? u) \in \mathcal{C}$$

Examples of simplifications

1. $\mathrm{senc}(n, k) \vdash^? \mathrm{senc}(x, k)$
2. $\mathrm{senc}(\mathrm{senc}(t_1, k), k) \vdash^? \mathrm{senc}(x, k)$ (two opportunities for R_2)
3. $S \vdash^? x \ \wedge\ S, n \vdash^? y \ \wedge\ S, n, \mathrm{senc}(m, \mathrm{senc}(x, k)), \mathrm{senc}(y, k) \vdash^? m$
4. $S \vdash^? x \wedge S \vdash^? \langle x, x \rangle$
5. $n \vdash^? x \wedge n \vdash^? \mathrm{senc}(x, k)$

Constraint simplification proof (1)

Proposition (Validity)

If $\mathcal{C}$ *is a deducibility constraint system, and* $\mathcal{C} \rightsquigarrow_\sigma \mathcal{C}'$, *then* $\mathcal{C}'$ *is a deducibility constraint system.*

Constraint simplification proof (1)

Proposition (Validity)

If $\mathcal{C}$ is a deducibility constraint system, and $\mathcal{C} \rightsquigarrow_\sigma \mathcal{C}'$, then $\mathcal{C}'$ is a deducibility constraint system.

Proposition (Soundness)

If $\mathcal{C} \rightsquigarrow_\sigma \mathcal{C}'$ and $\theta \in \mathsf{Sol}(\mathcal{C}')$ then $\sigma\theta \in \mathsf{Sol}(\mathcal{C})$.

Proposition (Termination)

Simplifications are terminating, as shown by the termination measure $(v(\mathcal{C}), p(\mathcal{C}), s(\mathcal{C}))$ where:

- *$v(\mathcal{C})$ is the number of variables occurring in $\mathcal{C}$;*
- *$p(\mathcal{C})$ is the number of terms of the form $aenc(u, x)$ occurring on the left of constraints in $\mathcal{C}$;*
- *$s(\mathcal{C})$ is the total size of the right-hand sides of constraints in $\mathcal{C}$.*

Constraint simplification proof (2)

Left-minimality & Simplicity

A derivation Π of $T_i \vdash u$ is left-minimal if, whenever $T_j \vdash u$, Π is also a derivation of $T_j \vdash u$.
A derivation is simple it is non-repeating
and all its subderivations are left-minimal.

Proposition

If $T_i \vdash u$, then it has a simple derivation.

Lemma

Let $\mathcal{C} = \bigwedge_j T_j \vdash^? u_j$ be a constraint system, $\theta \in \mathsf{Sol}(\mathcal{C})$,
and i be such that $u_j \in \mathcal{X}$ for all $j < i$.
If $T_i\theta \vdash u$ with a simple derivation starting with an axiom or a decomposition, then there is $t \in \mathsf{subterm}(T_i) \setminus \mathcal{X}$ such that $t\theta = u$.

Constraint simplification proof (3)

Lemma

Let $\mathcal{C} = \bigwedge_j T_j \vdash^? u_j$, $\sigma \in \mathsf{Sol}(\mathcal{C})$.
Let i be a minimal index such that $u_i \notin \mathcal{X}$.
Assume that:

- T_i *does not contain two subterms* $t_1 \neq t_2$ *such that* $t_1\sigma = t_2\sigma$;
- T_i *does not contain any subterm of the form* $aenc(t, x)$;
- u_i *is a non-variable subterm of* T_i.

Then $T_i' \vdash u_i$, *where* $T_i' = T_i \cup \{x \mid (T \vdash^? x) \in \mathcal{C}, T \subsetneq T_i\}$.

Constraint simplification proof (3)

Lemma

Let $\mathcal{C} = \bigwedge_j T_j \vdash^? u_j$, $\sigma \in \mathsf{Sol}(\mathcal{C})$*.*
Let i *be a minimal index such that* $u_i \notin \mathcal{X}$*.*
Assume that:

- T_i *does not contain two subterms* $t_1 \neq t_2$ *such that* $t_1\sigma = t_2\sigma$*;*
- T_i *does not contain any subterm of the form* $aenc(t, x)$*;*
- u_i *is a non-variable subterm of* T_i*.*

Then $T_i' \vdash u_i$*, where* $T_i' = T_i \cup \{x \mid (T \vdash^? x) \in \mathcal{C}, T \subsetneq T_i\}$*.*

Proposition (Completeness)

If $\mathcal{C}$ *is unsolved and* $\theta \in \mathsf{Sol}(\mathcal{C})$*, there is* $\mathcal{C} \rightsquigarrow_\sigma \mathcal{C}'$ *and* $\theta' \in \mathsf{Sol}(\mathcal{C}')$ *such that* $\theta = \sigma\theta'$*.*

Concluding remarks

Improvements

- A complete strategy can yield a polynomial bound, hence a small attack property
- Equalities and disequalities may be added
- Several variants and extensions may be considered: sk instead of pub, signatures, xor, etc.

We have not answered the original question yet!

- Symbolic semantics, (dis)equality constraints
- The enumeration of all interleavings is too naive

Complexity

- Deciding whether a system has a solution is NP-hard
- Reminder: for a general theory, security is undecidable