Cryptographic Protocols
Formal and Computational Proofs
Introduction

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(based on slides by B. Blanchet & S. Delaune)

Cryptographic protocols

Cryptographic protocols

Communicating processes that use cryptographic primitives to meet security properties in a hostile environment.

Security properties (1)

- Secrecy: May an intruder learn some secret message?
- Authentication: Is the agent Alice really talking to Bob?
- Non-repudiation: Alice sends a message to Bob. Alice cannot later deny having sent this message. Bob cannot deny having received the message.
- Forward secrecy: past communications remain private if long-term keys are compromised.

Security properties: E-voting (2)

Eligibility: only legitimate voters can vote

Fairness: no early results can be obtained which could influence the remaining voters

Individual verifiability:
a voter can verify that her vote was really counted

Universal verifiability:
the published outcome really is the sum of all the votes

Security properties: E-voting (3)

Privacy: the fact that a particular voted in a particular way is not revealed to anyone

Receipt-freeness: a voter cannot prove that she voted in a certain way (this is important to protect voters from coercion)

Coercion-resistance: same as receipt-freeness, but the coercer interacts with the voter during the protocol, (e.g. by preparing messages)

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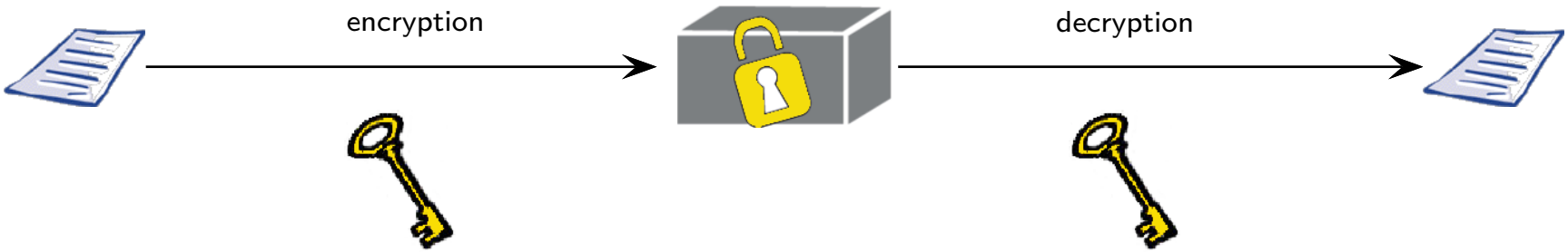
Symmetric encryption



$\longrightarrow$ Examples: Caesar encryption, DES, AES, ...

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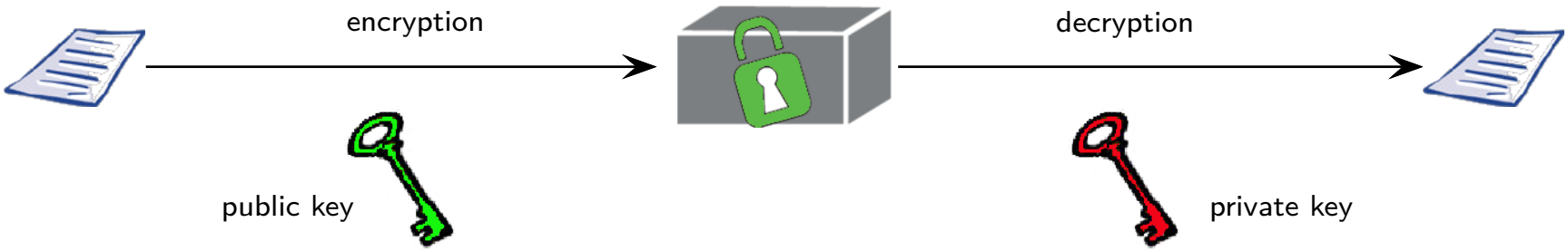
Asymmetric encryption



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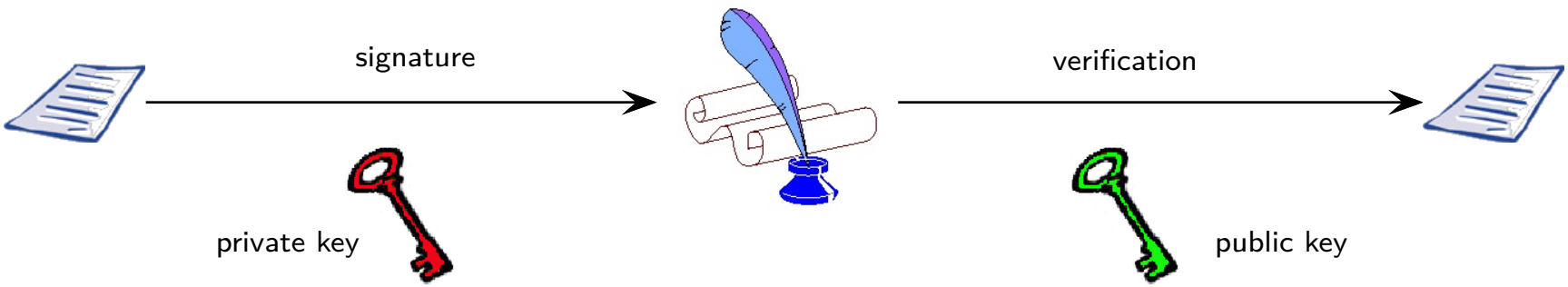
Signature



Credit Card Payment Protocol

Example: credit card payment

- The client Cl puts his credit card C in the terminal T.
- The merchant enters the amount M of the sale.

- The terminal authenticates the credit card.
- The client enters his PIN.
  If $M \geq 100\,€$, then in 20% of cases,
  - The terminal contacts the bank B.
  - The banks gives its authorisation.

More details

the Bank B , the Client Cl, the Credit Card C and the Terminal T

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- a private signature key – priv(B)
- a public key to verify a signature – pub(B)
- a secret key shared with the credit card – K_{CB}

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Credit Card

- some $Data$: name of the cardholder, expiry date ...
- a signature of the $Data$ – $\{hash(Data)\}_{\mathsf{priv}(B)}$
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Terminal

- the public key of the bank – $\mathsf{pub}(B)$

Payment protocol

the terminal T reads the credit card C:

$$1. \quad C \quad \rightarrow \quad T : \quad Data, \{hash(Data)\}_{\mathsf{priv}(B)}$$

Payment protocol

the terminal T reads the credit card C:

1. $C \rightarrow T : Data, \{hash(Data)\}_{priv(B)}$

the terminal T asks the code:

2. $T \rightarrow Cl : PIN?$
3. $Cl \rightarrow C : 1234$
4. $C \rightarrow T : ok$

Payment protocol

the terminal T reads the credit card C:

1. $C \rightarrow T$: $Data, \{hash(Data)\}_{priv(B)}$

the terminal T asks the code:

2. $T \rightarrow CI$: $PIN?$
3. $CI \rightarrow C$: 1234
4. $C \rightarrow T$: ok

the terminal T requests authorisation the bank B:

5. $T \rightarrow B$: $auth?$
6. $B \rightarrow T$: 4528965874123
7. $T \rightarrow C$: 4528965874123
8. $C \rightarrow T$: $\{4528965874123\}_{K_{CB}}$
9. $T \rightarrow B$: $\{4528965874123\}_{K_{CB}}$
10. $B \rightarrow T$: ok

Attack against credit cards

Initially, security was guaranteed by:

- cards hard to replicate,
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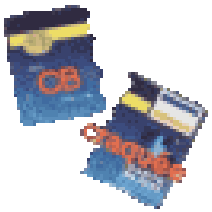
- secrecy of keys and protocol.

However, there are attacks!

- cryptographic attack: 320-bit keys are no longer secure,
- logical attack: no link between PIN and authentication,
- hardware attack: replication of cards.



$\rightarrow$ "YesCard" made by Serge Humpich (1997).

The "YesCard": how does it work?

Logical attack

$$
\begin{aligned}
1.C &\rightarrow T &&: \text{Data}, \{\text{hash}(\text{Data})\}_{\text{priv}(B)} \\
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$\rightarrow$ copy card data and certificate on card that accepts all PINs

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Logical attack

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$$
\begin{aligned}
1.C' &\rightarrow T &&: \text{XXX}, \{\text{hash}(\text{XXX})\}_{\text{priv}(B)} \\
2.T &\rightarrow CI &&: PIN? \\
3.CI &\rightarrow C' &&: 0000 \\
4.C' &\rightarrow T &&: ok
\end{aligned}
$$

Credit cards: going further

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Flaw fixed with the more recent Dynamic Data Authentication mode.
Each card has a public key (authentified) used to complete a challenge.

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New "attacks": contact-less cards

The same protocol is used through wireless communications:

$$\rightsquigarrow \text{Card data easily harvested!}$$

Included card holder name and record of purchase in France until 2013.

Conclusion: formal security not the main goal of credit cards.

The problem

In summary:

- Protocol design is error prone.
  Hard to clearly define threats and security properties.
- Flaws undetected by testing appear in presence of adversary.
- Errors can have serious consequences.

$\rightsquigarrow$ Formal methods and formal proofs are needed!

Active and successful research for several decades.

Models of protocols: communication assumptions

Attacker capabilities

- The attacker can intercept all messages sent on the network.
- He can compute messages using crypto primitives.
- He can send messages on the network.

A worst case scenario where
the attacker is (perhaps maliciously) executing the protocol.

Models of protocols: the formal model

The formal model, aka. symbolic or "Dolev-Yao model"
is due to Needham and Schroeder [1978] and Dolev and Yao [1983].

- Cryptographic primitives are blackboxes.
- Messages are terms on these primitives.
  $\hookrightarrow \{m\}_k$ encryption of the message m with key k,
  $\hookrightarrow \langle m_1, m_2 \rangle$ pairing of messages m_1 and m_2, ...
- The attacker is restricted to compute only using these primitives, according to some equations.
  $\hookrightarrow \mathrm{dec}(\{m\}_k, k) = m$
  $\hookrightarrow \mathrm{fst}(\langle x, y \rangle) = x$
  $\Rightarrow$ perfect cryptography assumption

One can add equations between primitives, but one makes the hypothesis that the only equalities are those given by the equations.

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Lends itself well to automatic proofs: AVISPA, ProVerif, ...

Verifying protocols in the formal model

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- Bounded number of sessions but unbounded messages
  $\Rightarrow$ constraint solving: CI-AtSe, integrated in AVISPA
- Unbounded messages and number of sessions $\Rightarrow$ undecidable
  - Interactive theorem proving: Isabelle [Paulson, JCS'98]
  - Approximations:
    - abstract interpretation [Monniaux'03], TA4SP integrated in AVISPA
    - typing [Abadi'99], [Gordon & Jeffrey '02]
  - Semi-decision procedures (and approximations): Proverif

Models of protocols: the computational model

The computational model has been developed in the early 80'
by Goldwasser, Micali, Rivest, Yao, and others.

- Messages are bitstrings.
- Cryptographic primitives are computations on bitstrings.
- The attacker is any probabilistic (polynomial-time) Turing machine.

More realistic than formal model, but until recently only manual proofs.

Proofs in the computational model

Proof by sequence of game reductions [Shoup, Bellare & Rogaway].
Games correspond to security property of protocol or primitive,
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Example: IND-CPA

- Generate public and private keys.
- Allow attacker to (polynomially) perform encryptions.
- Attacker chooses m_0, m_1.
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Automation

CryptoVerif, Certicrypt, F$\star$...

Example: RSA

It is possible to generate e, d and N such that $x^{ed} = x \mod N$.
Keep d private, make e and d public.

Encryption

$$\text{enc}(m) = m^e \qquad \text{dec}(c) = c^d \qquad \text{dec}(\text{enc}(m)) = m$$

What does it ensure? What problems remain?

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Blinding (for a random r)

$$\text{blind}(m) = mr^e \quad \text{unblind}(s) = sr^{-1} \quad \text{unblind}(\text{sign}(\text{blind}(m))) = \text{sign}(m)$$

Notice the more complex equation, and remaining problems.

Relationship between the two models

An attack in the formal model is an attack in the computational model.
What more can we say?

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Computational soundness (under very specific assumptions)

Proof in the formal model $\Rightarrow$ proof in the computational model

Approach pioneered by Abadi&Rogaway [2000]; many works since then.

Other issues

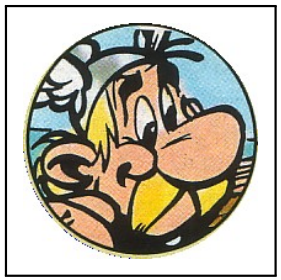
The computational model is still a model!
In particular, it ignores side channels which give additional information:
timing, power consumption, noise, physical attacks, etc.

Verifying protocols, regardless of the model, is useless if there are implementation flaws:

- Google's non-compliant SSO implem. found flawed in 2008.
- Heartbleed bug in 2012: buffer over-read in OpenSSL.
- SkipTLS: JSSE's SSL implem allows to skip crucial steps.
- Freak: trick SSL implementations to choose export-grade crypto.

Needham-Schroeder (public-key) Protocol

Needham-Schroeder's Protocol (1978)



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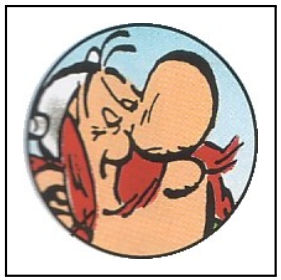

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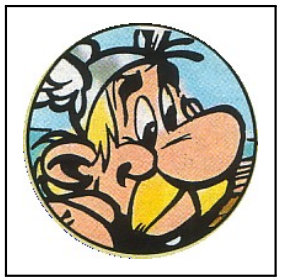

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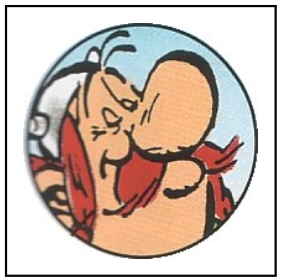

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Questions

- Is N_b secret between A and B ?
- When B receives $\{N_b\}_{\mathrm{pub}(B)}$, does this message really comes from A ?

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Attack

An attack was found 17 years after its publication! [Lowe 96]

Example: Man in the middle attack

Agent A

Intruder I

Agent B

Attack

- involving 2 sessions in parallel,
- an honest agent has to initiate a session with I.

$$
\begin{aligned}
A \to B &: \{A, N_a\}_{\mathrm{pub}(B)} \\
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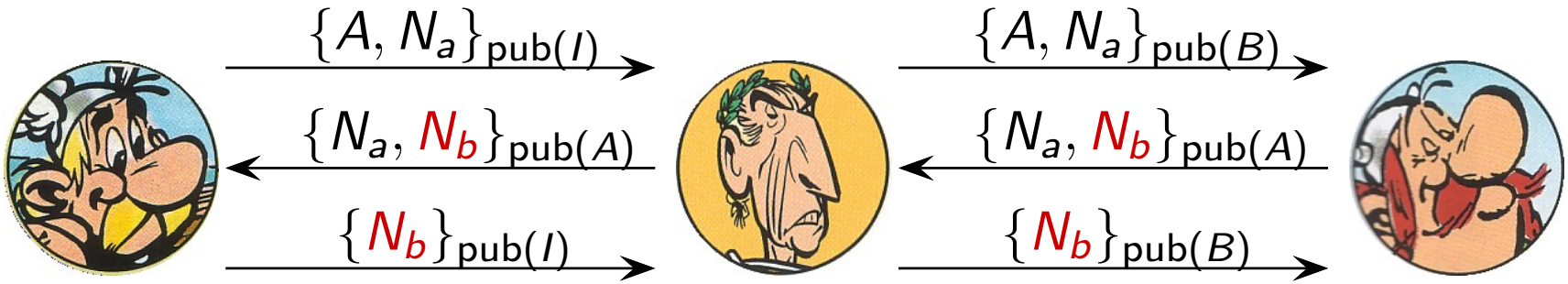

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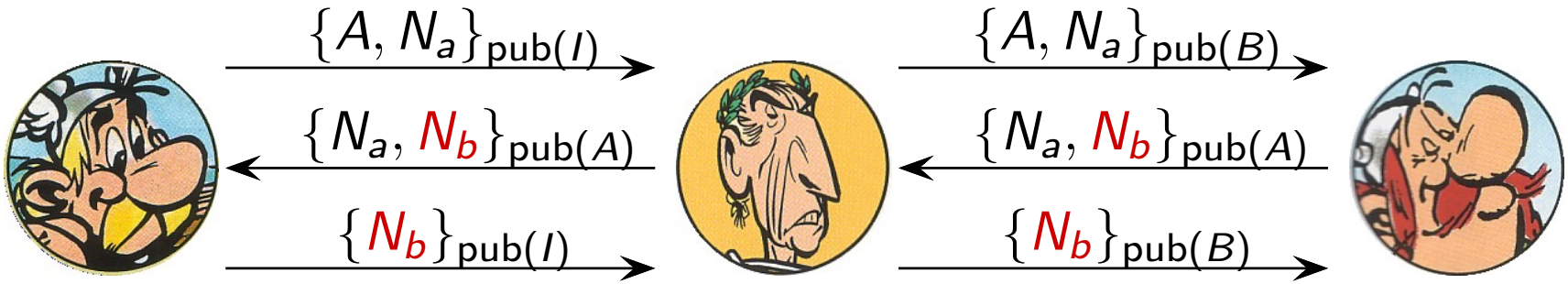

Attack

- the intruder knows N_b,
- When B finishes his session (apparently with A), A has never talked with B.

$$\begin{aligned} A \to B &: \{A, N_a\}_{pub(B)} \\ B \to A &: \{N_a, N_b\}_{pub(A)} \\ A \to B &: \{N_b\}_{pub(B)} \end{aligned}$$

Exercise

$$\begin{aligned} \mathrm{A} &\to \mathrm{B} \quad : \{A, N_a\}_{\mathrm{pub}(B)} \\ \mathrm{B} &\to \mathrm{A} \quad : \{N_a, N_b\}_{\mathrm{pub}(A)} \\ \mathrm{A} &\to \mathrm{B} \quad : \{N_b\}_{\mathrm{pub}(B)} \end{aligned}$$

Exercise

Propose a fix for the Needham-Schroeder protocol.

Conclusion

Difficulties: a big picture

- Communicating agents: concurrent, bounded or not
- Crypto primitives: symbolic vs. computational, algebraic properties
- Hostile environment: passive vs. active, deduction, equivalence

Outline

- Verification in the symbolic model (D. Baelde, 24h)
- Computational proofs (D. Pointcheval, 12h)
- Verification of protocol implementations (K. Bhargavan, 12h)

Conclusion

Outline

- Verification in the symbolic model (D. Baelde, 24h)
  - Formal definitions
  - The deduction problem
  - Symbolic model-checking
  - Indistinguishability
  - Proverif: the unbounded case, correspondences, diff-equiv.
  - Advanced topics: composition, typing, ...
- Computational proofs (D. Pointcheval, 12h)
- Verification of protocol implementations (K. Bhargavan, 12h)